Solution to Prob. 1

(a) Acceptable answers:
   i.) can guarantee linear phase over passband
       with symmetric FIR filter -- distortion less
       transmission requires linear phase as well as flat
       magnitude over passband
   ii.) IIR filters may have poles close to the
        unit circle ⇒ implying a "long" impulse response
        ⇒ implying "longer" transients

(b) \( H(\omega) = H_r(\omega) e^{-j\frac{(M-1)}{2}\omega} \) for symmetric
    FIR filter satisfying \( h(n) = h(M-1-n) \). Since
    \( H_r(\omega) > 0 \) for all \( \omega \) for \( h(n) = \{1, 2, 3, 4, 5, 4, 3, 2, 1\} \)
    it follows that
    \[ \angle H(\omega) = -\left(\frac{M-1}{2}\right)\omega = -\frac{9-1}{2}\omega = -4\omega \]
    slope = -4

(c) For \( M \) odd, \( L = \frac{M-1}{2} \). For \( M = 21 \), \( L = 15 \).
    From pg.
Solns. to Prob. 1 (cont.)

(d) Acceptable answers:

(i) Can't design a highpass filter by sampling the impulse response of an analog highpass filter \( \Rightarrow \) there will be severe aliasing.

(ii) Difficult to guarantee equi-ripple characteristic in stopband as there always some degree of aliasing since no analog impulse response is truly bandlimited.

(e) (i) insures resulting digital filter may be implemented as a difference equation

(ii) insures BIBO stable analog filter is mapped to a BIBO stable digital filter

(iii) insures equi-ripple characteristic of analog filter is preserved through the bilinear mapping
Solution to Problem 2

(a)

(b)

This follows since \( y_1(n) = \frac{1}{3} X_{a1} \left( \frac{n}{6B} \right) e^{-j \frac{2\pi}{3} n} \)

\[ \Rightarrow x_1(n) = \frac{1}{3} X_{a1} \left( \frac{3n}{6B} \right) e^{-j \frac{2\pi}{3} (3n)} = x_1(n) \]

Similar for \( x_2(n) \) and \( x_3(n) \)
Solution to Problem 3

\[ s = \frac{Z^{-1}}{Z + 1} \quad \Rightarrow \quad j\omega_2 = \frac{e^{j\omega} - 1}{e^{j\omega} + 1} \quad \Rightarrow \quad \omega_2 = \tan \left( \frac{\omega}{2} \right) \]

\[ \omega_2 = \tan \left( \frac{\omega}{2} \right) = \tan \left[ 2 \tan^{-1} \left( \frac{1}{2} \right) \right] \]

\[ \Rightarrow \omega_2 = \frac{\sqrt{2}}{2} \]

(b) \[ H_a(s) = \frac{\sqrt{2}}{s^2 + \sqrt{2} (\sqrt{2}) s^2 + (\sqrt{2})^2} \]

\[ H(z) = \frac{\sqrt{2}}{s^2 + 2s + 2} = \left( \frac{\frac{Z^{-1}}{Z + 1}}{Z^{-1} + 2 \frac{Z^{-1}}{Z + 1} + 2} \right) \]

\[ H(z) = \frac{\sqrt{2}}{(\frac{Z^{-1}}{Z + 1})^2 + 2 \frac{Z^{-1}}{Z + 1} + 2} \]

\[ \sqrt{2} \left( Z^2 + 2Z + 1 \right) \]

\[ \frac{\sqrt{2}}{Z^2 - 2Z + 1 + 2 (Z^2 - 1) + 2 (Z^2 + 2Z + 1)} \]

\[ \frac{\sqrt{2}}{5} \frac{Z^2 + 2Z + 1}{Z^2 + 2Z + 1} = \frac{\sqrt{2}}{5} \frac{Z^2 + 2Z + 1}{\frac{2}{5}Z^2 + \frac{2}{5}Z + \frac{1}{5}} \]

\[ Y(z) = \frac{\sqrt{2}}{5} \frac{1 + 2Z^{-1} + Z^{-2}}{1 + \frac{2}{5}Z^{-1} + \frac{1}{5}Z^{-2}} \]

\[ \frac{Y(n)}{X(n)} = \frac{\sqrt{2}}{5} \frac{y(n-1) + \frac{1}{5} y(n-2) + \frac{\sqrt{2}}{5} (x(n) + 2x(n-1) + x(n-2))}{x(n)} \]

(c) Yes. Butterworth filter is stable, and stability is preserved through bilinear transform.

Poles of \( H(z) \): \( \frac{-2}{5} \pm j\frac{3}{10} \Rightarrow |z| = \frac{3}{10} < 1 \)}