EE638 (CC760-M) DSP I Exam 3 Sol'ns. Fall '95

Solution to Prob. 1

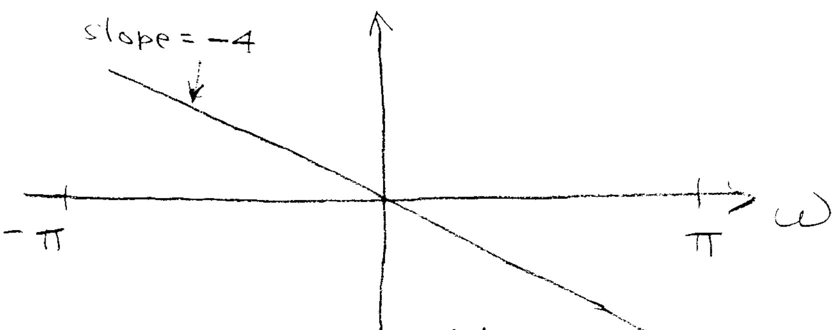
(a) Acceptable answers:

i.) can guarantee linear phase over passband with symmetric FIR filter -- distortionless transmission requires linear phase as well as flat magnitude over passband

ii.) IIR filters may have poles close to the unit circle ⇒ implying a "long" impulse response ⇒ implying "longer" transients

(b) $H(\omega) = H_r(\omega)\, e^{-j\left(\frac{M-1}{2}\right)\omega}$ for symmetric FIR filter satisfying $h(n) = h(M-1-n)$. Since $H_r(\omega) > 0$ for all ω for $h(n) = \{1, 2, 3, 4, \underset{\uparrow}{5}, 4, 3, 2, 1\}$, it follows that

$$\angle H(\omega) = -\left(\frac{M-1}{2}\right)\omega = -\frac{9-1}{2}\omega = -4\omega$$



(c) For M odd, $L = \frac{M-1}{2}$. For $M = 31$, $L = 15$.

From pg.

Sol'ns, to Prob. 1 (cont.)

(d) Acceptable answers:

i.) Can't design a highpass filter by sampling the impulse response of an analog highpass filter => there will be severe aliasing

ii.) Difficult to guarantee equi-ripple characteristic in stopband as there always some degree of aliasing since no analog impulse response is truly bandlimited

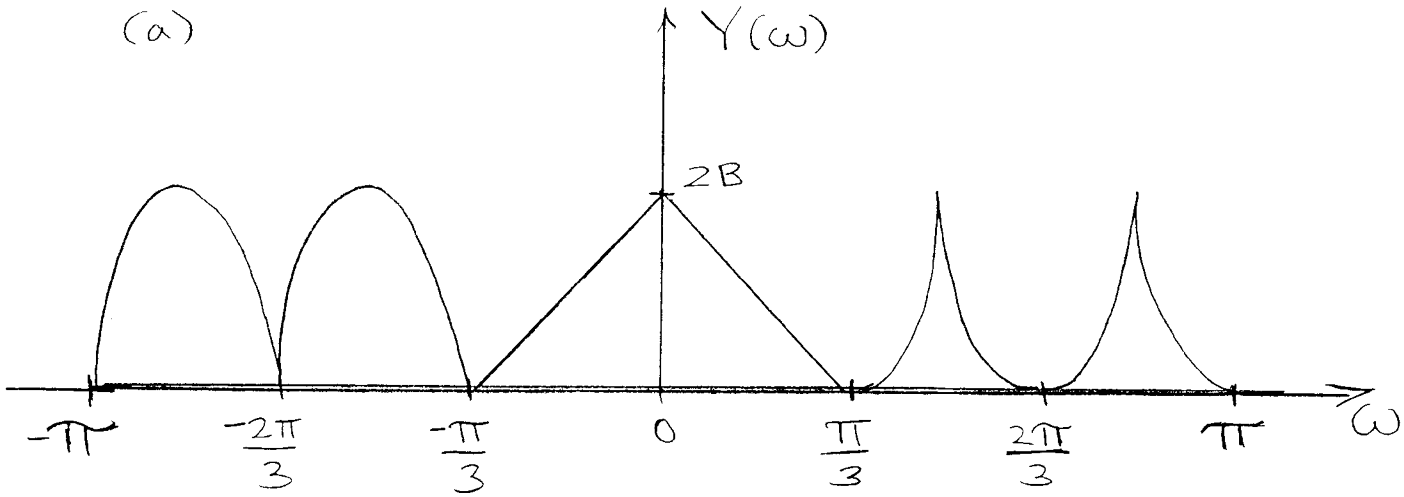
(e) (i) insures resulting digital filter may be implemented as a difference equation

