EE538 Digital Signal Processing I — Session 13
Exam 1 — Live: Wed., Sept. 18, 2002

Cover Sheet

Test Duration: 50 minutes.
Coverage: Sessions 1-10.
Open Book but Closed Notes.
Calculators **not** allowed.
This test contains **three** problems.
All work should be done in the blue books provided.
You must show all work for each problem to receive full credit.
Do **not** return this test sheet, just return the blue books.

Prob. No.	**Topic(s)**	**Points**
1.	DT Correlation	30
2.	LTI Systems (Causal): Properties, Frequency Response Interconnection, Pole-Zero Diagram	30
3.	Digital Upsampling	40

Problem 1. [30 points]

Consider a very simplistic CDMA system with only two users assigned the following length 4 *orthogonal* complex-value codes, respectively, which are two rows of a 4-point DFT matrix:

$$\text{User 1's code:} \quad c_1[n] = \{1, -j, -1, j\}$$

$$\text{User 2's code:} \quad c_2[n] = \{1, j, -1, -j\}$$

Consider transmitting a block of two QAM symbols for each of the two users,

$$\text{User 1's two info. symbols:} \quad b_1[n] = \{b_1[0], b_1[1]\}$$

$$\text{User 2's two info. symbols:} \quad b_2[n] = \{b_2[0], b_2[1], \}$$

where $b_k[n]$ can take on one of four different complex values (for each value of k and n):

1. 1+j representing the bit pair {1,1} ;
2. -1+j representing the bit pair {0,1} ;
3. -1-j representing the bit pair {0,0} ;
4. 1-j representing the bit pair {1,0} ;

The transmitted code-division multiplexed block may be mathematically expressed as

$$x[n] = \sum_{k=1}^{2} \sum_{m=0}^{1} b_k[m] c_k[n - 4m], \qquad n = 0, 1, ..., 7$$

Given that the received block has the following numerical values

$$x[n] = \{\underbrace{2j}_{\uparrow}, -2j, -2j, 2j, -2j, 2j, 2j, -2j\}$$

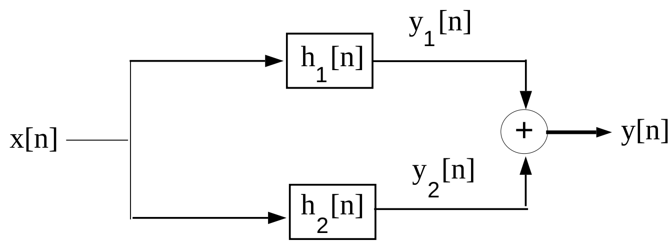
where the first entry above is the value of $x[0]$, determine the numerical values of $b_1[n], n = 0, 1$ and $b_2[n], n = 0, 1$. Your answer should consist of 4 numerical values all together. Show all work in arriving at your answer. Note that code 1 is orthogonal to code 2 and that the two users are synchronized.

Problem 2. [30 points]

Consider the causal, second-order difference equation below where it is noted that $.9025 = .95^2$.

$$y[n] = -0.95\ y[n-1] - 0.9025\ y[n-2] + x[n] +\ x[n-1] +\ x[n-2]$$

Consider implementing this second-order difference equation as two first-order systems in parallel (one pole each) as shown in the diagram.



(a) Determine and write the first-order difference equation for each of the two first-order systems in parallel. The upper first-order system has impulse response $h_1[n]$ and is described by the difference equation

$$y_1[n] = a_1^{(1)}\ y_1[n-1] + b_0^{(1)} x[n] + b_1^{(1)}\ x[n-1]$$

The lower first-order system has impulse response $h_2[n]$ and is described by the difference equation

$$y_2[n] = a_1^{(2)}\ y_2[n-1] + b_0^{(2)} x[n] + b_1^{(2)}\ x[n-1]$$

Determine the numerical values of $a_1^{(i)}$, $b_0^{(i)}$, and $b_1^{(i)}$, $i = 1, 2$ – six values total.

(b) For EACH of the two first-order systems, $i = 1, 2$, do the following:

(i) Plot the pole-zero diagram.

(ii) State and plot the region of convergence for $H_i(z)$.

(iii) Determine the DTFT of $h_i[n]$ and plot the magnitude $|H_i(\omega)|$ over the interval $-\pi < \omega < \pi$ showing as much detail as possible. In particular, explicitly point out if there are any values of ω for which $|H_i(\omega)|$ is exactly zero.

Problem 3. [40 points]

