EE538 Digital Signal Processing I **Session 14**
Exam 1 **Live: Fri., Sept. 26, 2002**

Cover Sheet

Test Duration: 50 minutes.
Coverage: Sessions 1-10.
Open Book but Closed Notes.
Calculators allowed.
This test contains **three** problems.
All work should be done in the blue books provided.
You must show all work for each problem to receive full credit.
Do **not** return this test sheet, just return the blue books.

Prob. No.	**Topic(s)**	**Points**
1.	DT Correlation	30
2.	Deriving DT Model from CT Signal/System Digital Upsampling Frequency Response of LTI System	35
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Problem 1. [30 points]

Consider a very simplistic CDMA system with only two users assigned the following length 4 *orthogonal* codes, respectively, which are two rows of a 4x4 Walsh-Hadamard matrix:

$$\text{User 1's code:} \quad c_1[n] = \{1, -1, 1, -1\}$$

$$\text{User 2's code:} \quad c_2[n] = \{1, 1, -1, -1\}$$

Consider transmitting a block of two PAM symbols for each of the two users,

$$\text{User 1's two info. symbols:} \quad b_1[n] = \{b_1[0], b_1[1]\}$$

$$\text{User 2's two info. symbols:} \quad b_2[n] = \{b_2[0], b_2[1], \}$$

where $b_k[n]$ can take on one of four different real values (for each value of k and n):

1. 3 representing the bit pair {1,1} ;
2. 1 representing the bit pair {0,1} ;
3. -1 representing the bit pair {0,0} ;
4. -3 representing the bit pair {1,0} ;

The transmitted code-division multiplexed block may be mathematically expressed as

$$x[n] = \sum_{k=1}^{2} \sum_{m=0}^{1} b_k[m] c_k[n - 4m], \qquad n = 0, 1, ..., 7$$

Given that the received block has the following numerical values

$$x[n] = \{\underbrace{4}_{\uparrow}, -2, 2, -4, -4, -2, 2, 4\}$$

where the first entry above is the value of $x[0]$, determine the numerical values of $b_1[n], n = 0, 1$ and $b_2[n], n = 0, 1$. Your answer should consist of 4 numerical values all together. Show all work in arriving at your answer. Note that code 1 is orthogonal to code 2 and that the two users are synchronized.

Problem 2. [35 points]

The analog signal $x_a(t)$ is reconstructed from its samples $x[n] = x_a(nT_s)$ according to the following equation

$$x_r(t) = \sum_{k=-\infty}^{\infty} x[k]\; g(t - kT_s) \qquad \text{where: } g(t) = \frac{\sin\left(\pi \frac{t}{T_s}\right)}{\pi \frac{t}{T_s}}$$

Samples of the reconstructed signal at L times the original sampling rate may be obtained via the following discrete-time system.

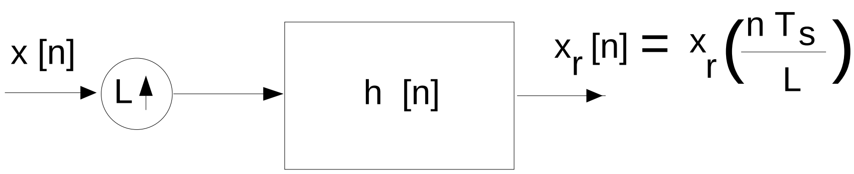


Figure 1.

Your primary task in this problem is to determine the appropriate filter impulse response $h[n]$ for different values of L so that the output of the system above is what you would have obtained if you had sampled the reconstructed signal at L times the original sampling rate as specified. **NOTE:** Correct answer for $h[n]$ is different for each part (for each value of L.)

(a) For the case of $L = 1$, write a closed-form expression for the filter $h[n]$. Plot the magnitude of the DTFT of $h[n]$, $H(\omega)$, over $-\pi < \omega < \pi$.

(b) For the case of $L = 2$, write a closed-form expression for the filter $h[n]$. Plot the magnitude of the DTFT of $h[n]$, $H(\omega)$, over $-\pi < \omega < \pi$. The case of $L = 2$ in Figure 1 may be efficient implementated as in the block diagram in Figure 2 below.

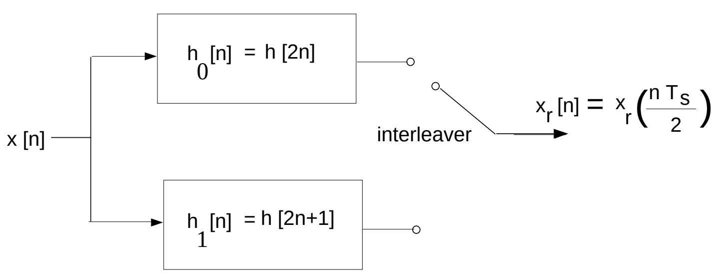


Figure 2.

Using your answer for $h[n]$ for this part (b), do the following:

(i) Write an expression for $h_0[n] = h[2n]$. Plot the magnitude of the DTFT of $h_0[n]$, $|H_0(\omega)|$, over $-\pi < \omega < \pi$.

(ii) Write an analytical expression for $h_1[n] = h[2n + 1]$. Plot the magnitude of the DTFT of $h_1[n]$, $|H_1(\omega)|$, over $-\pi < \omega < \pi$.

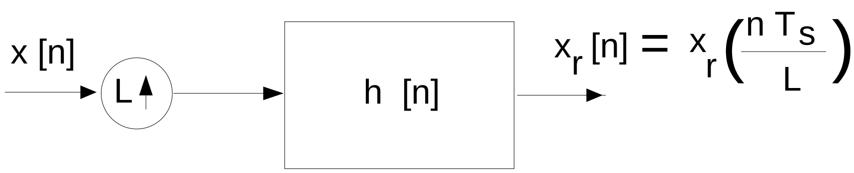
(c) For the case of $L = 4$, write a closed-form expression for the filter $h[n]$. Plot the magnitude of the DTFT of $h[n]$, $H(\omega)$, over $-\pi < \omega < \pi$.

Problem 3. [35 points]

The analog signal $x_a(t)$ is reconstructed from its samples $x[n] = x_a(nT_s)$ according to the following equation

$$x_r(t) = \sum_{k=-\infty}^{\infty} x[k]\; g(t - kT_s) \qquad \text{where: } g(t) = \Lambda\left(\frac{t}{T_s}\right) = \begin{cases} 1 - \frac{|t|}{T_s} & \text{for } |t| < T_s \\ 0 & \text{for } |t| > T_s \end{cases}$$

Samples of the reconstructed signal at L times the original sampling rate may be obtained via the following discrete-time system.



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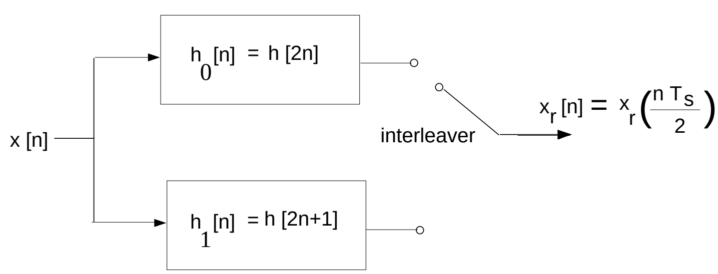

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