Solution to Prob. 1

$F_s = 8\times10^3$ Hz

(a) $\theta_o = 2\pi \dfrac{1\times10^3}{8\times10^3} = \dfrac{\pi}{4}$

$r_o = R_o = 1 \Rightarrow$ on unit circle

$L = \dfrac{1.99\times10^3 - 1\times10^3}{10} + 1 = 99 + 1 = 100$

$\phi_o = \dfrac{2\pi}{100}$

(b) $N + L - 1 = N + 100 - 1 = N + 99$

Sol'n. to Prob. 2

- impulse response of ideal (noncausal) Hilbert Transformer:

$$h_d'(n) = 2\frac{\sin^2\left(\frac{\pi}{2}n\right)}{\pi n} \quad \text{(from the Text)}$$

- to achieve causal linear phase FIR Hilbert Transformer,

$$h_d(n) = 2\frac{\sin^2\left(\frac{\pi}{2}\left(n-\frac{M-1}{2}\right)\right)}{\pi\left(n-\frac{M-1}{2}\right)}$$

- need $h(n) = h_d(n)\,w(n)$ to be anti-symmetric, i.e., $h(n) = -h(M-1-n)$ since frequency response of ideal Hilbert Transformer is purely imaginary $H_d'(\omega) = \begin{cases} -j, & 0<\omega<\pi \\ j, & -\pi<\omega<0 \end{cases}$
- thus: $w(n) = w_1(n) = \sin\left(\frac{\pi}{M}(n+.5)\right) \quad 0 \le n \le M-1$
- $\left.\begin{array}{l} h_d(n) = -h_d(M-1-n) \\ w_1(n) \text{ is symmetric, } w_1(n) = w_1(M-1-n) \end{array}\right\}$ product of anti-symmetric and symmetric is anti-symmetric
- $w_2(n)$ is anti-symmetric, $w_2(n) = -w_2(M-1-n)$
- product of anti-symmetric with anti-symmetric is symmetric

Sol'n. to Problem 3

$$s = \frac{z-1}{z+1} \Rightarrow s(z+1) = z-1$$

$$z(s-1) = -s-1$$

$$\Rightarrow z = \frac{1+s}{1-s}$$

- first pole:

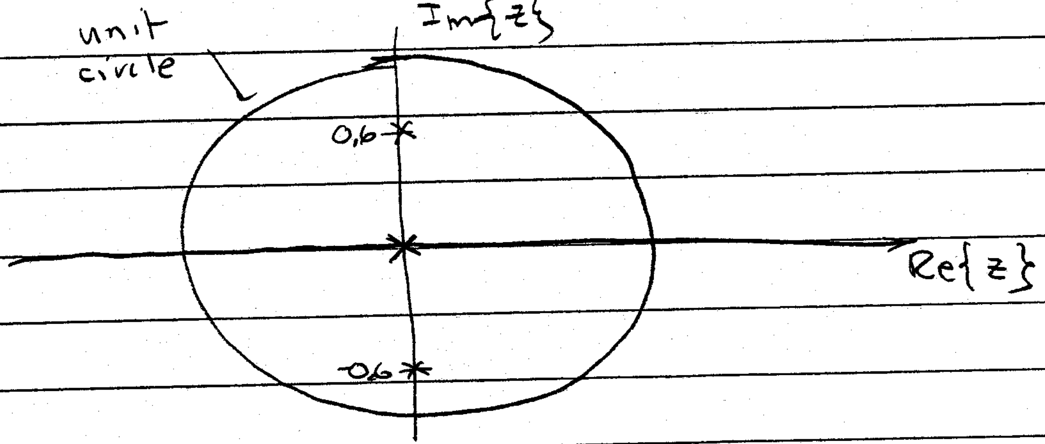
$$z_1 = \frac{1+s_1}{1-s_1} = \frac{1+e^{-j\frac{2\pi}{3}}}{1-e^{-j\frac{2\pi}{3}}} = \frac{e^{-j\frac{\pi}{3}}\, 2\cos\left(\frac{\pi}{3}\right)}{e^{-j\frac{\pi}{3}}\, 2j\sin\left(\frac{\pi}{3}\right)}$$

