EE538 Exam 1 Fall 2003 Solutions

Sol'n to Prob. 1:

$x[n]=$	$\{4 \quad -2 \quad 2 \quad -4$	$-4 \quad -2 \quad 2 \quad 4\}$
$*\ c_1[n]$	$\{1 \quad -1 \quad 1 \quad -1\}$	$\{1 \quad -1 \quad 1 \quad -1\}$
	$= 12$	$= -4$
	$b_1[0] = \frac{12}{4} = 3$	$b_1[1] = \frac{-4}{4} = -1$
	bits: $\{1,1\}$	bits: $\{0,0\}$
$*\ c_2[n]$	$\{1, 1, -1, -1\}$	$\{1, 1, -1, -1\}$
	$b_2[0] = \frac{4}{4} = 1$	$b_2[1] = \frac{-12}{4} = -3$
	bits: $\{0,1\}$	bits: $\{1,0\}$

Prob. 2 Sol'n.

$$x_r[n] = x_r\left(n\frac{T_s}{L}\right) = \sum_{k=-\infty}^{\infty} x[k]\, \underbrace{g\left(n\frac{T_s}{L} - kT_s\right)}_{g\left((n-kL)\frac{T_s}{L}\right)}$$

Thus, defining: $h[n] = g\left(n\frac{T_s}{L}\right)$

we have:

$$x_r[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-kL]$$

$$= \left\{\sum_{k=-\infty}^{\infty} x[k]\, \delta[n-kL]\right\} * h[n]$$

Hence:

$$x[n] \longrightarrow \boxed{L\uparrow} \longrightarrow \boxed{h[n]} \longrightarrow x_r[n] = x_r\left(n\frac{T_s}{L}\right)$$

where: $h[n] = g\left(n\frac{T_s}{L}\right)$

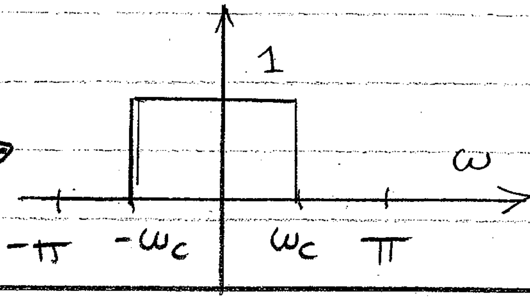
For Prob. 2 $\Rightarrow$ $g(t) = \sin\left(\pi t/T_s\right) / \pi t/T_s$

Thus: $h[n] = \dfrac{L\sin\left(\frac{\pi}{L} n\right)}{\pi n}$

Sol'n to Prob. 2 (cont.)

From Table in Text (can also use CTFT–DTFT relationship)

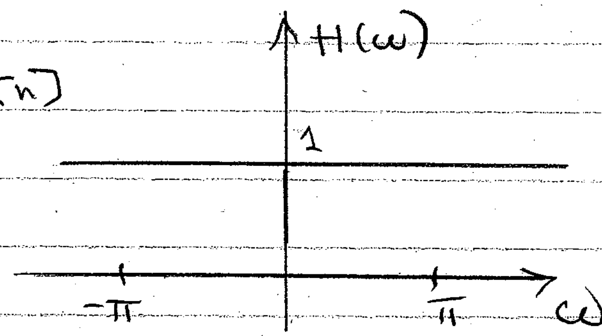
$$\frac{\sin(\omega_c n)}{\pi n} \xleftrightarrow{\text{DTFT}}$$



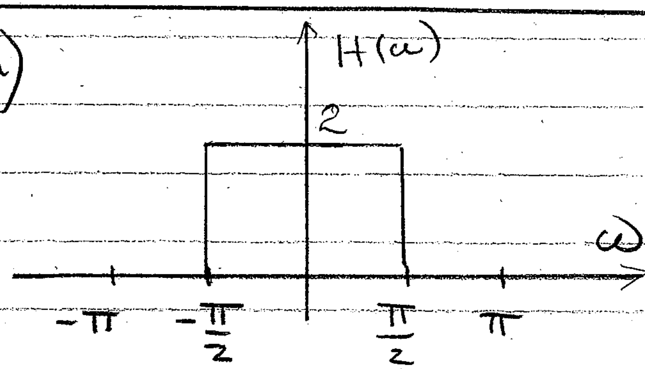
Thus:

(a) $h[n] = \frac{\sin(\pi n)}{\pi n} = \delta[n]$

$L = 1$



(b) $L=2$ $h[n] = 2\frac{\sin\left(\frac{\pi}{2}n\right)}{\pi n}$

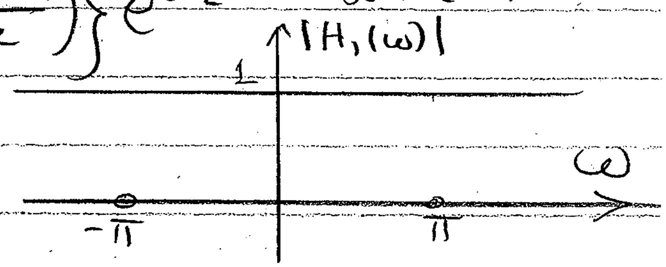


(i) $h_0[n] = h[2n] = \delta[n]$

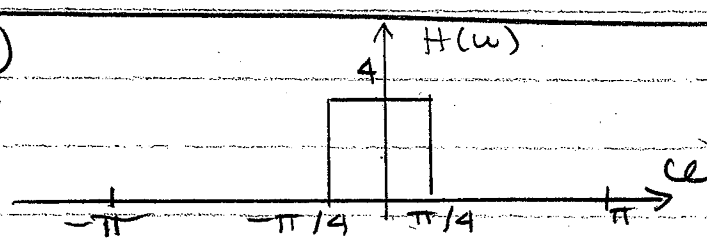
— see plot for (a)

