Cover Sheet

Test Duration: 60 minutes.
Coverage: Chapters 1-5.
Open Book but Closed Notes.
Calculators NOT allowed.
This test contains two problems.
All work should be done in blue books.
You must show all work for each problem to receive full credit.
Do not return the exam itself; just your blue book.

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<th>Prob. No.</th>
<th>Topic(s)</th>
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<td>1.</td>
<td>LTI Systems: Properties, Transfer Functions, Frequency Response</td>
<td>55</td>
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<td>2.</td>
<td>DT Autocorrelation, Cross-Correlation in terms of Convolution</td>
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Problem 1. [55 points] Consider a DT LTI system with the transfer function below, with a single pole at \( z = a \), where \( a \) is real-valued and satisfies \( 0 < a < 1 \), and a single zero at \( z = 1/a \). Consider computing the frequency response of this system graphically.

\[
H(z) = \frac{(z - \frac{1}{a})}{(z - a)}
\]  

(a) To this end, first find an expression for the length \( N(\omega) \) of the vector that connects the zero at \( z = 1/a \) to the point \( z = e^{j\omega} \) on the unit circle, where \( 0 \leq \omega \leq \pi \).

(b) Next, find an expression for the length \( D(\omega) \) of the vector that connects the pole at \( z = a \) to the point \( z = e^{j\omega} \) on the unit circle, where \( 0 \leq \omega \leq \pi \).

(c) Show that the ratio of the answer to (a) to the answer to (b), \( N(\omega)/D(\omega) \), is a constant that does not depend on the frequency \( \omega \).

(d) Determine a closed-form expression for the impulse response \( h[n] \) of this system (in terms of \( a \)).

(e) Determine a closed-form expression for the autocorrelation sequence \( r_{hh}[\ell] \) for the impulse response \( h[n] \) of the system. Simplify as much as possible and show all work.

(f) Draw a block diagram for this system that uses only a single delay unit.

\[
y[n] = ay[n - 1] + x[n] - \frac{1}{a}x[n - 1]
\]  

(g) The signal below is input to the system above. (i) Plot the energy density spectrum for the output \( y[n] \) AND (ii) find the total energy in \( y[n] \) (in terms of \( a \)).

\[
x[n] = \left\{ \frac{\sin(\frac{\pi n}{\pi n})}{\pi n} \right\}
\]
Problem 2. [45 points]

(a) Let $x[n]$ and $y[n]$ be real-valued sequences both of which are even-symmetric: $x[n] = x[-n]$ and $y[n] = y[-n]$. Under these conditions, prove that $r_{xy}[\ell] = r_{yx}[\ell]$ for all $\ell$.

(b) Express the autocorrelation sequence $r_{zz}[\ell]$ for the complex-valued signal $z[n] = x[n] + jy[n]$ where $x[n]$ and $y[n]$ are real-valued sequences, in terms of $r_{xx}[\ell]$, $r_{xy}[\ell]$, $r_{yx}[\ell]$, and $r_{yy}[\ell]$.

(c) Determine a closed-form expression for the autocorrelation sequence $r_{xx}[\ell]$ for the signal $x[n]$ below.

\[
x[n] = \left\{ \sin\left(\frac{\pi n}{2}\right) \right\}
\]  

(d) Determine a closed-form expression for the autocorrelation sequence $r_{yy}[\ell]$ for the signal $y[n]$ below.

\[
y[n] = (-1)^n x[n] = (-1)^n \left\{ \sin\left(\frac{\pi n}{2}\right) \right\}
\]

(e) Determine a closed-form expression for the autocorrelation sequence $r_{zz}[\ell]$ for the complex-valued signal $z[n]$ formed with $x[n]$ and $y[n]$ defined above as the real and imaginary parts, respectively, as defined below. You must show all work and simplify as much as possible.

\[
z[n] = x[n] + jy[n]
\]

(f) Plot $r_{zz}[\ell]$. 