$$= -j\frac{1}{\tan\left(\frac{\pi}{3}\right)} \approx -0.6j$$

- second pole: $z_2 = \frac{1+s_2}{1-s_2} = \frac{1-1}{1+1} = 0$

- third pole:

$$z_3 = \frac{1+s_3}{1-s_3} = \frac{1+e^{j\frac{2\pi}{3}}}{1-e^{j\frac{2\pi}{3}}} = z_1^* = +0.6j$$



- poles are inside the unit circle so that the resulting causal IIR filter is stable

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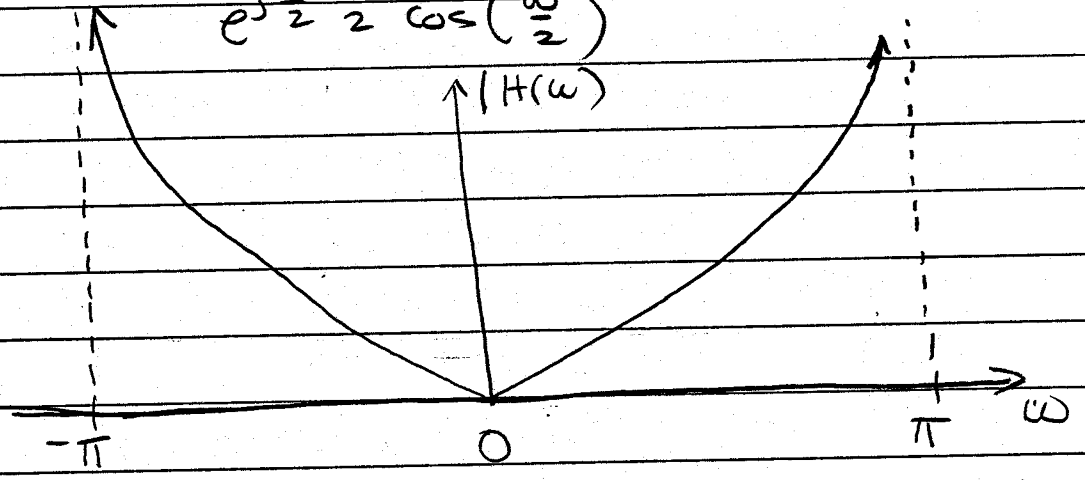
Solution to Prob. 4

$$H(z) = \frac{z-1}{z+1} \Rightarrow H(\omega) = \frac{e^{j\omega}-1}{e^{j\omega}+1}$$

• simplifying:

