Cover Sheet

Test Duration: 60 minutes.
Coverage: Chaps 1-5.
Open Book but Closed Notes.
Calculators allowed.
This test contains three problems.
All work should be done in the blue books provided.
You must show all work for each problem to receive full credit.
Do not return this test sheet, just return the blue books.

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Problem 1. [30 points]
Consider a very simplistic CDMA system with only two users assigned the following length 4 orthogonal codes, respectively, which are two rows of a 4x4 Walsh-Hadamard matrix:

User 1’s code: \( c_1[n] = \{1, -1, 1, -1\} \)

User 2’s code: \( c_2[n] = \{1, 1, -1, -1\} \)

Consider transmitting a block of two PAM symbols for each of the two users,

User 1’s two info. symbols: \( b_1[n] = \{b_1[0], b_1[1]\} \)

User 2’s two info. symbols: \( b_2[n] = \{b_2[0], b_2[1], \} \)

where \( b_k[n] \) can take on one of four different real values (for each value of \( k \) and \( n \):

1. 3 representing the bit pair \( \{1,1\} \);
2. 1 representing the bit pair \( \{1,0\} \);
3. -1 representing the bit pair \( \{0,1\} \);
4. -3 representing the bit pair \( \{0,0\} \);

The transmitted code-division multiplexed block may be mathematically expressed as

\[
x[n] = \sum_{k=1}^{2} \sum_{m=0}^{1} b_k[m]c_k[n - 4m], \quad n = 0, 1, \ldots, 7
\]

Given that the received block has the following numerical values

\[
x[n] = \{-4, -2, 2, 4, 4, -2, 2, -4\}
\]

where the first entry above is the value of \( x[0] \), determine the numerical values of \( b_1[n], n = 0, 1 \) and \( b_2[n], n = 0, 1 \). Your answer should consist of 4 numerical values all together. Show all work in arriving at your answer. Note that code 1 is orthogonal to code 2 and that the two users are synchronized.
Problem 2. [35 points]

The analog signal $x_a(t)$ is reconstructed from its samples $x[n] = x_a(nT_s)$ according to the following equation

$$x_r(t) = \sum_{k=-\infty}^{\infty} x[k] g(t - kT_s) \quad \text{where: } g(t) = \cos \left( \frac{\pi t}{2T_s} \right) \left( u(t + T_s) - u(t - T_s) \right)$$

Samples of the reconstructed signal at $L$ times the original sampling rate may be obtained via the following discrete-time system.

![Block diagram](image)

Figure 1.

Your primary task in this problem is to determine the appropriate filter impulse response $h[n]$ for different values of $L$ so that the output of the system above is what you would have obtained if you had sampled the reconstructed signal at $L$ times the original sampling rate as specified. **NOTE:** Correct answer for $h[n]$ is different for each part (for each value of $L$.)

(a) For the case of $L = 1$, write a closed-form expression for the filter $h[n]$. Plot the magnitude of the DTFT of $h[n]$, $H(\omega)$, over $-\pi < \omega < \pi$.

(b) For the case of $L = 2$, write a closed-form expression for the filter $h[n]$. Plot the magnitude of the DTFT of $h[n]$, $H(\omega)$, over $-\pi < \omega < \pi$. The case of $L = 2$ in Figure 1 may be efficiently implemented as in the block diagram in Figure 2 below.

![Block diagram](image)

Figure 2.

Using your answer for $h[n]$ for this part (b), do the following:

(i) Write an expression for $h_0[n] = h[2n]$. Plot the magnitude of the DTFT of $h_0[n]$, $|H_0(\omega)|$, over $-\pi < \omega < \pi$.

(ii) Write an analytical expression for $h_1[n] = h[2n + 1]$. Plot the magnitude of the DTFT of $h_1[n]$, $|H_1(\omega)|$, over $-\pi < \omega < \pi$.

(c) For the case of $L = 4$, write a closed-form expression for the filter $h[n]$. Plot the magnitude of the DTFT of $h[n]$, $H(\omega)$, over $-\pi < \omega < \pi$. 
Problem 3. [35 points] Consider the transmission of a pulse amplitude-modulated signal

\[ x(t) = \sum_{k=-\infty}^{\infty} b[k]p(t - kT_o) \]

where \( b[n] \) are the information-bearing symbols being transmitted which may be viewed as a discrete-time sequence. In binary phase-shift keying, \( b[n] \) is either "+1" or "-1" for all \( n \). \( 1/T_o \) is the bit rate and \( p(t) \) is the pulse symbol waveform below

\[ p(t) = u(t + \frac{T_o}{4}) - u(t - \frac{T_o}{4}) \]

which is a rectangular pulse that is "turned on" for half of the bit interval.

At the receiver, \( x(t) \) arrives by both a direct path and a multipath reflection having the same strength as the direct path but at a delay of \( T_o \) and phase-shifted by \( \theta \). Denoting continuous time convolution as \( * \), the received signal, \( y(t) \), may be modeled as:

\[ y(t) = x(t) * g(t) \]

where \( g(t) \) is described below using \( \delta(t) \) to denote the Dirac Delta function.

\[ g(t) = \delta(t) + e^{j\theta} \delta(t - T_o) \]  \hspace{1cm} (1)

Sampling \( y(t) \) at the bit rate, \( F_s = \frac{1}{T_o} \), it is easily shown that the resulting sequence \( y[n] = y(nT_o) \) may be modeled as having been generated by the following discrete-time system.

```
symbol sequence \hspace{1cm} b[n] \hspace{1cm} [h[n]] \hspace{1cm} y[n] = y(nT_o)
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Determine \( h[n] \) and plot the magnitude of its DTFT \( H(\omega) = \sum_{n=-\infty}^{\infty} h[n]e^{-j\omega n} \) over \(-\pi \leq \omega \leq \pi\) for each of the following three values of \( \theta \):

(i) \( \theta = 0 \)
(ii) \( \theta = \frac{\pi}{4} \)
(iii) \( \theta = \frac{3\pi}{4} \)

In each case, \( H(\omega) \) will be exactly equal to zero for one specific value of \( \omega \). Determine this frequency in each case.