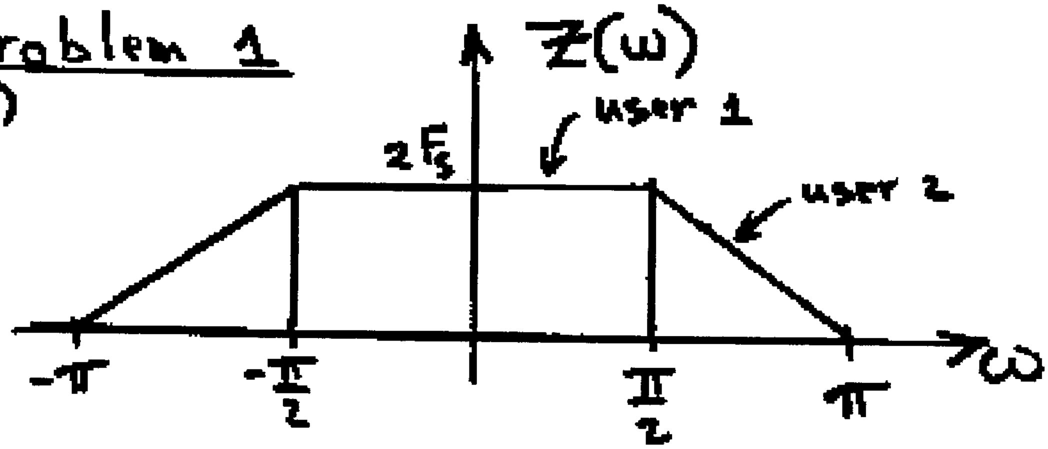
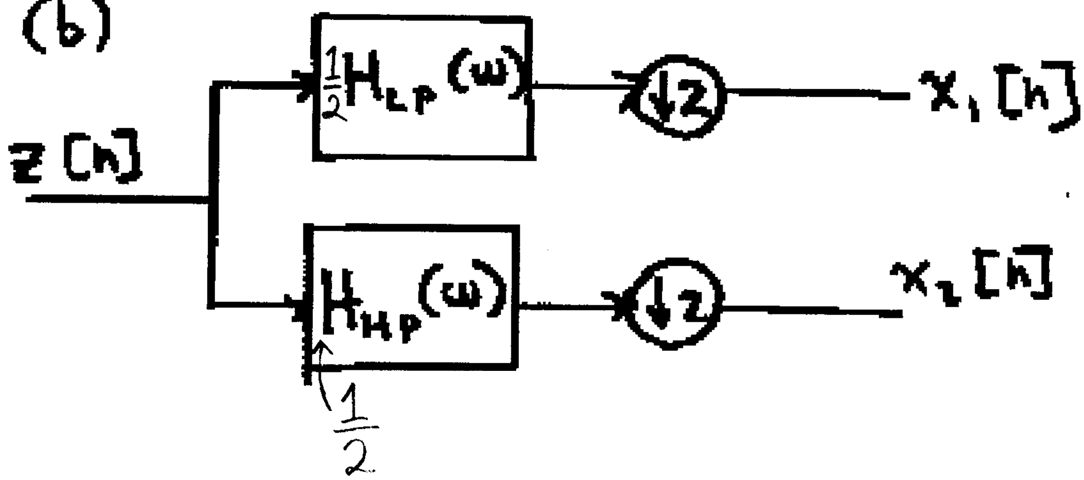
Sol'n. to Exam 2

Problem 1

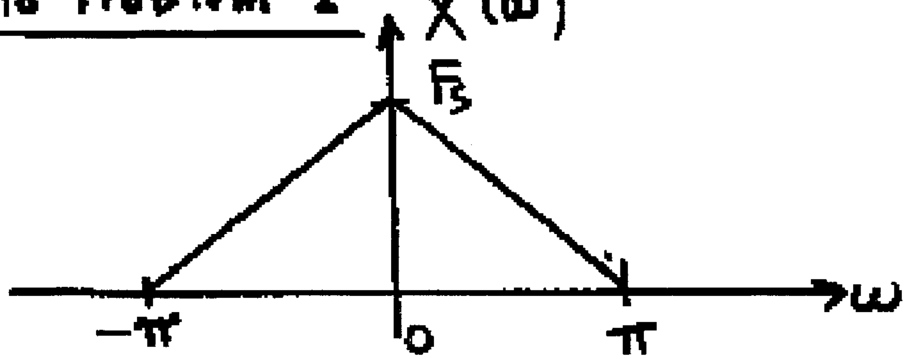
(a)



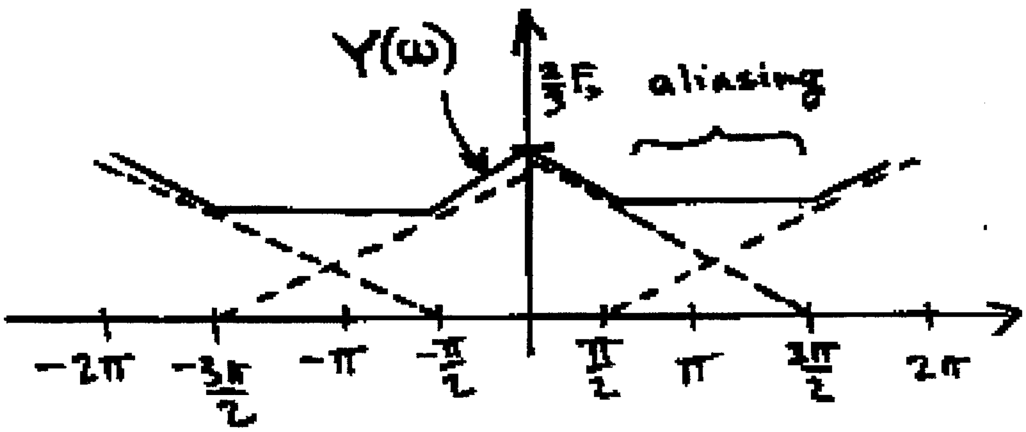
(b)



