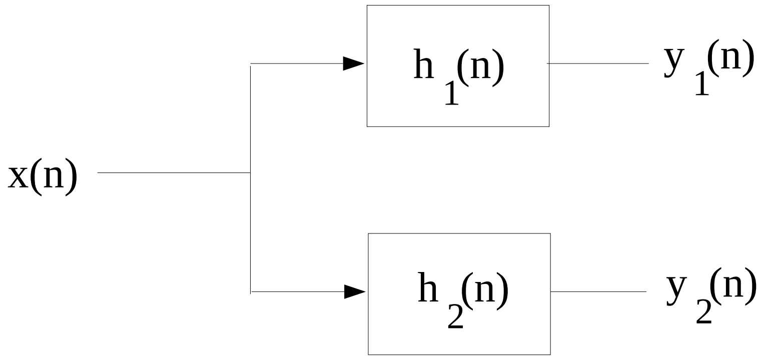
EE538 Digital Signal Processing I 20 September 2001
Exam 1

Cover Sheet

Test Duration: 75 minutes.
Open Book but Closed Notes.
Calculators **not** allowed.
This test contains **three** problems.
All work should be done in the blue books provided.
You must show all work for each problem to receive full credit.
Do **not** return this test sheet, just return the blue books.

Prob. No.	**Topic(s)**	**Points**
1.	Convolution and LTI Systems	30
2.	Z-Transform (ZT); Relationship between ZT and DTFT	40
5.	Digital Upsampling	30

Problem 1. [30 points]



Consider the system above where the causal signal $x(n)$ is simultaneously input to two different causal FIR filters of length $M = 2$ with respective impulses responses $h_1(n)$ and $h_2(n)$. That is, both $h_1(n)$ and $h_2(n)$ are only nonzero for $n = 0$ and $n = 1$. We don't know anything about the input signal $x(n)$ except that it is causal. Yet, it is possible to determine $h_1(n)$ and $h_2(n)$ (to within a scalar multiple) given the respective outputs $y_1(n)$ and $y_2(n)$.

(a) Show that $y_1(n) * h_2(n) = y_2(n) * h_1(n)$, which may be alternatively expressed as

$$y_1(n) * h_2(n) - y_2(n) * h_1(n) = 0$$

(b) Exploiting this relationship and arbitrarily assigning $h_1(0) = 1$ without loss of generality, determine the numerical values of $h_1(1)$, $h_2(0)$, and $h_2(1)$ given the following output values.

$$\begin{array}{llll} y_1(0) = 1 & y_1(1) = 4 & y_1(2) = 7 & y_1(3) = 10 \\ y_2(0) = 3 & y_2(1) = 5 & y_2(2) = 7 & y_2(3) = 9 \end{array}$$

Note: This is not all of the output values for both $y_1(n)$ and $y_2(n)$. However, it is enough information to determine both $h_1(n)$ and $h_2(n)$ which, again, are only nonzero for $n = 0$ and $n = 1$. Again, you are given $h_1(0) = 1$. SHOW ALL WORK.

Problem 2. [40 points]

Consider that we wish to filter a long stream of data with a filter having the following impulse response

$$h[n] = e^{j\frac{2\pi}{N}\ell n}\{u[n] - u[n-N]\}$$

where ℓ is an integer between 0 and $N-1$. Convolution with this impulse response may be effected via the following difference equation

$$y[n] = \sum_{k=0}^{N-1} e^{j\frac{2\pi}{N}\ell k} x[n-k]$$

This implementation is seen to require N multiplications and $N-1$ additions per output point.

(a) It can be shown that we can achieve exactly the same input-output (I/O) relationship via the following difference equation which requires only 1 multiplication and two additions per output point.

$$y[n] = a_1\, y[n-1] + x[n] - x[n-D]$$

Determine the values of a_1 and D in terms of ℓ and N so that this IIR system has exactly the same I/O relationship as the FIR filter above. SHOW ALL WORK – NO CREDIT FOR ANSWERS THAT ARE NOT SUBSTANTIATED BY WORK.

