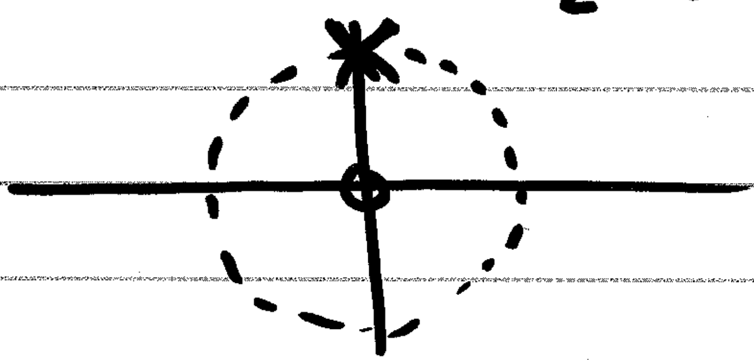
Sol'n to Prob. 1 Exam 1

(a) $h[n] = (j)^n u[n] = e^{j\frac{\pi}{2}n} u[n]$

BIBO Stability? $|a^n| = |a|^n$

$$\sum_n |h[n]| = \sum_{n=0}^{\infty} 1 = \infty$$ NOT Stable!

(c) $H(z) = \left.\frac{z}{z-a}\right|_{a=j} = \frac{z}{z-j}$



(b) $x[n] = e^{j\frac{\pi}{2}n} u[n]$

or $= e^{j\frac{\pi}{2}n} \quad \forall n$

(b) (cont.)

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k]$$

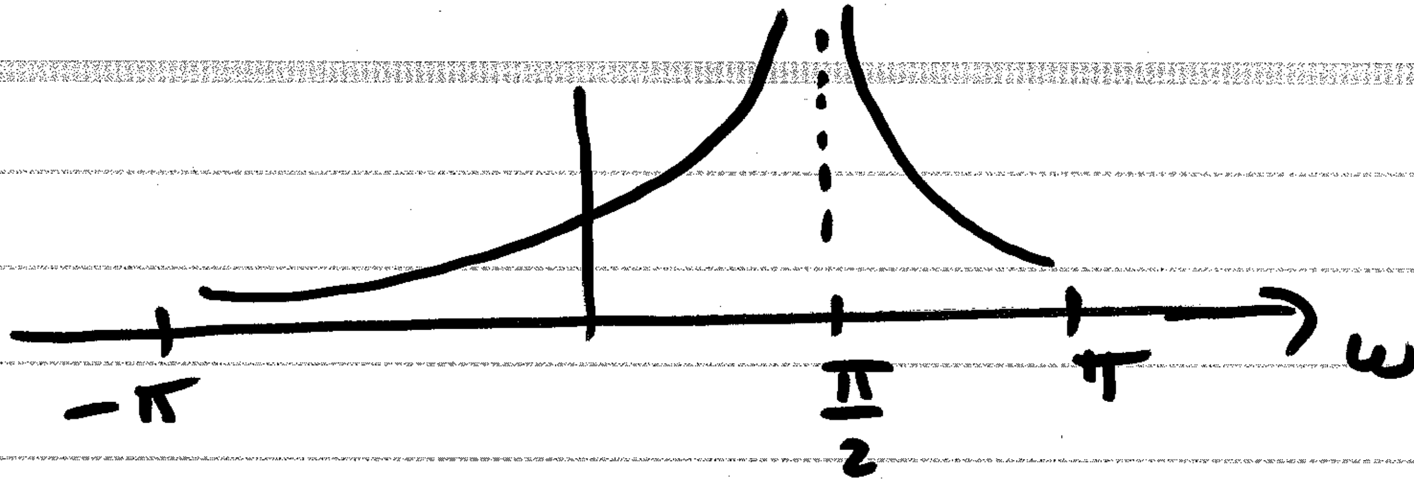
$$= \sum_{k=0}^{n} e^{j\frac{\pi}{2}k}\, e^{j\frac{\pi}{2}(n-k)}$$

$$= \left\{\sum_{k=0}^{n} 1\right\} e^{j\frac{\pi}{2}n} = (n+1)\, e^{j\frac{\pi}{2}n}$$

