

Sect. 5.3 Correlation Functions and Spectra at the output of LTI Systems

Addendum to
Module 2 on
Autocorrelation and Cross-Correlation

All of this is in
Sect. 5.3 of the text book

Plus Energy Density Spectrum

⑤
Summarizing three main properties of auto correlation sequence $r_{xx}[\ell] = x[\ell] * x^*[-\ell]$

1. $r_{xx}[-\ell] = r_{xx}^*[\ell]$

2. $|r_{xx}[\ell]| \leq r_{xx}[0]$

3. $\sum_{\ell=-\infty}^{\infty} r_{xx}[\ell] e^{-j\omega\ell} \geq 0$

for all ω

From 1. , $S_{xx}(\omega) = \sum_{\ell=-\infty}^{\infty} r_{xx}[\ell] e^{-j\omega\ell}$

is real-valued for all ω

$$S'_{xx}(\omega) = X(\omega)X^*(\omega) = |X(\omega)|^2$$

\Rightarrow energy density spectrum

I/O Relationships for LTI System



$$r_{yx}[\ell] = r_{xx}[\ell] * h[\ell] \quad \left. \vphantom{r_{yx}[\ell]} \right\} \begin{array}{l} \text{cross-} \\ \text{correlation} \\ \text{between input} \\ \text{and output} \end{array}$$

$$r_{yy}[\ell] = r_{xx}[\ell] * r_{hh}[\ell]$$

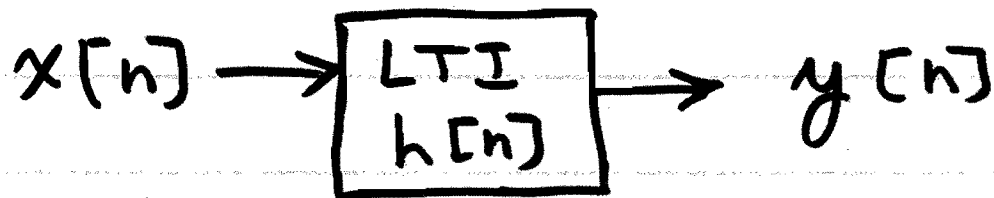
Energy Density Spectrum:

$$r_{xx}[\ell] \xleftrightarrow{\text{DTFT}} S_{xx}(\omega) = \mathcal{F}\{r_{xx}[\ell]\}$$

Props. (ii) + (iii): $S_{xx}(\omega) = |X(\omega)|^2 \geq 0 \quad \forall \omega$

since $x^*[-\ell] \xleftrightarrow{\text{DTFT}} X^*(\omega)$

I/O Relationships for Energy Density Spectrum



$$r_{xx}[l] \xleftrightarrow{\text{DTFT}} \sum_{xx}(\omega) = |X(\omega)|^2$$

$$r_{yy}[l] \xleftrightarrow{\text{DTFT}} \sum_{yy}(\omega) = |Y(\omega)|^2$$

$$\sum_{yy}(\omega) = |H(\omega)|^2 \sum_{xx}(\omega)$$

Parseval's Theorem:

$$\text{Energy} = \sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} |X(\omega)|^2 d\omega$$

$$= r_{xx}[0] = E_x = \frac{1}{2\pi} \int_{-\pi}^{\pi} \sum_{xx}(\omega) d\omega$$