EE 538 Exam 1 Fall 2008 Solutions

Prob. 1 This is a simple example of OVSF (Orthogonal Variable Spreading Factor) codes used in Third Generation Cell phones (3G)

User 1's first symbol & second symbol

$$\{6 \quad 0 \quad 4 \quad -2 \;\vdots\; -2 \quad 0 \quad -4 \quad -2\}$$

.* or $\odot$ (Matlab)

$$1 \quad -1 \quad 1 \quad -1 \;\vdots\; 1 \quad -1 \quad 1 \quad -1$$

$$6+0+4+2 = 12 \qquad -2+0-4+2 = -4$$

$$b_1[0] = \frac{12}{4} = 3 \qquad b_1[1] = \frac{-4}{4} = -1$$

$$\{1,1\} \qquad \{0,1\}$$

User 2:

$$\{6, 0, \vdots\, 4, -2 \,\vdots\, -2, 0, \vdots\, -4, -2\}$$

.* or $\odot$

$$1 \quad 1 \;\vdots\; 1 \quad 1 \;\vdots\; 1 \quad 1 \;\vdots\; 1 \quad 1$$

$$6 \qquad 2 \qquad -2 \qquad -6$$

$$b_2[0] = \frac{6}{2} = 3 \qquad b_2[1] = \frac{2}{2} = 1 \qquad b_2[2] = \frac{-2}{2} = -1 \Rightarrow \{0,1\}$$

$$b_2[3] = \frac{-6}{2} = -3$$

$$\{1,1\} \qquad \{1,0\} \qquad \{0,0\}$$

Problem 2

(a) $y[n] = x[n-1] = x[n] * \delta[n-1]$

Thus, we require:

$$h_1[n] * g_1[n] + h_2[n] * g_2[n] = \delta[n-1]$$

$$\{\underset{\uparrow}{\tfrac{1}{2}}, \tfrac{1}{2}\} * \{g_1[0], g_1[1]\} + \{\underset{\uparrow}{\tfrac{1}{2}}, -\tfrac{1}{2}\} * \{g_2[0], g_2[1]\}$$

$$= \{0, \underset{\uparrow}{1}, 0\}$$

$g_1[0] = \frac{1}{2}$

$\underline{n=0}$: $\quad \frac{1}{2}\left(\frac{1}{2}\right) + \frac{1}{2} g_2[0] = 0 \qquad g_2[0] = -\frac{1}{2}$

$\underline{n=1}$: $\quad \frac{1}{2}\left(\frac{1}{2}\right) + \frac{1}{2} g_1[1] \boxed{-} \frac{1}{2} \underbrace{g_2[0]}_{-\frac{1}{2}} \boxed{+} \frac{1}{2} g_2[1] = 1$

multiply by 2 on both sides:

$$\frac{1}{2} + g_1[1] + \frac{1}{2} + g_2[1] = 2$$

$$g_1[1] + g_2[1] = 1$$

$\underline{n=2}$: $\quad \frac{1}{2} g_1[1] - \frac{1}{2} g_2[1] = 0$

$$g_1[1] = g_2[1]$$

$g_1[1] = \frac{1}{2}$

$g_2[1] = \frac{1}{2}$

$g_1[n] = \left\{\frac{1}{2}, \frac{1}{2}\right\} \qquad g_2[n] = \left\{-\frac{1}{2}, \frac{1}{2}\right\}$

CHECK: $G_1(\omega) = \frac{1}{2} + \frac{1}{2} e^{-j\omega} \qquad G_2(\omega) = -\frac{1}{2} + \frac{1}{2} e^{-j\omega}$

$= e^{-j\frac{\omega}{2}} \cos\left(\frac{\omega}{2}\right) \qquad = -e^{-j\frac{\omega}{2}}\, j \sin\left(\frac{\omega}{2}\right)$

