EE538 Digital Signal Processing I
Exam 1 Session 12. Live: Thurs, Sept. 28, 2006

Cover Sheet

Test Duration: 75 minutes.
Coverage: Chaps 1-5, 10.2,10.3
Open Book but Closed Notes.
Calculators NOT allowed.
This test contains **TWO** problems.
All work should be done in the blue books provided.
You must show all work for each problem to receive full credit.
Do **not** return this test sheet, just return the blue books.

Problem 1. [50 points]

(a) Consider the symmetric sequence below which is one for $-2 \leq m \leq 2$ and zero for $|m| > 2$. Is this is a valid autocorrelation sequence? Justify your answer.

$$r_{xx}[m] = u[m+2] - u[m-3]$$

(b) Consider the symmetric sequence below. Is this is a valid autocorrelation sequence? Justify your answer.

$$r_{xx}[m] = (3 - |m|)(u[m+2] - u[m-3])$$

(c) Let $r_{xx}[m]$ denote the autocorrelation sequence for the DT signal $x[n]$. Let $y[n] = x[n - n_o]$, where n_o is an integer. Let $r_{yy}[m]$ denote the autocorrelation sequence for the DT signal $y[n]$. Derive an expression relating $r_{yy}[m]$ and $r_{xx}[m]$. That is, how is $r_{yy}[m]$ related to $r_{xx}[m]$?

(d) Let $r_{xx}[m]$ denote the autocorrelation sequence for the DT signal $x[n]$. Let $y[n] = e^{j(\omega_0 n + \theta)}x[n]$, where ω_o is some frequency and θ is some phase value. Let $r_{yy}[m]$ denote the autocorrelation sequence for the DT signal $y[n]$. Derive an expression relating $r_{yy}[m]$ and $r_{xx}[m]$. That is, how is $r_{yy}[m]$ related to $r_{xx}[m]$?

(e) Consider that the signal $x[n]$ below, where $a = \frac{1}{2}$, is the input to the LTI system described by the difference equation in equation (2) below.

$$x[n] = a^n u[n] - \frac{1}{a} a^{n-1} u[n-1] \quad (1)$$

$$y[n] = \frac{1}{4} y[n-1] + x[n] \quad (2)$$

(i) Determine a closed-form analytical expression for the auto-correlation $r_{xx}[m]$ for $x[n]$, when $a = \frac{1}{2}$. (Hint: examine the Z-Transform of $x[n]$.)

(ii) Determine a closed-form analytical expression for the cross-correlation $r_{yx}[m]$ between the input and output.

(iii) Determine a closed-form analytical expression for the auto-correlation $r_{yy}[m]$ for the output $y[n]$.

Problem 2. [50 points]

Consider the transmission of a pulse amplitude-modulated signal

$$s(t) = \sum_{k=-\infty}^{\infty} x[k]p(t - kT_s)$$

where $x[n]$ are the information-bearing symbols being transmitted which may be viewed as a discrete-time sequence. In binary phase-shift keying, $x[n]$ is either "+1" or "-1" for all n. $1/T_s$ is the symbol rate and $p(t)$ is the pulse symbol waveform below

$$p(t) = \cos\left(\frac{\pi t}{2T_s}\right)\ (u(t + T_s) - u(t - T_s))$$

At the receiver, $s(t)$ arrives by both a direct path and a multipath reflection having the same strength and phase as the direct path but at a delay of T_s. Denoting continuous time convolution as $*$, the received signal, $x_r(t)$, may be modeled as:

$$x_r(t) = s(t) * g(t)$$

where $g(t)$ is described below using $\delta(t)$ to denote the Dirac Delta function.

$$g(t) = \delta(t) + \delta(t - T_s) \tag{3}$$

Samples of the received signal at L times the symbol rate rate may be obtained via the following discrete-time system.

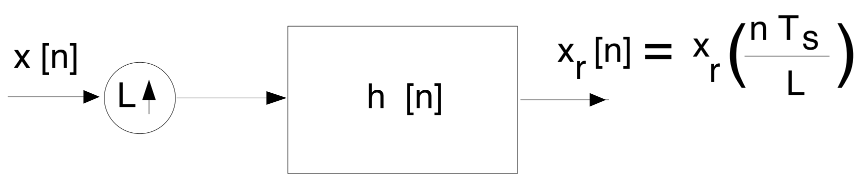

Figure 1.

Your primary task in this problem is to determine the appropriate filter impulse response $h[n]$ for different values of L so that the output of the system above is what you would have obtained if you had sampled the received signal at L times the symbol rate as specified. **NOTE:** Correct answer for $h[n]$ is different for each part (for each value of L.)

(a) For the case of $L = 1$, write a closed-form expression for the filter $h[n]$ (the nonzero values). Plot the magnitude of the DTFT of $h[n]$, $H(\omega)$, over $-\pi < \omega < \pi$.

(b) For the case of $L = 2$, write a closed-form expression for the filter $h[n]$ (the nonzero values). Plot the magnitude of the DTFT of $h[n]$, $H(\omega)$, over $-\pi < \omega < \pi$. The case of $L = 2$ in Figure 1 may be alternatively implemented as in the block diagram in Figure 2 on the top of the next page.

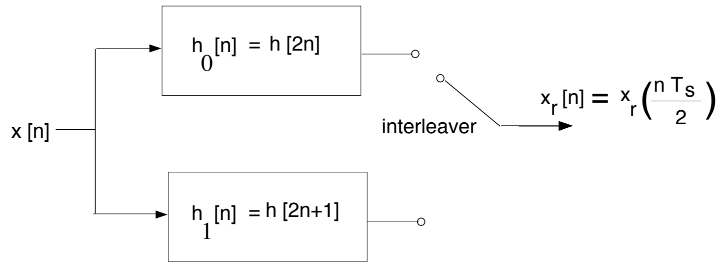

Figure 2.

Using your answer for $h[n]$ for this part (b), do the following:

(i) Write an expression for $h_0[n] = h[2n]$. Plot the magnitude of the DTFT of $h_0[n]$, $|H_0(\omega)|$, over $-\pi < \omega < \pi$.

(ii) Write an analytical expression for $h_1[n] = h[2n+1]$. Plot the magnitude of the DTFT of $h_1[n]$, $|H_1(\omega)|$, over $-\pi < \omega < \pi$.

(c) For the case of $L = 3$, write a closed-form expression for the filter $h[n]$.