EE538 Digital Signal Processing I

Exam 1 **Live: Wed., Sept. 28, 2005**

Cover Sheet

Test Duration: 60 minutes.
Coverage: Chaps 1-5.
Open Book but Closed Notes.
Calculators allowed.
This test contains **three** problems.
All work should be done in the blue books provided.
You must show all work for each problem to receive full credit.
Do **not** return this test sheet, just return the blue books.

Prob. No.	Topic(s)	Points
1.	DT Autocorrelation, Cross-Correlation	30
2.	Digital Upsampling Frequency Response, Efficient implementation	35
3.	Sampling Theory, CTFT-DTFT Relationship, DT model from a CT signal DT Frequency Selective Filtering	35

Problem 1. [30 points]

Consider a very simplistic CDMA system with only two users assigned the following length 4 *orthogonal* codes, respectively, which are two rows of a 4x4 Walsh-Hadamard matrix:

$$\text{User 1's code:} \quad c_1[n] = \{1, -1, 1, -1\}$$

$$\text{User 2's code:} \quad c_2[n] = \{1, 1, -1, -1\}$$

Consider transmitting a block of two PAM symbols for each of the two users,

$$\text{User 1's two info. symbols:} \quad b_1[n] = \{b_1[0], b_1[1]\}$$

$$\text{User 2's two info. symbols:} \quad b_2[n] = \{b_2[0], b_2[1], \}$$

where $b_k[n]$ can take on one of four different real values (for each value of k and n):

1. 3 representing the bit pair {1,1} ;
2. 1 representing the bit pair {1,0} ;
3. -1 representing the bit pair {0,1} ;
4. -3 representing the bit pair {0,0} ;

The transmitted code-division multiplexed block may be mathematically expressed as

$$x[n] = \sum_{k=1}^{2} \sum_{m=0}^{1} b_k[m] c_k[n - 4m], \qquad n = 0, 1, ..., 7$$

Given that the received block has the following numerical values

$$x[n] = \{\underset{\uparrow}{\underbrace{-4}}, -2, 2, 4, 4, -2, 2, -4\}$$

where the first entry above is the value of $x[0]$, determine the numerical values of $b_1[n], n = 0, 1$ and $b_2[n], n = 0, 1$. Your answer should consist of 4 numerical values all together. Show all work in arriving at your answer. Note that code 1 is orthogonal to code 2 and that the two users are synchronized.

Problem 2. [35 points]

The analog signal $x_a(t)$ is reconstructed from its samples $x[n] = x_a(nT_s)$ according to the following equation

$$x_r(t) = \sum_{k=-\infty}^{\infty} x[k]\; g(t - kT_s) \qquad \text{where: } g(t) = \cos\left(\frac{\pi t}{2T_s}\right)\;(u(t+T_s) - u(t-T_s))$$

Samples of the reconstructed signal at L times the original sampling rate may be obtained via the following discrete-time system.

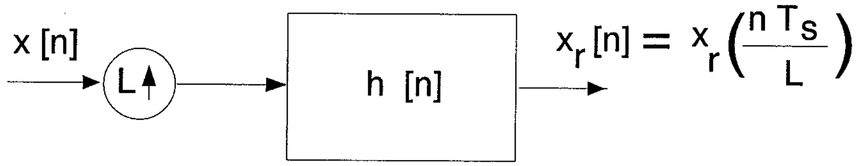


Figure 1.

Your primary task in this problem is to determine the appropriate filter impulse response $h[n]$ for different values of L so that the output of the system above is what you would have obtained if you had sampled the reconstructed signal at L times the original sampling rate as specified. **NOTE:** Correct answer for $h[n]$ is different for each part (for each value of L.)

(a) For the case of $L = 1$, write a closed-form expression for the filter $h[n]$. Plot the magnitude of the DTFT of $h[n]$, $H(\omega)$, over $-\pi < \omega < \pi$.

(b) For the case of $L = 2$, write a closed-form expression for the filter $h[n]$. Plot the magnitude of the DTFT of $h[n]$, $H(\omega)$, over $-\pi < \omega < \pi$. The case of $L = 2$ in Figure 1 may be efficient implementated as in the block diagram in Figure 2 below.

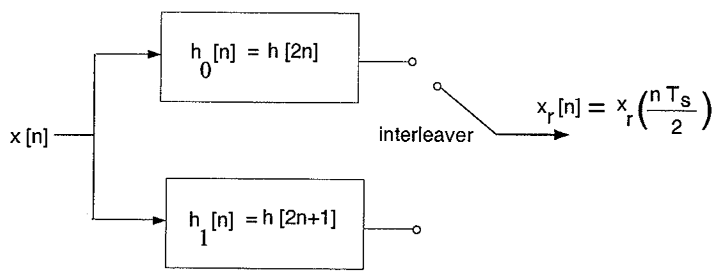


Figure 2.

Using your answer for $h[n]$ for this part (b), do the following:

(i) Write an expression for $h_0[n] = h[2n]$. Plot the magnitude of the DTFT of $h_0[n]$, $|H_0(\omega)|$, over $-\pi < \omega < \pi$.

(ii) Write an analytical expression for $h_1[n] = h[2n+1]$. Plot the magnitude of the DTFT of $h_1[n]$, $|H_1(\omega)|$, over $-\pi < \omega < \pi$.

(c) For the case of $L = 4$, write a closed-form expression for the filter $h[n]$. Plot the magnitude of the DTFT of $h[n]$, $H(\omega)$, over $-\pi < \omega < \pi$.

Problem 3. [35 points] Consider the transmission of a pulse amplitude-modulated signal

$$x(t) = \sum_{k=-\infty}^{\infty} b[k]p(t - kT_o)$$

where $b[n]$ are the information-bearing symbols being transmitted which may be viewed as a discrete-time sequence. In binary phase-shift keying, $b[n]$ is either "+1" or "-1" for all n. $1/T_o$ is the bit rate and $p(t)$ is the pulse symbol waveform below

$$p(t) = u(t + \frac{T_o}{4}) - u(t - \frac{T_o}{4})$$

which is a rectangular pulse that is "turned on" for half of the bit interval.

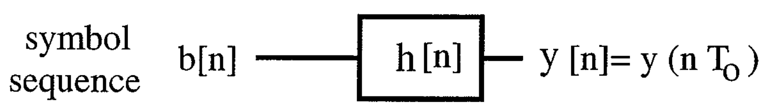
At the receiver, $x(t)$ arrives by both a direct path and a multipath reflection having the same strength as the direct path but at a delay of T_o and phase-shifted by θ. Denoting continuous time convolution as $*$, the received signal, $y(t)$, may be modeled as:

$$y(t) = x(t) * g(t)$$

where $g(t)$ is described below using $\delta(t)$ to denote the Dirac Delta function.

$$g(t) = \delta(t) + e^{j\theta}\delta(t - T_o) \tag{1}$$

Sampling $y(t)$ at the bit rate, $F_s = \frac{1}{T_o}$, it is easily shown that the resulting sequence $y[n] = y(nT_o)$ may be modeled as having been generated by the following discrete-time system.



Determine $h[n]$ and plot the magnitude of its DTFT $H(\omega) = \sum_{n=-\infty}^{\infty} h[n]e^{-j\omega n}$ over $-\pi \leq \omega \leq \pi$ for **each** of the following three values of θ:

(i) $\theta = 0$

(ii) $\theta = \frac{\pi}{4}$

(iii) $\theta = \frac{3\pi}{4}$

In each case, $H(\omega)$ will be exactly equal to zero for one specific value of ω. Determine this frequency in each case.