Since $g_1[n] = h_1[n]$ and $g_2[n] = -h_2[n]$

AND $$h[n] = h_1[n] * g_1[n] + h_2[n] * g_2[n]$$

$$H(\omega) = H_1^2(\omega) \quad H_2^2(\omega)$$

minus

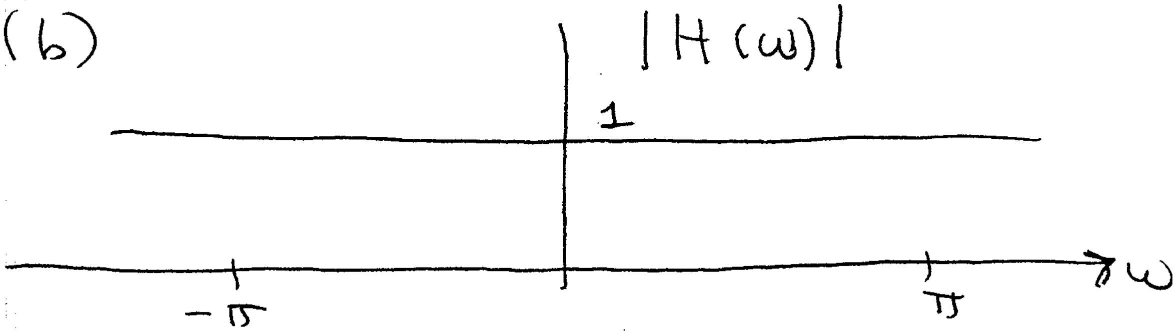
$$H(\omega) = e^{-j\omega} \cos^2\left(\frac{\omega}{2}\right) - (j)^2 e^{-j\omega} \sin^2\left(\frac{\omega}{2}\right)$$

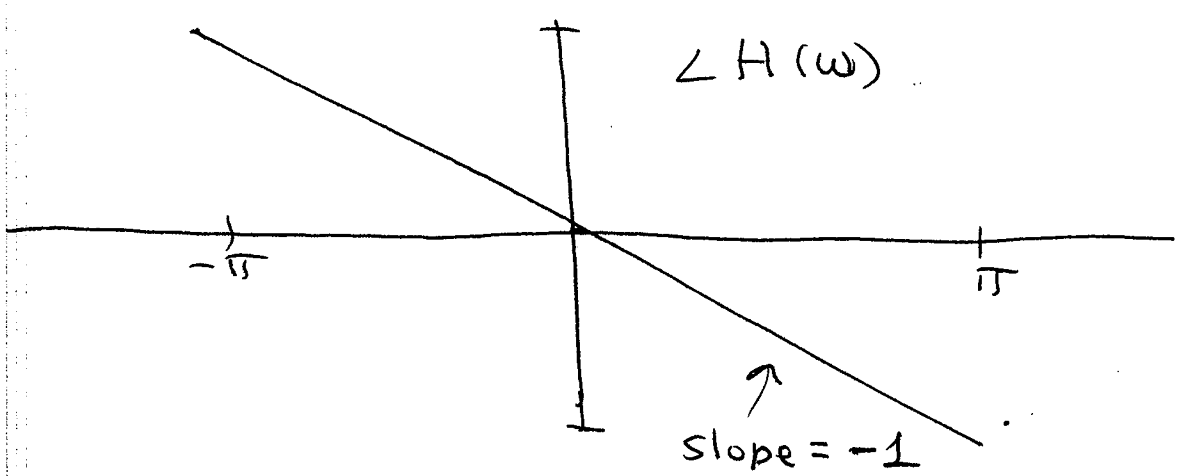
$$= e^{-j\omega} \left\{ \cos^2\left(\frac{\omega}{2}\right) + \sin^2\left(\frac{\omega}{2}\right) \right\}$$

END OF CHECK

$$= e^{-j\omega} \quad \Rightarrow \text{Thus: } h[n] = \delta[n-1]$$

(b)





3a

Alternatively, via Z-Transform

$$H_1(z)\,G_1(z) + H_2(z)\,G_2(z) = \mathcal{Z}\{\delta[n-1]\}$$

$$\left(\tfrac{1}{2}+\tfrac{1}{2}z^{-1}\right)\left(\tfrac{1}{2}+g_1[1]z^{-1}\right)+\left(\tfrac{1}{2}-\tfrac{1}{2}z^{-1}\right)\left(g_2[0]+g_2[1]z^{-1}\right) = z^{-1} + 0 + 0z^{-2}$$

$$\tfrac{1}{4} + \tfrac{1}{2}g_2[0] = 0 \qquad (z^{-0}\text{ eqn})$$

$$\tfrac{1}{4} + \tfrac{1}{2}g_1[1] + \tfrac{1}{2}g_2[1] - \tfrac{1}{2}g_2[0] = 1 \qquad (z^{-1}\text{ eqn})$$

$$\tfrac{1}{2}g_1[1] - \tfrac{1}{2}g_2[1] = 0 \qquad (z^{-2}\text{ eqn})$$

Three eqns in three unknowns obtained by equating like powers of z on both sides

Prob. 3 Sol'n.

