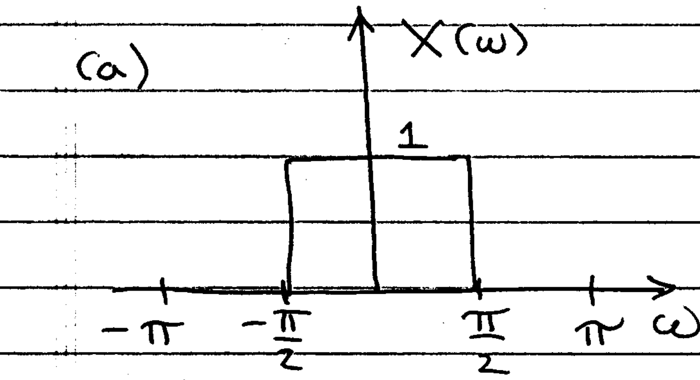
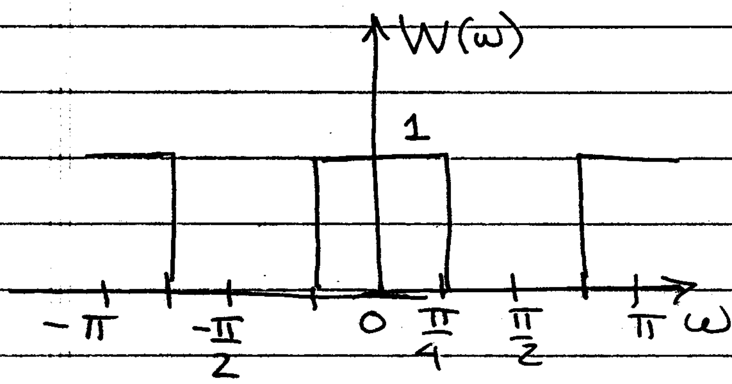
EE538 DSP I EXAM 2 Fall 1999

Solution to Problem 1

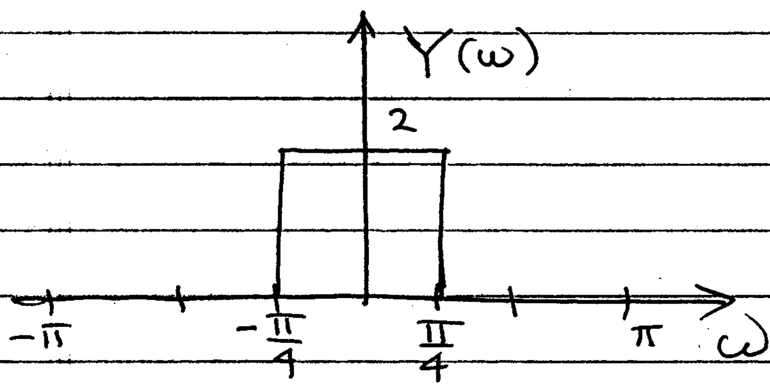
(a)



F_s = twice Nyquist



$\cancel{W(\omega) = X(2\omega)}$



(b)

$$y[n] = 2\,\frac{\sin\left(\frac{\pi}{4}n\right)}{\pi n}$$

$$= \frac{\sin\left(\frac{\pi}{2}\,\frac{n}{2}\right)}{\pi\,\frac{n}{2}}$$

$$= x\left[\frac{n}{2}\right] \qquad -\infty < n < \infty$$

Solution to Prob. 1 (cont.)

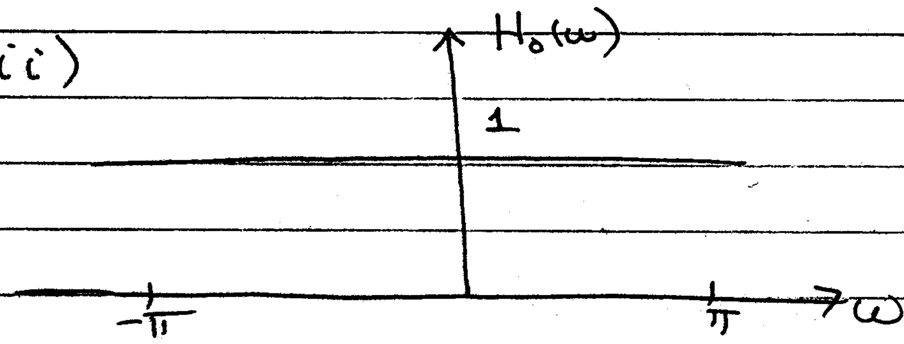
(c) $h_0[n] = h_{LP}[2n]$

(i)
$$= \frac{\sin\left(\frac{\pi}{2}2n\right)}{\frac{\pi}{2}2n} \cdot \frac{\cos\left(\frac{\pi}{4}2n\right)}{1-\frac{(2n)^2}{4}}$$

$$= \frac{\sin(\pi n)}{\pi n} \frac{\cos\left(\frac{\pi}{2}n\right)}{1-n^2}$$

$$= \delta[n]$$

(ii)



(iii) $y_0[n] = x[n] * h_0[n]$

$= x[n] * \delta[n]$

$= x[n]$ YES

(iv) Don't need to do the convolution of $x[n]$ with $h_0[n]$ since $h_0[n] = \delta[n]$

