# EE538 Digital Signal Processing I

**Exam 1** **Sept. 21, 2007**

## Cover Sheet

Test Duration: 60 minutes.
Coverage: Chaps 1-5.
Open Book but Closed Notes.
Calculators allowed.
This test contains **three** problems.
All work should be done in the blue books provided.
You must show all work for each problem to receive full credit.
Do **not** return this test sheet, just return the blue books.

| **Prob. No.** | **Topic(s)** | **Points** |
|---|---|---|
| 1. | DT Autocorrelation, Cross-Correlation | 30 |
| 2. | Interconnection of LTI Systems<br>Frequency Response, DTFT | 35 |
| 3. | Sampling Theory, CTFT-DTFT Relationship,<br>DT model from a CT signal<br>DT Frequency Selective Filtering | 35 |

**Problem 1.** [30 points]

Consider a very simplistic CDMA system with only two users, with one user assigned a length 4 code and the other user assigned a length 2 code, as specified below. Note that user 2's code is orthogonal to user 1's code AND if you shift user 2's code over by 2 it is still orthogonal to user 1's code.

$$\text{User 1's code:} \quad c_1[n] = \{1, -1, 1, -1\}$$
$$\text{User 2's code:} \quad c_2[n] = \{1, 1\}$$

Consider that user 1 transmits a block of two PAM symbols, while user 2 simultaneously (synchronously) transmits a block of four PAM symbols

$$\text{User 1's two info. symbols:} \quad b_1[n] = \{b_1[0], b_1[1]\}$$
$$\text{User 2's four info. symbols:} \quad b_2[n] = \{b_2[0], b_2[1], b_2[2], b_2[3], \}$$

where $b_k[n]$ can take on one of four different real values (for each value of $k$ and $n$):

1. 3 representing the bit pair {1,1} ;
2. 1 representing the bit pair {1,0} ;
3. -1 representing the bit pair {0,1} ;
4. -3 representing the bit pair {0,0} ;

The transmitted code-division multiplexed block may be mathematically expressed as

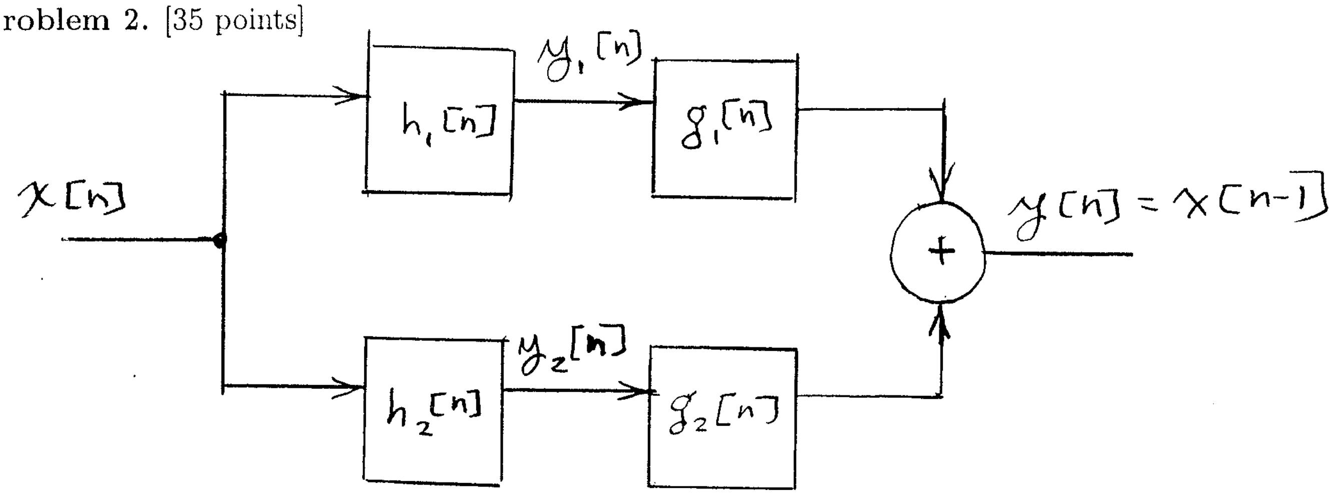
$$x[n] = \sum_{m=0}^{1} b_1[m]c_1[n-4m] + \sum_{m=0}^{3} b_2[m]c_2[n-2m], \qquad n = 0, 1, ..., 7$$