$$y[n] = \sum_{k=-\infty}^{\infty} b[k]\, p(\underbrace{nT_0 - \tau - kT_0}_{(n-k)T_0 - \tau})$$

define: $p[n] = p(nT_0 - \tau)\Big|_{\tau = \frac{T_0}{2}} = h[n]$

$$h[n] = \frac{\sin\left(\pi \frac{(nT_0 - \frac{T_0}{2})}{T_0}\right)}{\pi \frac{(nT_0 - \frac{T_0}{2})}{T_0}} = \frac{\sin\left[\pi\left(n - \frac{1}{2}\right)\right]}{\pi\left(n - \frac{1}{2}\right)}$$

This $h[n]$ corresponds to a positive half-sample delay back in the analog time domain

If we apply a negative half-sample delay, then

$$f[n] = h[n] * g[n] = \delta[n]$$

so that $z[n] = b[n]$

$$\Rightarrow \quad g[n] = \frac{\sin\left[\pi\left(n + \frac{1}{2}\right)\right]}{\pi\left(n + \frac{1}{2}\right)} \quad \Big\} \text{ ANSWER}$$

There are several ways that we can show this.

Note: $h[n] = \dfrac{\sin\left[\frac{\pi}{2}(2n-1)\right]}{\frac{\pi}{2}(2n-1)}$

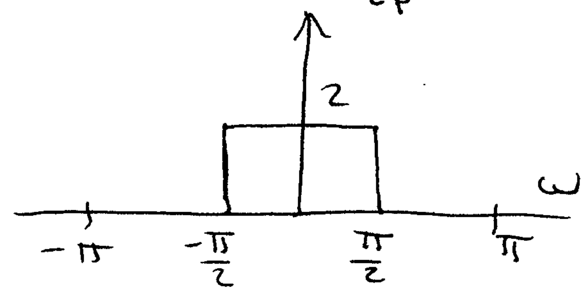
~~[illegible]~~ define: $h_{LP}[n] = \dfrac{\sin\left(\frac{\pi}{2}n\right)}{\frac{\pi}{2}n}$

Then $H(\omega) = \frac{1}{2} H_{LP}\left(\frac{\omega}{2}\right) e^{-j\frac{\omega}{2}}$

for $-\pi < \omega < \pi$

(from derivation done in class on morning of exam)

Thus: $H(\omega) = e^{-j\frac{\omega}{2}}$ since $H_{LP}(\omega)$



note: in class & old exams derived DTFT of $h_{LP}[2n+1] = f[n]$ when $h_{LP}[n] = \frac{\sin(\frac{\pi}{2}n)}{\frac{\pi}{2}n} \Rightarrow$ in this exam $h[n] = h_{LP}[2n-1] = f[n-1]$

Thus, we need $G(\omega) = e^{+j\frac{\omega}{2}}$ over $-\pi < \omega < \pi$

From class, it follows that

$$g[n] = \frac{\sin\left[\frac{\pi}{2}(2n+1)\right]}{\frac{\pi}{2}(2n+1)} = \frac{\sin\left[\pi\left(n+\frac{1}{2}\right)\right]}{\pi\left(n+\frac{1}{2}\right)}$$

ALTERNATIVELY: If τ had been zero, then $y[n] = b[n] * h[n]$

where $h[n] = p(nT_0) = \frac{\sin\left(\pi \frac{nT_0}{T_0}\right)}{\pi \frac{nT_0}{T_0}}$

$$= \frac{\sin(\pi n)}{\pi n} =$$

$$= \delta[n]$$

So, since there was a half-sample delay to the right, if we induce a half-sample delay to the left, there will be no timing offset and $y[n] = b[n]$