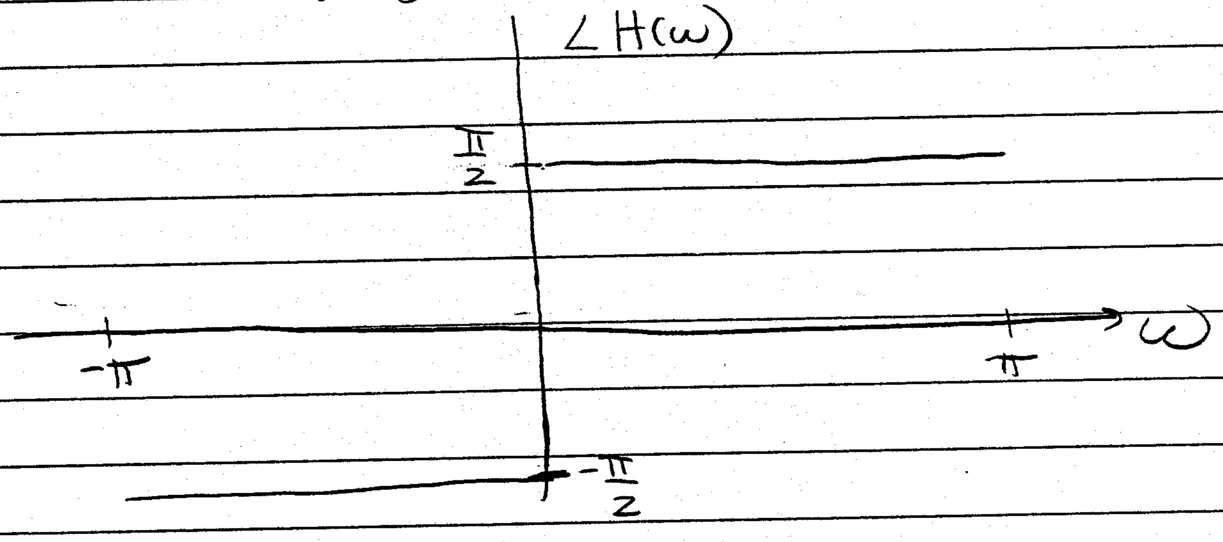
(a) $$H(\omega) = \frac{2e^{j\frac{\omega}{2}}\, 2j\sin\left(\frac{\omega}{2}\right)}{e^{j\frac{\omega}{2}}\, 2\cos\left(\frac{\omega}{2}\right)} = 2j\tan\left(\frac{\omega}{2}\right)$$

(b)



• system is not stable, pole on unit circle at $z=-1$
⇒ causes frequency response at $\omega=\pi$ to go to infinity

(c)



Yes, frequency response is purely imaginary

(d) for $\omega \ll 1$, $\sin\left(\frac{\omega}{2}\right) \approx \frac{\omega}{2}$ and $\cos\left(\frac{\omega}{2}\right) \approx 1$

· thus: $H(\omega) \approx j\omega$ for $\omega \ll 1$

Solution to Problem 5

$w(n) = w_1(n) + w_2(n)$

$$w_1(n) = \left\{ \frac{1}{2} - \frac{1}{2}\cos\left(\frac{4\pi}{M}(n+.5)\right)\right\}\left\{u(n) - u(n-M)\right\}$$

$$\text{DTFT}\left\{ u(n) - u(n-M)\right\} = e^{-j\omega\frac{(M-1)}{2}} \frac{\sin\left(\frac{M}{2}\omega\right)}{\sin\left(\frac{1}{2}\omega\right)}$$

and:

$$\cos(\omega_0 n + \theta)\, x(n) \overset{\text{DTFT}}{\longleftrightarrow} \frac{e^{j\theta}}{2} X(\omega - \omega_0) + \frac{e^{-j\theta}}{2} X(\omega + \omega_0)$$

note: $-\frac{1}{2} \cdot \frac{1}{2} e^{j\frac{2\pi}{M}} \cdot e^{-j\frac{(M-1)}{2}\left(-\frac{4\pi}{M}\right)}$

$$= -\frac{1}{4} e^{j\frac{2\pi}{M}} e^{j2\pi} e^{-j\frac{2\pi}{M}} = -\frac{1}{4}$$

So $$\text{DTFT}\{w_1(n)\} = e^{-j\frac{(M-1)}{2}\omega}\left\{ \frac{1}{2}\frac{\sin\left(\frac{M}{2}\omega\right)}{\sin\left(\frac{1}{2}\omega\right)} - \frac{1}{4}\frac{\sin\left(\frac{M}{2}\left(\omega - \frac{4\pi}{M}\right)\right)}{\sin\left(\frac{1}{2}\left(\omega - \frac{4\pi}{M}\right)\right)} - \frac{1}{4}\frac{\sin\left(\frac{M}{2}\left(\omega + \frac{4\pi}{M}\right)\right)}{\sin\left(\frac{1}{2}\left(\omega + \frac{4\pi}{M}\right)\right)}\right\}$$