Given that the received block has the following numerical values

$$x[n] = \{\underbrace{6}_{\uparrow}, 0, 4, -2, -2, 0, -4, -2\}$$

where the first entry above is the value of $x[0]$, determine the numerical values of $b_1[n], n = 0, 1$ and $b_2[n], n = 0, 1, 2, 3$. Your answer should consist of 6 numerical values all together. Show all work in arriving at your answer.

**Problem 2.** [35 points]




The respective difference equation for each of the two systems in parallel above may be expressed as

$$y_1[n] = \frac{1}{2}x[n] + \frac{1}{2}x[n-1]$$

$$y_2[n] = \frac{1}{2}x[n] - \frac{1}{2}x[n-1]$$

Consider the respective outputs of these two systems as the inputs to a pair of length two FIR filters with impulse responses, $g_1[n]$ and $g_2[n]$, respectively, as shown in the diagram.

(a) Determine the values of $g_1[n]$, $n = 0, 1$ and $g_2[n]$, $n = 0, 1$ such that the difference equation for the overall system is simply

$$y[n] = x[n-1]$$

That is, determine length two FIR filters $g_1[n]$ and $g_2[n]$ so that the output is simply the input delayed by one (for any input.) **You** need to solve this by setting $g_1[0] = \frac{1}{2}$ and then setting up a linear system of three equations in three unknowns to solve for $g_1[1]$, $g_2[0]$, and $g_2[1]$.

(b) Let $H(\omega)$ denote the frequency response of the overall system equal to the DTFT of $h[n]$ below:

$$h[n] = h_1[n] * g_1[n] + h_2[n] * g_2[n]$$

Plot both the magnitude $H(\omega)$ and the phase $\angle H(\omega)$ (two separate plots) over $-\pi < \omega < \pi$.

**Problem 3.** [35 points] Consider the transmission of a pulse amplitude-modulated signal

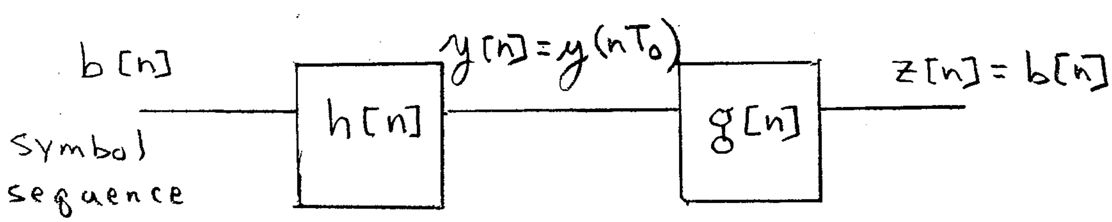
$$y(t) = \sum_{k=-\infty}^{\infty} b[k]p(t - \tau - kT_o)$$

where $b[n]$ are the information-bearing symbols being transmitted which may be viewed as a discrete-time sequence. In binary phase-shift keying, $b[n]$ is either "+1" or "-1" for all $n$. $1/T_o$ is the bit rate and $p(t)$ is the pulse symbol waveform below

$$p(t) = \frac{\sin\left(\pi \frac{t}{T_o}\right)}{\pi \frac{t}{T_o}}$$

$\tau$ represents a timing offset. For this problem, $\tau = \frac{T_o}{2}$.

Sampling $y(t)$ at the bit rate, $F_s = \frac{1}{T_o}$, the resulting sequence $y[n] = y(nT_o)$ may be modeled as having been generated by the following discrete-time system.




Determine the impulse response $h[n]$. (The answer is an analytical expression.) In order to compensate for the timing offset $\tau = \frac{T_o}{2}$, the DT signal $y[n]$ is input to a filter with impulse response $g[n]$. Determine the impulse response $g[n]$ such that the final output is the bit sequence $b[n]$. (The answer is an analytical expression.)