(b) Consider the specific case of $\ell = 2$ and $N = 4$:

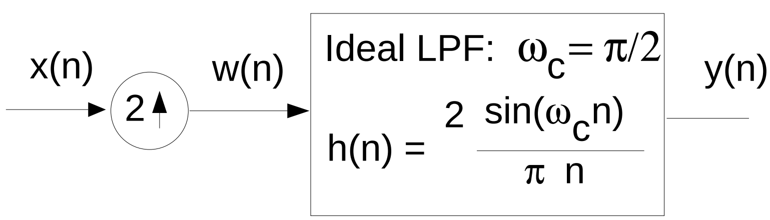
(i) State the numerical values of a_1 and D for this case so that the FIR filter and the IIR filter have the same I/O relationship.

(ii) Plot the pole-zero diagram for the system. Show the region of convergence.

(iii) Is this a lowpass, bandpass, or highpass filter? Briefly explain why.

(iv) Plot the impulse response of the system (Stem plot).

Problem 3. [30 points]

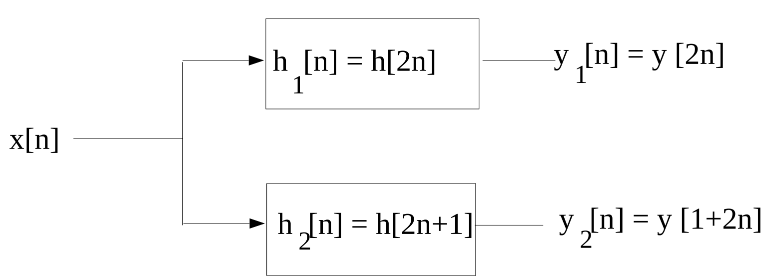


The DT signal $x(n)$ is input to the system above to effect digital upsampling by a factor of two, where

$$w(n) = \begin{cases} x(\frac{n}{2}), & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$$

$$h(n) = \frac{\sin(\frac{\pi}{2}n)}{\frac{\pi}{2}n}, \qquad -\infty < n < \infty,$$

Instead of having to actually run zeroes through the filter, we know that we can compute the even and odd samples of the output signal $y(n)$ above as shown below.



(a) Let $H_1(\omega)$ and $H_2(\omega)$ denote the DTFT's of $h_1(n)$ and $h_2(n)$, respectively. Plot the magnitude and phase (separate plots) of $H_1(\omega)$ over $-\pi < \omega < \pi$. Also plot the magnitude and phase (separate plots) of $H_2(\omega)$ over $-\pi < \omega < \pi$. Your answer to this part of the question should be four plots.

(b) Repeat part (a) for a nonideal lowpass filter with impulse response

$$h_{LP}[n] = \frac{\sin(\frac{\pi}{2}n)}{\frac{\pi}{2}n} \frac{\cos(\frac{\pi}{4}n)}{1 - \frac{n^2}{4}}, \qquad -\infty < n < \infty,$$

having the raised-cosine spectrum plotted below for which $H_{LP}(\omega) = 2$ for $|\omega| \leq \frac{\pi}{4}$, $H_{LP}(\omega) = 0$ for $\frac{3\pi}{4} \leq |\omega| \leq \pi$, and $H_{LP}(\omega)$ has a cosine roll-off from 2 at $\omega_p = \frac{\pi}{4}$ to 0 at $\omega_s = \frac{3\pi}{4}$. Let $H_1(\omega)$ and $H_2(\omega)$ denote the DTFT's of $h_1(n) = h_{LP}[2n]$ and $h_2(n) = h_{LP}[1 + 2n]$, respectively. Plot the magnitude and phase (separate plots) of $H_1(\omega)$ over $-\pi < \omega < \pi$. Also plot the magnitude and phase (separate plots) of $H_2(\omega)$ over $-\pi < \omega < \pi$. Your answer to this part of the question should also be four plots.