(d) $H(z) = \dfrac{1}{1-jz^{-1}} = \dfrac{Y(z)}{X(z)}$

$$y[n] = j\, y[n-1] + x[n]$$

(e)



(f) $x[n] = 1 + (-j)^n + (-1)^n$

$= e^{j0n} + e^{-j\frac{\pi}{2}n} + e^{j\pi n}$

$y[n] = H(\omega)\Big|_{\omega=0}(1) + H(\omega)\Big|_{\omega=-\frac{\pi}{2}} e^{-j\frac{\pi}{2}n}$

$+ H(\omega)\Big|_{\omega=\pi} e^{j\pi n}$

$$H(\omega) = H(z)\Big|_{z=e^{j\omega}}$$

$$= \frac{1}{1-j\,e^{-j\omega}}$$

$$y[n] = \frac{1}{1-j} + \frac{1}{2}\, e^{-j\frac{\pi}{2}n}$$

$$+ \frac{1}{1+j}\, e^{j\pi n}$$

(g) $x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n]$

$$r_{yx}[\ell] = r_{xx}[\ell] * h[\ell]$$

(5)

$$x[n] = \left(\frac{1}{2}\right)^n u[n]$$

$$r_{xx}[\ell] = \frac{1}{1-\left(\frac{1}{2}\right)^2} \left(\frac{1}{2}\right)^{|\ell|}$$

$$r_{yx}[\ell] = \frac{4}{3}\left(\frac{1}{2}\right)^{|\ell|} * j^{\ell} u[\ell]$$

$$= \frac{4}{3}\left(\frac{1}{2}\right)^{\ell} u[\ell] * (j)^{\ell} u[\ell]$$

$$+ \frac{4}{3}\left(\frac{1}{2}\right)^{-\ell} u[-\ell-1] * (j)^{\ell} u[\ell]$$

⑥

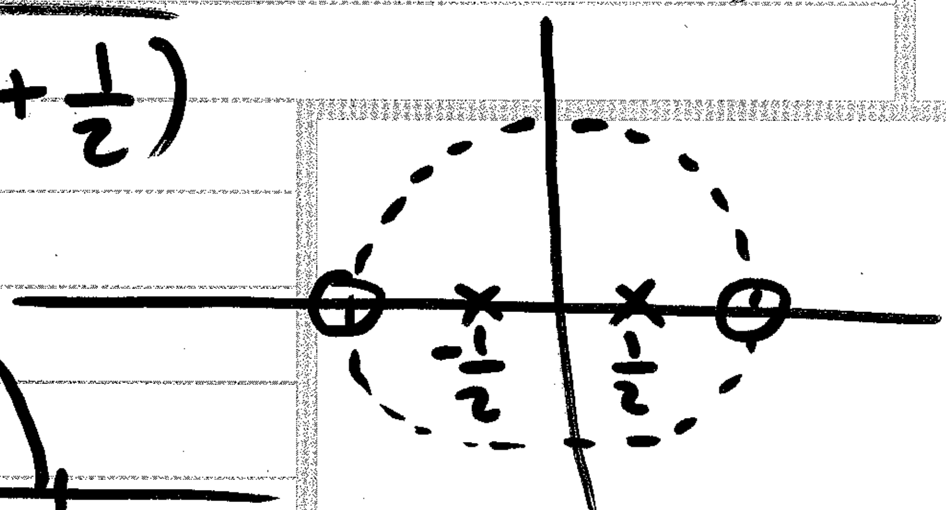
Prob. 2 sol'n:

