EE538 Digital Signal Processing I — Fall 2009
Exam 1 — Tuesday, Oct. 6, 2009

Cover Sheet

Test Duration: 60 minutes.
Coverage: Chapters 1-5.
Open Book but Closed Notes.
Calculators NOT allowed.
This test contains **two** problems.
All work should be done in blue books.
You must show all work for each problem to receive full credit.
Do **not** return the exam itself; just your blue book.

Prob. No.	**Topic(s)**	**Points**
1.	LTI Systems: Properties, Transfer Functions, Frequency Response	55
2.	DT Autocorrelation, Cross-Correlation Correlation in terms of Convolution	45

Problem 1. [55 points] Consider a DT LTI system with the transfer function below, with a single pole at $z = a$, where a is real-valued and satisfies $0 < a < 1$, and a single zero at $z = 1/a$. Consider computing the frequency response of this system graphically.

$$H(z) = \frac{(z - \frac{1}{a})}{(z - a)} \tag{1}$$

(a) To this end, first find an expression for the length $N(\omega)$ of the vector that connects the zero at $z = 1/a$ to the point $z = e^{j\omega}$ on the unit circle, where $0 \leq \omega \leq \pi$.

(b) Next, find an expression for the length $D(\omega)$ of the vector that connects the pole at $z = a$ to the point $z = e^{j\omega}$ on the unit circle, where $0 \leq \omega \leq \pi$.

(c) Show that the ratio of the answer to (a) to the answer to (b), $N(\omega)/D(\omega)$, is a constant that does not depend on the frequency ω.

(d) Determine a closed-form expression for the impulse response $h[n]$ of this system (in terms of a).

(e) Determine a closed-form expression for the autocorrelation sequence $r_{hh}[\ell]$ for the impulse response $h[n]$ of the system. Simplify as much as possible and show all work.

(f) Draw a block diagram for this system that uses only a single delay unit.

$$y[n] = ay[n-1] + x[n] - \frac{1}{a}x[n-1] \tag{2}$$

(g) The signal below is input to the system above. (i) Plot the energy density spectrum for the output $y[n]$ AND (ii) find the total energy in $y[n]$ (in terms of a).

$$x[n] = \left\{ \frac{\sin(\frac{3\pi}{4}n)}{\pi n} \right\} \tag{3}$$

Problem 2. [45 points]

(a) Let $x[n]$ and $y[n]$ be real-valued sequences both of which are even-symmetric: $x[n] = x[-n]$ and $y[n] = y[-n]$. Under these conditions, prove that $r_{xy}[\ell] = r_{yx}[\ell]$ for all ℓ.

(b) Express the autocorrelation sequence $r_{zz}[\ell]$ for the complex-valued signal $z[n] = x[n] + jy[n]$ where $x[n]$ and $y[n]$ are real-valued sequences, in terms of $r_{xx}[\ell]$, $r_{xy}[\ell]$, $r_{yx}[\ell]$, and $r_{yy}[\ell]$.

(c) Determine a closed-form expression for the autocorrelation sequence $r_{xx}[\ell]$ for the signal $x[n]$ below.

$$x[n] = \left\{ \frac{\sin(\frac{\pi}{2}n)}{\pi n} \right\} \tag{4}$$

(d) Determine a closed-form expression for the autocorrelation sequence $r_{yy}[\ell]$ for the signal $y[n]$ below.

$$y[n] = (-1)^n x[n] = (-1)^n \left\{ \frac{\sin(\frac{\pi}{2}n)}{\pi n} \right\} \tag{5}$$

(e) Determine a closed-form expression for the autocorrelation sequence $r_{zz}[\ell]$ for the complex-valued signal $z[n]$ formed with $x[n]$ and $y[n]$ defined above as the real and imaginary parts, respectively, as defined below. *You must show all work and simplify as much as possible.*

$$z[n] = x[n] + jy[n] \tag{6}$$

(f) Plot $r_{zz}[\ell]$.