(ii) $h_1[n] = h[2n+1]$

$$H_1(\omega) = \frac{1}{2}\left\{H\left(\frac{\omega}{2}\right) - H\left(\frac{\omega-2\pi}{2}\right)\right\}e^{j\frac{\omega}{2}}$$ where $H(\omega)$



(c) $L=4$ $h[n] = 4\frac{\sin\left(\frac{\pi}{4}n\right)}{\pi n}$



Sol'n to Prob. 3 Make use of sol'n to Prob. 2

(a) $h[n] = 1 - \left|\frac{nT_s}{T_s}\right|, \quad |nT_s| < T_s$

$= \delta[n] \qquad H(\omega) = 1 \quad \forall \omega$

(b) $h[n] = 1 - \left|\frac{nT_s}{2T_s}\right|, \quad |nT_s| < T_s$

$= 1 - \frac{|n|}{2}, \quad |n| \leq 2$

$= \{.5, \underset{\uparrow}{1}, .5\}$

$= \frac{1}{2} \{\underset{\uparrow}{1}, 1\} * \{1, \underset{\uparrow}{1}\}$

Time-Reversal Prop DTFT:

$x[-n] \longleftrightarrow X(-\omega)$

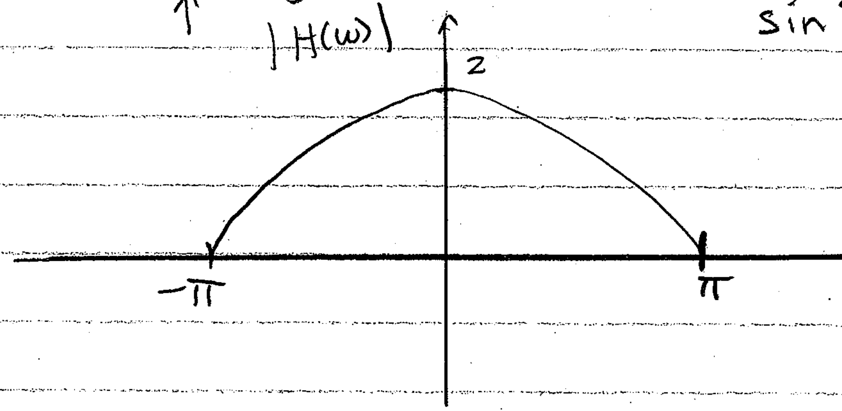
Recall: $u[n] - u[n-L] \overset{DTFT}{\longleftrightarrow} \dfrac{\sin\left(\frac{L}{2}\omega\right)}{\sin\left(\frac{1}{2}\omega\right)} e^{-j\frac{(L-1)\omega}{2}}$

$= \{\underset{\uparrow}{1}, 1, \ldots, 1\}$

Combining this DTFT pair and the time-reversal property of the DTFT

$h[n] = \{.5, \underset{\uparrow}{1}, .5\} \overset{DTFT}{\longleftrightarrow} H(\omega) = \dfrac{\sin^2(\omega)}{\sin^2\left(\frac{\omega}{2}\right)}$

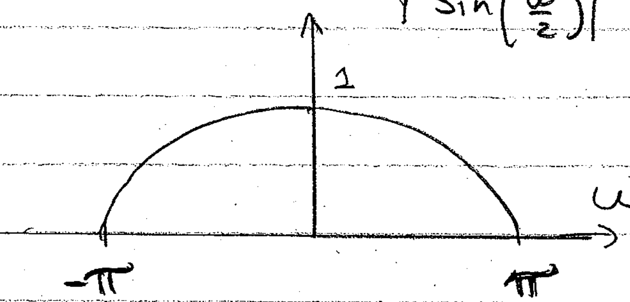
$\frac{\omega}{2} = \pi$



Sol'n to Prob. 3 (cont.)

(i) $h_0[n] = h[2n] = \delta[n]$ $\quad H_0(\omega) = 1 \ \forall \omega$

(ii) $h_1[n] = \{.5, \underset{\uparrow}{.5}\}$ $\quad |H_1(\omega)| = .5 \left| \frac{\sin(\omega)}{\sin(\frac{\omega}{2})} \right|$



(c) $h[n] = 1 - \left| \frac{nT_s}{4T_s} \right|, \ |nT_s| < T_s$

$= 1 - \left| \frac{n}{4} \right|, \ |n| \leq 4$

$= \{.25, .5, .75, \underset{\uparrow}{1}, .75, .5, .25\}$

$= \frac{1}{4} \cdot \{\underset{\uparrow}{1}, 1, 1, 1\} * \{1, 1, 1, \underset{\uparrow}{1}\}$

Thus:

$$H(\omega) = \frac{1}{4} \left\{ \frac{\sin(2\omega)}{\sin(\frac{\omega}{2})} \right\}^2$$