(a) $y[n] = .25\,y[n-2] + x[n] - x[n-2]$

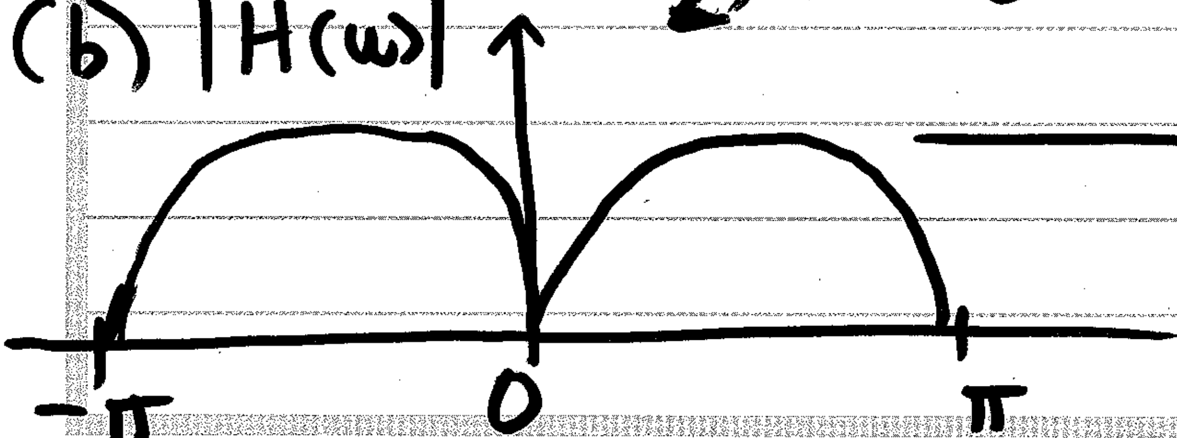
$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 - z^{-2}}{1 - .25\,z^{-2}} = \frac{z^2 - 1}{z^2 - \frac{1}{4}}$$

$$= \frac{(z-1)(z+1)}{(z-\frac{1}{2})(z+\frac{1}{2})}$$

(a)



(b) $|H(\omega)|$



$$\left(\frac{1}{2}z + b_1^{(1)}\right)\left(z - \frac{1}{2}\right) + \left(b_0^{(2)} z + b_1^{(2)}\right) \cdot \left(z + \frac{1}{2}\right)$$

$$= \left(\frac{1}{2} + b_0^{(2)}\right) z^2$$

$$+ \left[\left(b_1^{(1)} - \frac{1}{4}\right) + \left(\frac{1}{2} b_0^{(2)} + b_1^{(2)}\right)\right] z$$

$$+ -\frac{1}{2} b_1^{(1)} + \frac{1}{2} b_1^{(2)}$$

$$= z^2 + 0z - 1$$

$$H(z) = \frac{z^2 - 1}{(z - \frac{1}{2})(z + \frac{1}{2})} = H_1(z) + H_2(z)$$

$$= \frac{b_0^{(1)} + b_1^{(1)} z^{-1}}{1 - a_1^{(1)} z^{-1}} + \frac{b_0^{(2)} + b_1^{(2)} z^{-1}}{1 - a_1^{(2)} z^{-1}}$$

$$= \frac{b_0^{(1)} z + b_1^{(1)}}{z + \underbrace{\tfrac{1}{2}}_{-a_1^{(1)}}} + \frac{b_0^{(2)} z + b_1^{(2)}}{z - \underbrace{\tfrac{1}{2}}_{-a_1^{(2)}}}$$

