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Exam 3 Solutions (1)

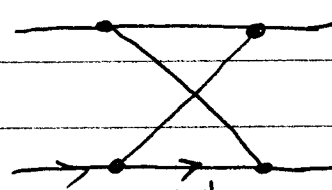
Problem 1 After 1^{st} step in deriving radix 2 FFT, we have

$$X_N(k) = F_0(k) + W_N^k F_1(k)$$

$$X_N(k+\tfrac{N}{2}) = F_0(k) - W_N^k F_1(k)$$

$$k = 0, 1, \ldots, \tfrac{N}{2}-1$$

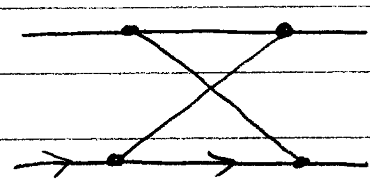
$2 = F_0(0)$ $\quad X_8(0) = 2+2 = 4$



$2 = F_1(0)$ $\quad X_8(4) = 2-2 = 0$

$W_8^0 = 1$ $\quad -1$

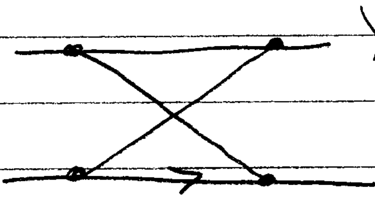
$2 = F_0(1)$ $\quad X_8(1) = 2 + \frac{\sqrt{2}}{\sqrt{2}}(1+j)(1-j) = 4$



$\sqrt{2}(1+j) = F_1(1)$ $\quad X_8(5) = 2 - 2 = 0$

$W_8^1 = \frac{1}{\sqrt{2}}(1-j)$ $\quad -1$

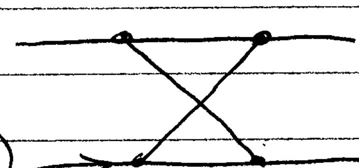
$0 = F_0(2)$ $\quad X_8(2) = 0$



$0 = F_1(2)$ $\quad X_8(6) = 0$

$W_8^2 = -j$ $\quad -1$

$2 = F_0(3)$ $\quad X_8(3) = 2 - \frac{\sqrt{2}}{\sqrt{2}}(1-j)(1+j) = 0$



$\sqrt{2}(1-j) = F_1(3)$ $\quad X_8(7) = 2+2 = 4$

$W_8^3 = -\frac{1}{\sqrt{2}}(1+j)$

Prob. 1 (cont.)

$$X_8(k) = \{4, 4, 0, 0, 0, 0, 0, 4\}$$
$$= 4\delta(k) + 4\delta(k-1) + 4\delta(k-7)$$

(b) recall: classic DFT pair derived in class:

$$\cos\left(2\pi \frac{k_0}{N} n\right) \xleftrightarrow[N]{DFT} \frac{N}{2}\delta(k-k_0) + \frac{N}{2}\delta(k-(N-k_0))$$

thus:

$$\cos\left(\frac{2\pi}{8}(1)n\right) \longleftrightarrow \frac{8}{2}\delta(k-1) + \frac{8}{2}\delta(k-7)$$

- answer: $\omega_0 = \frac{2\pi}{8} = \frac{\pi}{4}$

- Note: also:

$$1 \xleftrightarrow[N]{DFT} N\delta(k)$$

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Problem 2

$y[n] = x[n] * h[n]$

$$= \{\underset{\uparrow}{1}, 2, 3, 4, 3, 2, 1\}$$

length $= 4 + 4 - 1 = 7$

• ~~since computing 5 pt. DFT's,~~ will lead to time-aliasing

$y_p[n] = y[n] + y[n+5]$, $n = 0, 1, 2, 3, 4$

$y_p[0] = y[0] + y[5] = 1 + 2 = 3$

$y_p[1] = y[1] + y[6] = 2 + 1 = 3$

$y_p[n] = y[n]$ for $n = 2, 3, 4$ } only first 2 pts. are aliased

$$y_p[n] = \{\underset{\uparrow}{3}, 3, 3, 4, 3\}$$

• could also circularly convolve $x[n]$ and $h[n]$ with 1 zero appended to each

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Problem 3

$$w_T[n] = \frac{1}{5}\{u[n]-u[n-5]\} * \frac{1}{5}\{u[n]-u[n-5]\}$$

$$= w_{R_5}[n] * w_{R_5}[n]$$

$$W_{R_5}(\omega) = \frac{1}{5}\sum_{n=0}^{4} e^{-j\omega n} = \frac{1}{5} e^{-j\frac{(5-1)}{2}\omega} \frac{\sin\left(\frac{5}{2}\omega\right)}{\sin\left(\frac{\omega}{2}\right)}$$