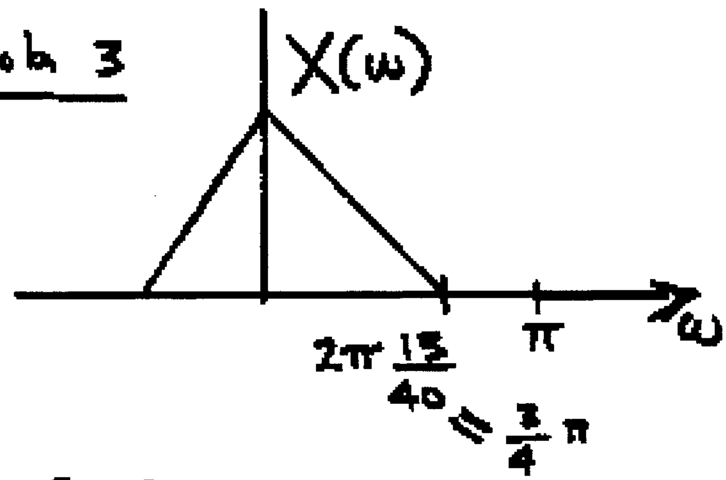
Soln. to Problem 2



• new sampling rate: $F_{S_{New}} = \frac{2}{3} F_{S_{old}}$



<u>Sol'n. to Prob 3</u>



(a) $\omega_{p1} = \frac{1}{2}\left(\frac{3}{4}\pi\right) = \frac{3}{8}\pi$

$\omega_{s1} = \pi - \frac{3}{8}\pi = \frac{5}{8}\pi$

(b) $\omega_{p2} = \frac{1}{2}\left(\frac{3}{8}\pi\right) = \frac{3}{16}\pi$

$\omega_{s2} = \pi - \frac{3}{16}\pi = \frac{13}{16}\pi$

Sol'n. to Prob. 4

(a) $\Omega_c = c \tan\left(\frac{\omega_c}{2}\right)$

$$1 = c \tan\left(\frac{1}{2}\,\frac{2\pi}{5}\right)$$

$$= c \tan\left(\frac{\pi}{5}\right) \approx c\,\frac{1}{\sqrt{2}}$$

$$\Rightarrow c = \sqrt{2}$$

(b) $$H(z) = \left.\frac{1}{s^2 + \sqrt{2}\,s + 1}\right|_{s = \sqrt{2}\,\frac{z-1}{z+1}}$$

$$= \frac{1}{2\,\frac{(z-1)^2}{(z+1)^2} + 2\,\frac{z-1}{z+1} + 1} \cdot \frac{(z+1)^2}{(z+1)^2}$$

$$H(z) = \frac{(z+1)^2}{2(z-1)^2 + 2(z^2-1) + (z+1)^2}$$

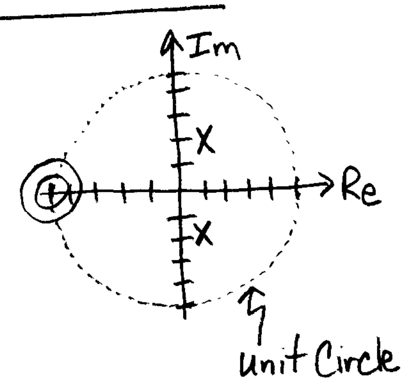
$$= \frac{(z+1)^2}{5z^2 - 2z + 1}$$

(c) zeroes: at $z = -1$ (Double zero)

poles: $5z^2 - 2z + 1 = 5\left(z^2 - \frac{2}{5}z + \frac{1}{5}\right)$

poles at $\dfrac{-\left(\frac{-2}{5}\right) \pm \sqrt{\left(\frac{-2}{5}\right)^2 - 4\cdot 1\cdot\left(\frac{1}{5}\right)}}{2}$

$= \frac{1}{5} \pm j\frac{2}{5}$



The filter is causal. The poles are inside the unit circle. $\Rightarrow$ Stable.

(d) $H(z) = \dfrac{Y(z)}{X(z)} = \dfrac{z^2 + 2z + 1}{5z^2 - 2z + 1} = \dfrac{1 + 2z^{-1} + z^{-2}}{5 - 2z^{-1} + z^{-2}}$

$$Y(z)\left(5 - 2z^{-1} + z^{-2}\right) = X(z)\left(1 + 2z^{-1} + z^{-2}\right)$$

$$5y[n] - 2y[n-1] + y[n-2] = x[n] + 2x[n-1] + x[n-2]$$

$$\Rightarrow y[n] = \frac{1}{5}\left\{2y[n-1] - y[n-2] + x[n] + 2x[n-1] + x[n-2]\right\}$$