- eqn coeffs on both sides:

$$b_0^{(2)} = \frac{1}{2} \qquad \Big| \; z^2$$

$$b_1^{(1)} - \frac{1}{4} + \frac{1}{2}\left(\frac{1}{2}\right) + b_1^{(2)} = 0 \qquad \Big| \; z^1$$

$$-\frac{1}{2} b_1^{(1)} + \frac{1}{2} b_1^{(2)} = -1 \qquad \Big| \; z^0$$

$$b_1^{(1)} + b_1^{(2)} = 0$$

$$-b_1^{(1)} + b_1^{(2)} = -2$$

$$b_1^{(2)} = -1$$

$$b_1^{(1)} = 1$$

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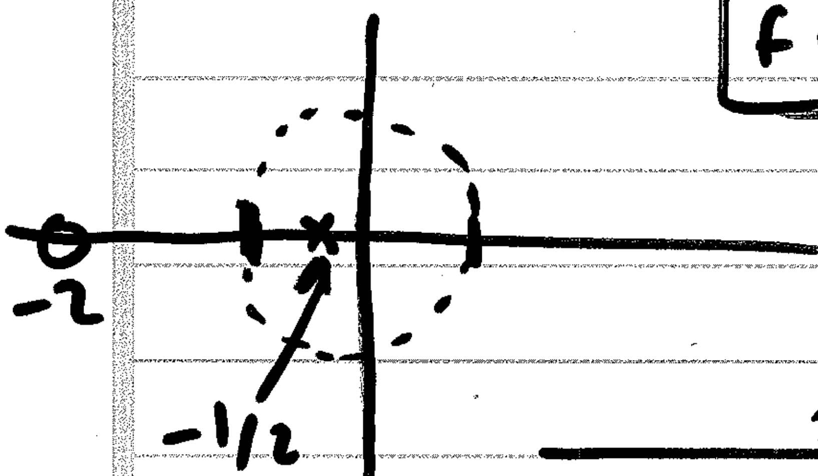
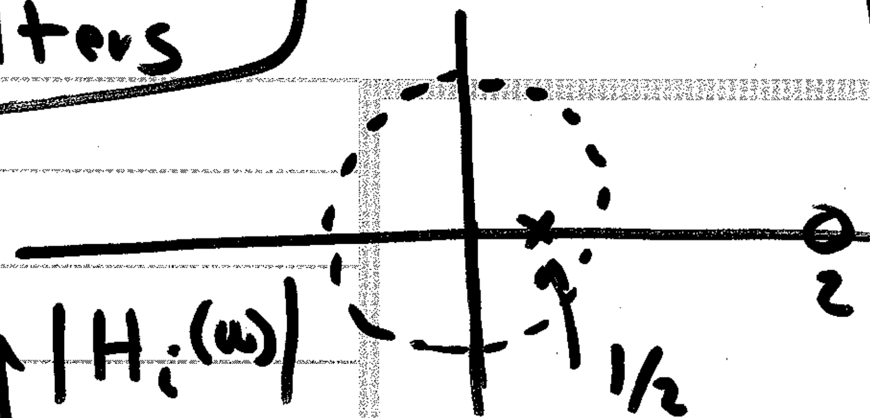
$$H_1(z) = \frac{\frac{1}{2}z+1}{z+\frac{1}{2}} = \frac{1}{2}\,\frac{(z+2)}{z+\frac{1}{2}}$$

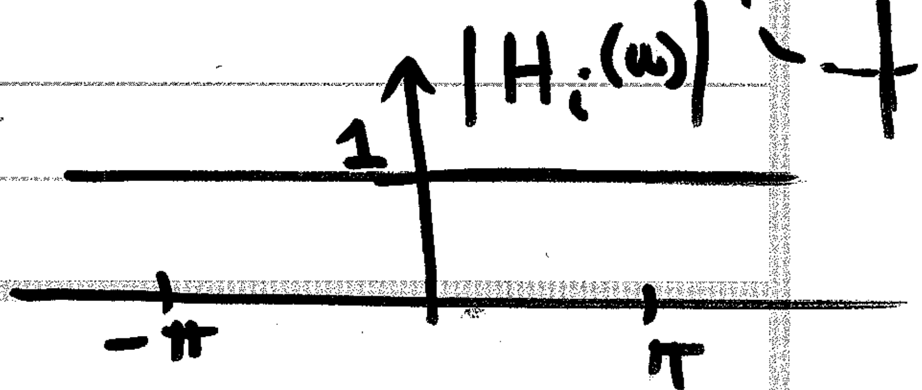
$$H_2(z) = \frac{\frac{1}{2}z-1}{z-\frac{1}{2}} = \frac{1}{2}\,\frac{(z-2)}{z-\frac{1}{2}}$$