Need only do convolution of $x[n]$ with $h_1[n]$

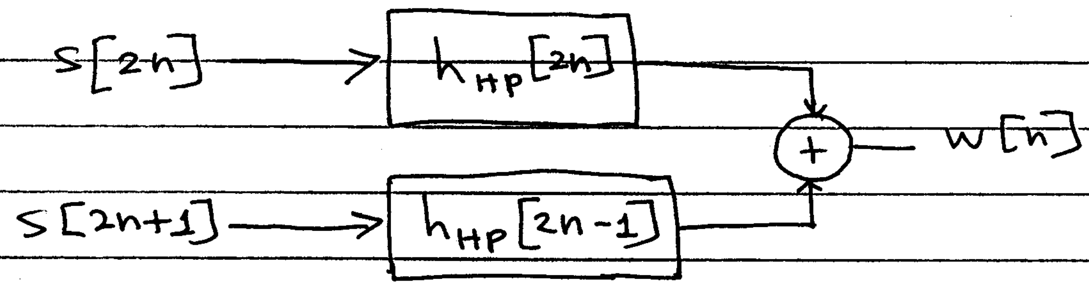
This makes sense since we're upsampling by a factor of 2, $y[n] = x_a\left(n\frac{T_s}{2}\right)$ where $T_s = \frac{1}{4W}$

We know $y_0[n] = y[2n] = x_a\left(2n\frac{T_s}{2}\right) = x[n]$

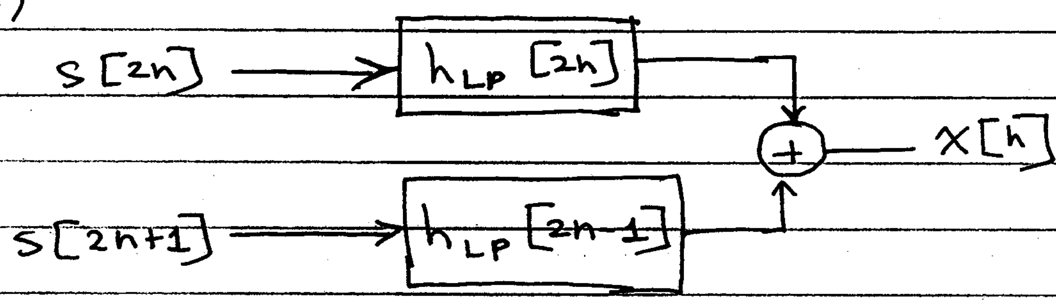
It's the "in-between" values $y_1[n] = y[2n+1]$ $= x_a\left(nT_s + \frac{T_s}{2}\right)$ that we need to compute

Solution to Problem 2

(a)



(b)



Solution to Problem 3

(a) $$\omega_{p0} = 2\pi \frac{20K}{50K} = \frac{4\pi}{5}$$

Stage 1

$$\omega_{p1} = \frac{\omega_{p0}}{4} = \frac{\pi}{5}$$

$$\omega_{s1} = \frac{2\pi}{4} - \omega_{p1} = \cancel{\frac{4\pi}{5}}\ \frac{\pi}{2} - \frac{\pi}{5} =$$

$$= \frac{5\pi}{10} - \frac{2\pi}{10} = \frac{3\pi}{10}$$

Stage 2

$$\omega_{p2} = \frac{\omega_{p1}}{4} = \frac{\pi}{20}$$

$$\omega_{s2} = \frac{2\pi}{4} - \omega_{p2} = \frac{\pi}{2} - \frac{\pi}{20} = \frac{10\pi}{20} - \frac{\pi}{20}$$

$$= 19\pi/20$$

EE538 DSP I Exam 2 Fall 1999

Solution to Prob. 4

4

(a) choose $c=1$ $\Rightarrow$ then do pre-warping

$$\Omega_c = \tan\left(\frac{\omega_c}{2}\right) = \tan\left(\frac{1}{2}\,\frac{\pi}{2}\right) = \tan\left(\frac{\pi}{4}\right) = 1$$

(b)
$$H(z) = H_a(s)\Big|_{s=\frac{z-1}{z+1}} = \frac{1}{s+1}\Big|_{s=\frac{z-1}{z+1}}$$

$$= \frac{1}{\frac{z-1}{z+1}+1} = \frac{z+1}{z-1+z+1} = \frac{z+1}{2z}$$

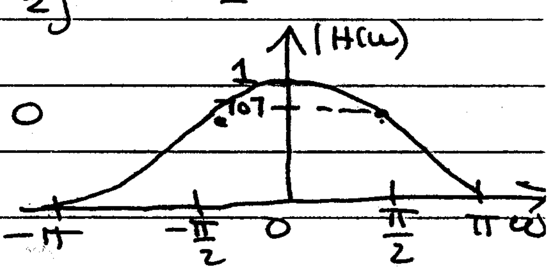
$$= \frac{1+z^{-1}}{2}$$

(c) Yes $\Rightarrow$ bilinear transform maps stable Butterworth analog filter to stable digital filter

(d)
$$\underbrace{H(0)}_{H(\omega)|_{\omega=0}} = \frac{z+1}{2z}\Big|_{z=1} = \frac{1+1}{2} = 1$$

$$H(\omega)\Big|_{\omega=\frac{\pi}{2}} = H(z)\Big|_{z=e^{j\frac{\pi}{2}}=j} = \frac{j+1}{2j} = \frac{1-j}{2} = \frac{1}{\sqrt{2}}e^{-j\frac{\pi}{4}}$$

$$H(\omega)\Big|_{\omega=\pi} = H(z)\Big|_{z=-1} = \frac{-1+1}{2(-1)} = 0$$



(e)
$$y[n] = \frac{1}{2}x[n] + \frac{1}{2}x[n-1]$$