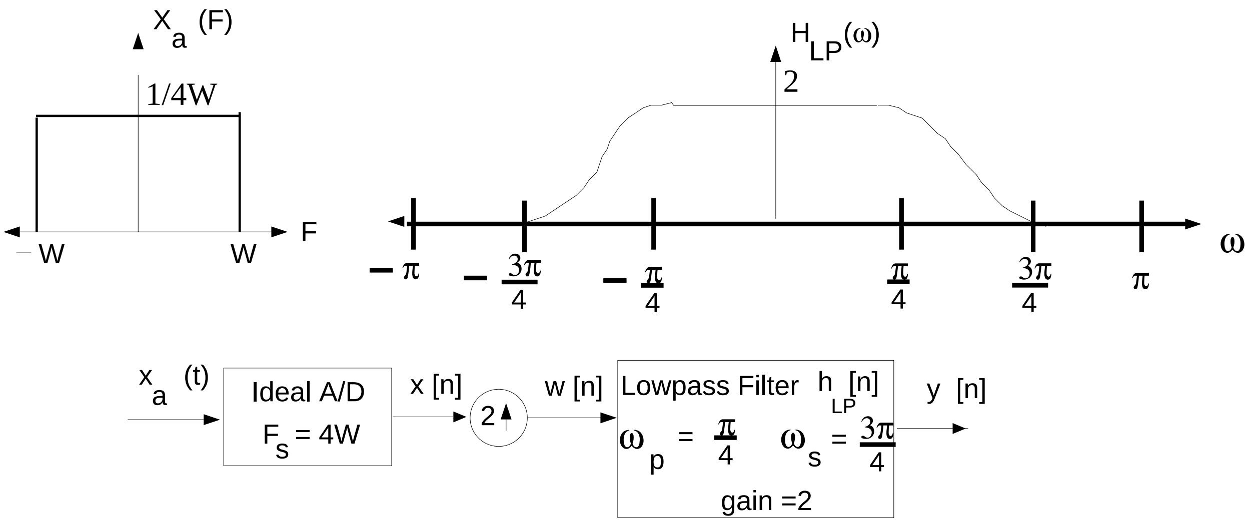


Figure 1.

The analog signal $x_a(t)$ with CTFT $X_a(F)$ shown above is input to the system above, where $x[n] = x_a(n/F_s)$ with $F_s = 4W$, and

$$h_{LP}[n] = \frac{\sin(\frac{\pi}{2}n)}{\frac{\pi}{2}n}\frac{\cos(\frac{\pi}{4}n)}{1-\frac{n^2}{4}}, \qquad -\infty < n < \infty,$$

such that $H_{LP}(\omega) = 2$ for $|\omega| \le \frac{\pi}{4}$, $H_{LP}(\omega) = 0$ for $\frac{3\pi}{4} \le |\omega| \le \pi$, and $H_{LP}(\omega)$ has a cosine roll-off from 1 at $\omega_p = \frac{\pi}{4}$ to 0 at $\omega_s = \frac{3\pi}{4}$. Finally, the zero inserts may be mathematically described as

$$w[n] = \begin{cases} x(\frac{n}{2}), & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$$

(a) Plot the magnitude of the DTFT of the output $y[n]$, $Y(\omega)$, over $-\pi < \omega < \pi$.

(b) Given that

$$x[n] = \frac{\sin(\frac{\pi}{2}n)}{\pi n} \qquad -\infty < n < \infty,$$

provide an analytical expression for $y[n]$ for $-\infty < n < \infty$ (similar to the expression for either $x[n]$ or $h_{LP}[n]$ above, for example.)

THIS PROBLEM IS CONTINUED ON THE NEXT PAGE.

(c) The up-sampling by a factor of 2 in Figure 1 can be efficiently done via the block diagram in Figure 2 below.

(i) Provide an analytical expression for $h_0[n] = h_{LP}[2n]$ for $-\infty < n < \infty$. Simplify as much as possible.

(ii) Plot the magnitude of the DTFT of $h_0[n]$, $|H_0(\omega)|$, over $-\pi < \omega < \pi$.

(iii) Provide an analytical expression for the output $y_0[n]$ for $-\infty < n < \infty$.

(iv) Plot the magnitude of the DTFT of $h_1[n]$, $|H_1(\omega)|$, over $-\pi < \omega < \pi$.

(v) Plot the phase of the DTFT of $h_1[n]$, $\angle H_1(\omega)$, over $-\pi < \omega < \pi$.

(vi) Provide an analytical expression for the output $y_1[n]$ for $-\infty < n < \infty$.

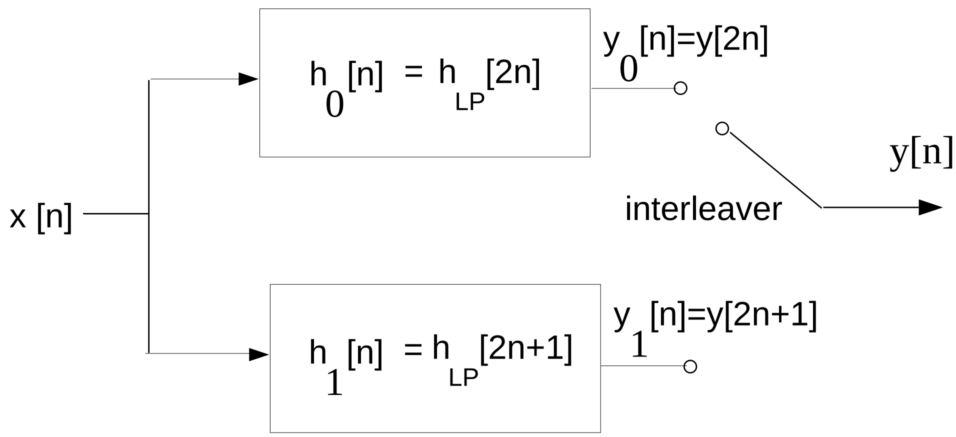


Figure 2.