$\boxed{\text{all-pass filters}}$

ROC: $|z| > \frac{1}{2}$







$i = 1, 2$

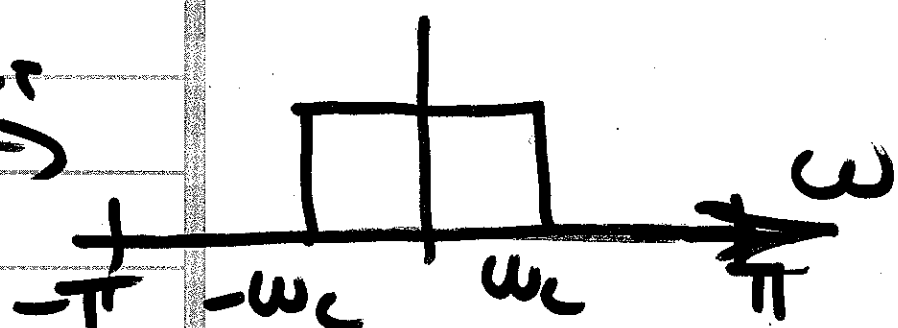
Prob 3.

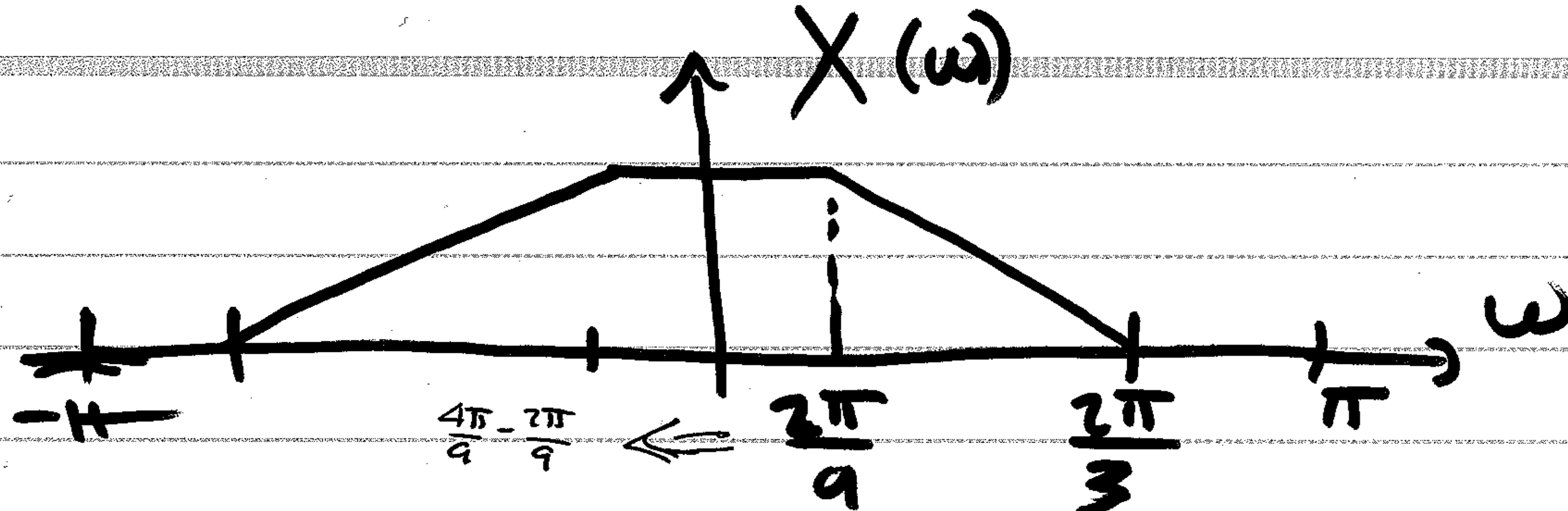
$$x[n] = x_a(nT_s) = x_a\left(n\frac{2\pi}{36}\right)$$