(ii) insures BIBO stable analog filter is mapped to a BIBO stable digital filter

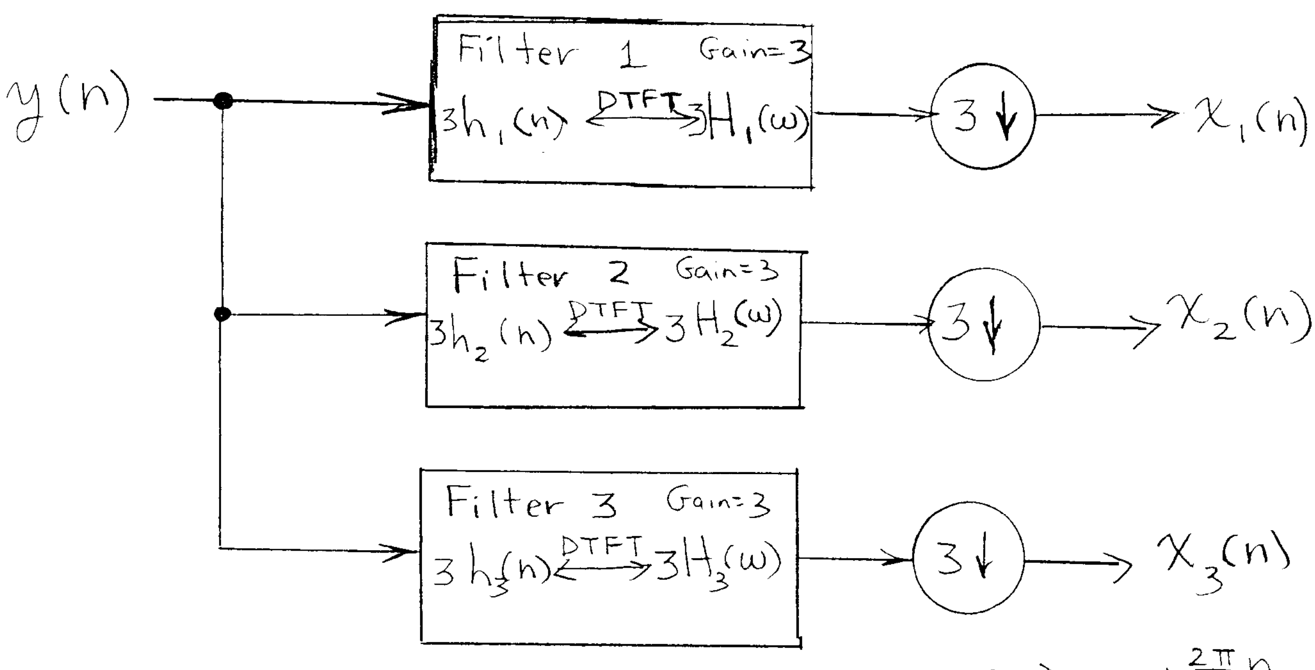
(iii) insures equi-ripple characteristic of analog filter is preserved through the bilinear mapping

Solution to Problem 2

(a)



(b)



- This follows since $y_1(n) = \frac{1}{3} x_{a1}\left(\frac{n}{6B}\right) e^{-j\frac{2\pi}{3}n}$

$$\Rightarrow x_1(n) = 3 \cdot \frac{1}{3} x_{a1}\left(\frac{3n}{6B}\right) e^{-j\frac{2\pi}{3}(3n)}$$

$$= x_1(n)$$

- Similar for $x_2(n)$ and $x_3(n)$

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Fall 296 (4)

Solution to Problem 3

$$s = \frac{z-1}{z+1} \Rightarrow j\Omega = \frac{e^{j\omega}-1}{e^{j\omega}+1} \Rightarrow \Omega = \tan\left(\frac{\omega}{2}\right)$$

$$\Omega_c = \tan\left(\frac{\omega_c}{2}\right) = \tan\left[\cancel{2}\,\frac{\tan^{-1}(\sqrt{2})}{\cancel{2}}\right]$$

$$\Rightarrow \Omega_c = \sqrt{2}$$

(b) $$H_a(s) = \frac{\sqrt{2}}{s^2 + \sqrt{2}(\sqrt{2})s + (\sqrt{2})^2}$$

$$H(z) = \frac{\sqrt{2}}{s^2+2s+2}\Bigg|_{s=\frac{z-1}{z+1}}$$

$$H(z) = \frac{\sqrt{2}}{\left(\frac{z-1}{z+1}\right)^2 + 2\,\frac{z-1}{z+1} + 2} \cdot \frac{(z+1)^2}{(z+1)^2}$$

$$= \frac{\sqrt{2}\,(z^2+2z+1)}{z^2-2z+1 + 2(z^2-1) + 2(z^2+2z+1)}$$

$$= \frac{\sqrt{2}\,(z^2+2z+1)}{5z^2+2z+1} = \frac{\sqrt{2}}{5}\,\frac{(z^2+2z+1)}{z^2+\frac{2}{5}z+\frac{1}{5}}$$

$$\frac{Y(z)}{X(z)} = \frac{\sqrt{2}}{5}\,\frac{(1+2z^{-1}+z^{-2})}{1+\frac{2}{5}z^{-1}+\frac{1}{5}z^{-2}}$$

(d) $$y(n) = -\frac{2}{5}y(n-1) + \frac{1}{5}y(n-2) + \frac{\sqrt{2}}{5}\big(x(n)+2x(n-1)+x(n-2)\big)$$

(c) Yes. Butterworth filter is stable, and stability is preserved thru bilinear transform

Poles of $H(z)$: $-\frac{2}{5} \pm j\sqrt{\frac{3}{20}} \Rightarrow |\cdot| = \frac{31}{100} = .31 < 1$