i	A_i	L_i	ω_i
1	1/2	M	0
2	-1/4	M	$4\pi/M$
3	-1/4	M	$-4\pi/M$

second part:

$$\text{DTFT}\left\{\underbrace{\left[\frac{1}{2} + \frac{1}{2}\cos\left[\frac{4\pi}{M}(n+.5)\right]\right]\left[u\left(n-\frac{M}{4}\right) - u\left(n - \frac{3M}{4}\right)\right]}_{\triangleq\, w_2(n)}\right\} = ?$$

$$w_3(n) = w_2\left(n + \frac{M}{4}\right) = \left\{\frac{1}{2} - \frac{1}{2}\cos\left[\frac{4\pi}{M}(n+.5)\right]\right\}\left\{u(n) - u\left(n - \frac{M}{2}\right)\right\}$$

since $\cos\left[\frac{4\pi}{M}\left(n + \frac{M}{4} + .5\right)\right] = \cos\left[\frac{4\pi}{M}(n+.5) + \pi\right]$

Sol'n. to Prob. 5 (cont.)

$$\mathrm{DTFT}\{w_3(n)\} = \mathrm{DTFT}\left\{ \underbrace{\left[\frac{1}{2} - \frac{1}{2}\cos\left[\frac{2\pi}{M/2}(n+.5)\right]\right]\left[u(n) - u\left(n-\frac{M}{2}\right)\right]}_{\text{Simply a Hanning window of length } \frac{M}{2}} \right\} = ?$$

$\Rightarrow$ done in class for length M

Hence:

$$\mathrm{DTFT}\{w_3(n)\} = e^{-j\frac{\left(\frac{M}{2}-1\right)}{2}\omega}\left\{\frac{1}{2}\frac{\sin\left(\frac{M}{4}\omega\right)}{\sin\left(\frac{1}{2}\omega\right)} + \frac{1}{4}\frac{\sin\left(\frac{M}{4}\left(\omega-\frac{4\pi}{M}\right)\right)}{\sin\left(\frac{1}{2}\left(\omega-\frac{4\pi}{M}\right)\right)} + \frac{1}{4}\frac{\sin\left(\frac{M}{4}\left(\omega+\frac{4\pi}{M}\right)\right)}{\sin\left(\frac{1}{2}\left(\omega+\frac{4\pi}{M}\right)\right)}\right\}$$

- now: $\mathrm{DTFT}\{w_2(n)\} = \mathrm{DTFT}\left\{w_3\left(n-\frac{M}{4}\right)\right\}$

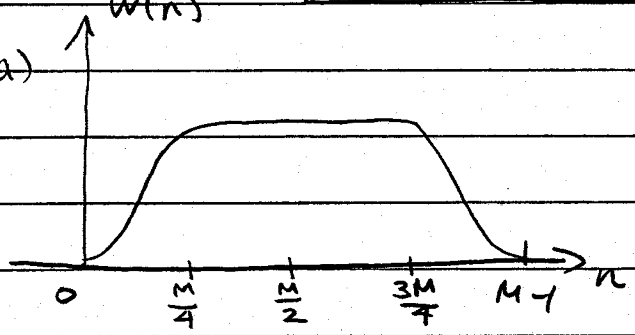
$$= e^{-j\frac{M}{4}\omega}\, W_3(\omega)$$

- note: $e^{-j\frac{\left(\frac{M}{2}-1\right)}{2}\omega} \cdot e^{-j\frac{M}{4}\omega} = e^{-j\left(\frac{M-1}{2}\right)\omega}$

- Thus:

i	A_i	L_i	ω_i
4	1/2	M/2	0
5	1/4	M/2	$4\pi/M$
6	1/4	M/2	$-4\pi/M$

(a)



$$\cos\left[\frac{4\pi}{M}(M-1-n+.5)\right]$$

$$= \cos\left[-\frac{4\pi}{M}(n+.5) + 4\pi\right]$$

$$= \cos\left[\frac{4\pi}{M}(n+.5)\right]$$