EE538 Digital Signal Processing I — Session 13
Exam 1 — Live: Wed., Sept. 22, 2004

Cover Sheet

Test Duration: 50 minutes.
Coverage: Sessions 1-12.
Open Book but Closed Notes.
Calculators allowed.
This test contains **three** problems.
All work should be done in the blue books provided.
You must show all work for each problem to receive full credit.
Do **not** return this test sheet, just return the blue books.

Prob. No.	**Topic(s)**	**Points**
1.	LTI Systems: Properties, Transfer Functions, Frequency Response DT Autocorrelation, Cross-Correlation	35
2.	Interconnection of LTI Systems: Transfer Functions, Frequency Response	35
3.	Sampling Theory, CTFT-DTFT Relationship, DT Frequency Selective Filtering	30

Problem 1. [35 points]

Consider a DT LTI system whose impulse response is

$$h[n] = (j)^n u[n]$$

(a) Is the system BIBO stable? Substantiate your answer mathematically.

(b) Find a bounded input signal $x[n]$ that produces an unbounded output from this system.

(c) Find the system transfer function $H(z)$ of this system and draw the pole-zero diagram.

(d) Write the difference equation for the LTI system having the impulse response above.

(e) Plot a rough sketch of the magnitude of the DTFT of $h[n]$, $|H(\omega)|$, over $-\pi < \omega < \pi$, showing as much detail as possible.

(f) Consider the input signal below which is a sum of sinewaves "turned on" for all time.

$$x[n] = 1 + (-j)^n + (-1)^n$$

Write a closed-form expression for the corresponding output $y[n]$. ALSO, plot a rough sketch of the magnitude of the DTFT of $y[n]$, $|Y(\omega)|$, over $-\pi < \omega < \pi$, showing as much detail as possible.

(g) Let $y[n]$ denote the output obtained with the input signal below relative to the LTI system with impulse response above.

$$x[n] = (0.5)^n u[n]$$

Write a closed-form expression for the cross-correlation $r_{yx}[\ell]$ between the output $y[n]$ and the input $x[n]$.

Problem 2. [35 points]

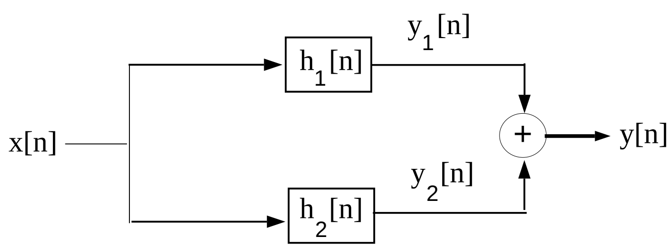
Consider the causal, second-order LTI system described by the difference equation below.

$$y[n] = 0.25\ y[n-2] + x[n] - x[n-2]$$

(a) Find the system transfer function $H(z)$ of this system and draw the pole-zero diagram.

(b) Plot the magnitude, $|H(\omega)|$, of the DTFT of the impulse response of the system over $-\pi < \omega < \pi$, showing as much detail as possible. In particular, explicitly point out if there are any values of ω for which $|H_i(\omega)|$ is exactly zero.

(c) Consider implementing the second-order difference equation above as two first-order systems (one pole each) in parallel as shown in the diagram.



The upper first-order system has impulse response $h_1[n]$ and is described by the difference equation

$$y_1[n] = a_1^{(1)}\ y_1[n-1] + b_0^{(1)} x[n] + b_1^{(1)}\ x[n-1]$$

The lower first-order system has impulse response $h_2[n]$ and is described by the difference equation

$$y_2[n] = a_1^{(2)}\ y_2[n-1] + b_0^{(2)} x[n] + b_1^{(2)}\ x[n-1]$$

Determine the numerical values of $a_1^{(i)}$, $b_0^{(i)}$, and $b_1^{(i)}$, $i = 1, 2$ – six values total. **NOTE:** Each of the two first-order systems has a single non-zero zero and a single non-zero pole. In order to get a unique answer, you are given that $b_0^{(1)} = \frac{1}{2}$, and you must find 5 numerical values: $a_1^{(1)}$, $b_1^{(1)}$, $a_1^{(2)}$, $b_0^{(2)}$, and $b_1^{(2)}$.

(d) For EACH of the two first-order systems, $i = 1, 2$, do the following:

(i) Plot the pole-zero diagram.

(ii) State and plot the region of convergence for $H_i(z)$.

(iii) Determine the DTFT of $h_i[n]$ and plot the magnitude $|H_i(\omega)|$ over the interval $-\pi < \omega < \pi$ showing as much detail as possible.

Problem 3. [30 points]

Consider the continuous-time signal $x(t) = \left\{ \frac{\sin(4t)}{\pi t} \right\} \left\{ \frac{\sin(8t)}{\pi t} \right\}$. A DT signal is obtained by sampling $x(t)$ according to $x[n] = x(nT_s)$ for $T_s = \frac{2\pi}{36}$.

(i) Plot the magnitude of the DTFT of $x[n]$ over $-\pi < \omega < \pi$.

(ii) $x[n]$ is passed through a DT linear system with impulse response $h_a[n] = (-1)^n \left\{ \frac{\sin(\frac{\pi}{3}n)}{\pi n} \right\}$ yielding the output $y_a[n]$. Plot magnitude of the DTFT of $y_a[n]$.

(iii) $x[n]$ is passed through a DT linear system with impulse response $h_b[n] = 2\cos\left(\frac{4\pi}{9}n\right) \left\{ \frac{\sin(\frac{2\pi}{9}n)}{\pi n} \right\}$ yielding the output $y_b[n]$. Plot magnitude of the DTFT of $y_b[n]$ over $-\pi < \omega < \pi$.

(iv) $x[n]$ is passed through a DT linear system with impulse response $h_c[n] = \frac{\sin(\frac{2\pi}{9}n)}{\pi n}$ yielding the output $y_c[n]$. Plot magnitude of the DTFT of $y_c[n]$ over $-\pi < \omega < \pi$.