$$= \frac{\sin\left[4n\frac{2\pi}{36}\right]}{\pi n \frac{2\pi}{36}} \cdot \frac{\sin\left[8n\frac{2\pi}{36}\right]}{\pi n \frac{2\pi}{36}}$$

$$= (18)^2 \, \frac{\sin\left[\frac{2\pi}{9}n\right]}{\pi n} \; \frac{\sin\left[\frac{4\pi}{9}n\right]}{\pi n}$$

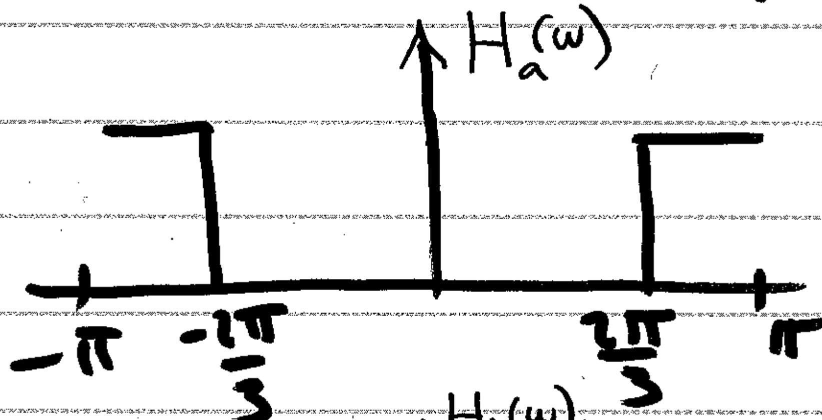
$$\frac{\sin(\omega_c n)}{\pi n} \overset{\text{DTFT}}{\longleftrightarrow}$$





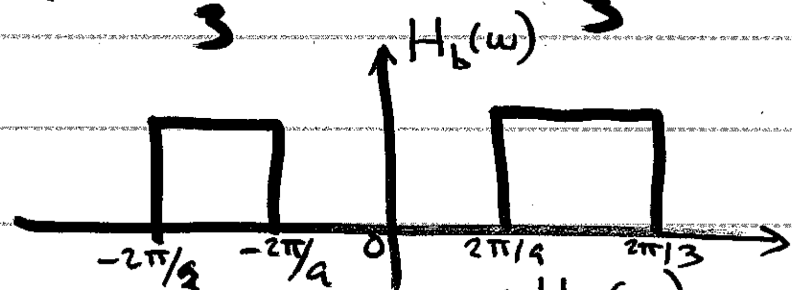
$\Rightarrow \frac{2\pi}{9} + \frac{4\pi}{9}$

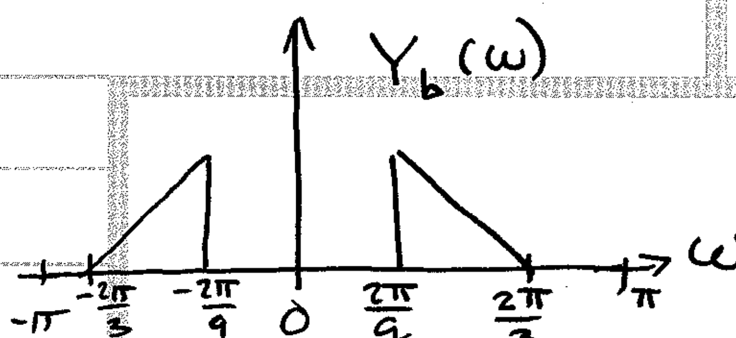
(i)



$\Rightarrow Y_a(\omega) = 0 \quad \forall \omega$

(ii)





(iii)