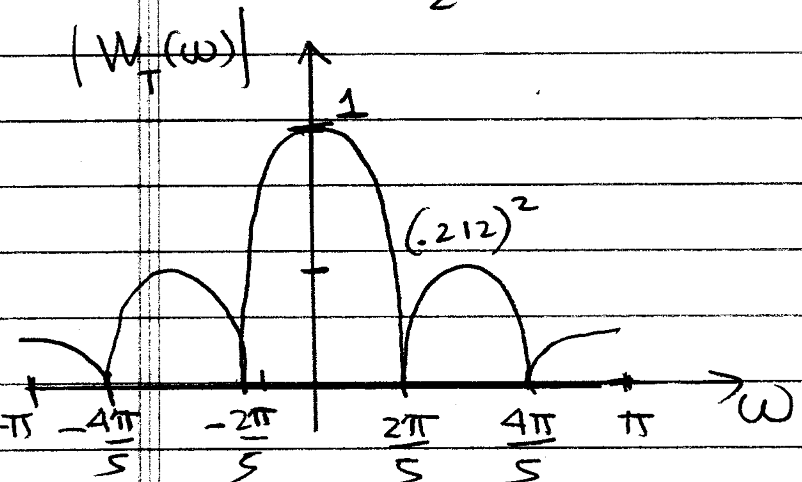
Thus:

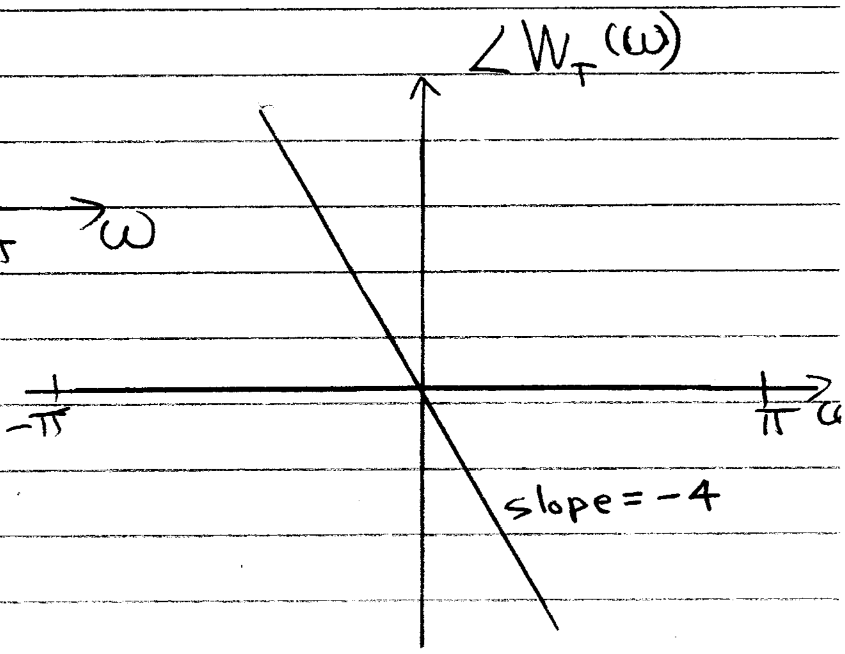
$$W_T(\omega) = W_{R_5}^2(\omega)$$

$$= \frac{1}{25} e^{-j4\omega} \left\{\frac{\sin\left(\frac{5}{2}\omega\right)}{\sin\left(\frac{\omega}{2}\right)}\right\}^2$$

• nulls: $\frac{5}{2}\omega_n = n\pi \Rightarrow \omega_{n_1} = \pm\frac{2\pi}{5}$

$$\omega_{n_2} = \pm\frac{4\pi}{5}$$





no pi discontinuities since $\{\cdot\}^2$ is strictly > 0

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Exam 3 Sol'ns. (5)

Prob. 3 (cont.)

(b) (i) ~~null-to-null~~ = $\frac{4\pi}{5}$

(ii) $W_T(\omega)$ has larger/wider mainlobe than $W_{R_9}(\omega)$

(iii) Why? The mainlobe width of $W_T(\omega)$ is that of DTFT of rectangular window of length 5 ⇒ which is wider than the mainlobe width of the DTFT of rectangular window of length 9

(c) (i) sidelobes of $W_T(\omega)$ are lower than sidelobes of $W_{R_9}(\omega)$

(ii) for either window, the max value of the DTFT is unity at $\omega=0$

~~Because of the squaring, the peak~~ sidelobe values are squared — and if $|x|<1$, then $|x^2|<|x|$

~~on a dB scale the peak sidelobes are reduced by a factor of 2~~