## Test Information

- **Test Duration:** 50 minutes.
- **Coverage:** Sessions 1-12.
- **Notes:** Open Book but Closed Notes.
- **Calculators:** allowed.
- **Number of Problems:** three

All work should be done in the blue books provided. You must show all work for each problem to receive full credit. Do **not** return this test sheet, just return the blue books.

### Problem Topics and Points

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Problem 1. [35 points]

Consider a DT LTI system whose impulse response is

\[ h[n] = (j)^n u[n] \]

(a) Is the system BIBO stable? Substantiate your answer mathematically.

(b) Find a bounded input signal \( x[n] \) that produces an unbounded output from this system.

(c) Find the system transfer function \( H(z) \) of this system and draw the pole-zero diagram.

(d) Write the difference equation for the LTI system having the impulse response above.

(e) Plot a rough sketch of the magnitude of the DTFT of \( h[n], |H(\omega)|, \) over \(-\pi < \omega < \pi\), showing as much detail as possible.

(f) Consider the input signal below which is a sum of sinewaves “turned on” for all time.

\[ x[n] = 1 + (\neg j)^n + (\neg 1)^n \]

Write a closed-form expression for the corresponding output \( y[n] \). ALSO, plot a rough sketch of the magnitude of the DTFT of \( y[n], |Y(\omega)|, \) over \(-\pi < \omega < \pi\), showing as much detail as possible.

(g) Let \( y[n] \) denote the output obtained with the input signal below relative to the LTI system with impulse response above.

\[ x[n] = (0.5)^n u[n] \]

Write a closed-form expression for the cross-correlation \( r_{yx}[\ell] \) between the output \( y[n] \) and the input \( x[n] \).
Problem 2. [35 points]
Consider the causal, second-order LTI system described by the difference equation below.

\[ y[n] = 0.25 y[n-2] + x[n] - x[n-2] \]

(a) Find the system transfer function \( H(z) \) of this system and draw the pole-zero diagram.

(b) Plot the magnitude, \( |H(\omega)| \), of the DTFT of the impulse response of the system over \(-\pi < \omega < \pi\), showing as much detail as possible. In particular, explicitly point out if there are any values of \( \omega \) for which \( |H_i(\omega)| \) is exactly zero.

(c) Consider implementing the second-order difference equation above as two first-order systems (one pole each) in parallel as shown in the diagram.

\[ \begin{align*}
  y_1[n] &= h_1[n] \\
  y_2[n] &= h_2[n] \\
  y[n] &= y_1[n] + y_2[n]
\end{align*} \]

The upper first-order system has impulse response \( h_1[n] \) and is described by the difference equation

\[ y_1[n] = a_1^{(1)} y_1[n-1] + b_0^{(1)} x[n] + b_1^{(1)} x[n-1] \]

The lower first-order system has impulse response \( h_2[n] \) and is described by the difference equation

\[ y_2[n] = a_1^{(2)} y_2[n-1] + b_0^{(2)} x[n] + b_1^{(2)} x[n-1] \]

Determine the numerical values of \( a_1^{(i)}, b_0^{(i)}, \) and \( b_1^{(i)} \), \( i = 1, 2 \) – six values total. **NOTE:** Each of the two first-order systems has a single non-zero zero and a single non-zero pole. In order to get a unique answer, you are given that \( b_0^{(1)} = \frac{1}{2} \), and you must find 5 numerical values: \( a_1^{(1)}, b_1^{(1)}, a_1^{(2)}, b_0^{(2)}, \) and \( b_1^{(2)} \).

(d) For EACH of the two first-order systems, \( i = 1, 2 \), do the following:

  (i) Plot the pole-zero diagram.

  (ii) State and plot the region of convergence for \( H_i(z) \).

  (iii) Determine the DTFT of \( h_i[n] \) and plot the magnitude \( |H_i(\omega)| \) over the interval \(-\pi < \omega < \pi\) showing as much detail as possible.
Problem 3. [30 points]

Consider the continuous-time signal \( x(t) = \left\{ \frac{\sin(4t)}{\pi t} \right\} \left\{ \frac{\sin(8t)}{\pi t} \right\} \). A DT signal is obtained by sampling \( x(t) \) according to \( x[n] = x(nT_s) \) for \( T_s = \frac{2\pi}{36} \).

(i) Plot the magnitude of the DTFT of \( x[n] \) over \(-\pi < \omega < \pi\).

(ii) \( x[n] \) is passed through a DT linear system with impulse response \( h_a[n] = (-1)^n \left\{ \frac{\sin(\frac{\pi}{3}n)}{\pi n} \right\} \) yielding the output \( y_a[n] \). Plot magnitude of the DTFT of \( y_a[n] \).

(iii) \( x[n] \) is passed through a DT linear system with impulse response \( h_b[n] = 2 \cos \left( \frac{4\pi}{9} n \right) \left\{ \frac{\sin(\frac{2\pi}{9} n)}{\pi n} \right\} \) yielding the output \( y_b[n] \). Plot magnitude of the DTFT of \( y_b[n] \) over \(-\pi < \omega < \pi\).

(iv) \( x[n] \) is passed through a DT linear system with impulse response \( h_c[n] = \frac{\sin(\frac{2\pi}{9} n)}{\pi n} \) yielding the output \( y_c[n] \). Plot magnitude of the DTFT of \( y_c[n] \) over \(-\pi < \omega < \pi\).