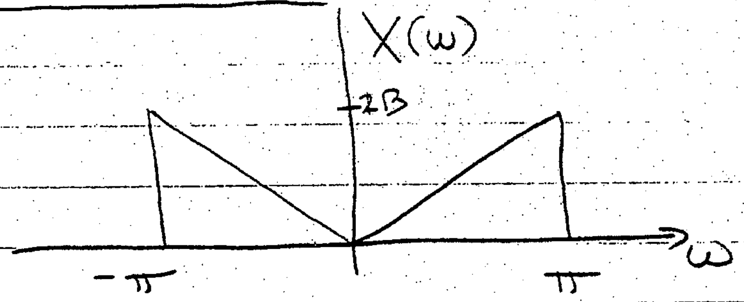
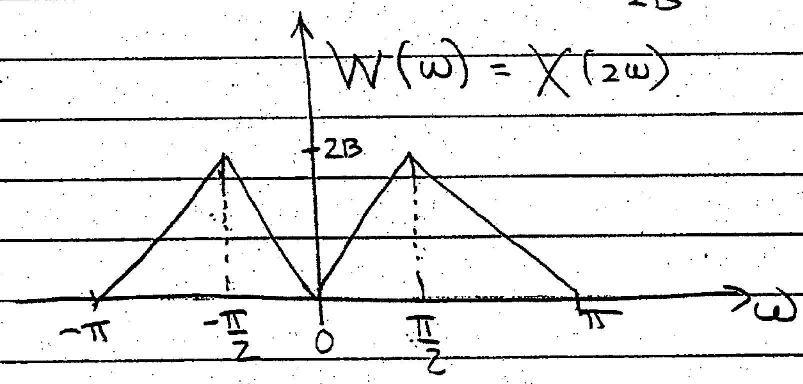
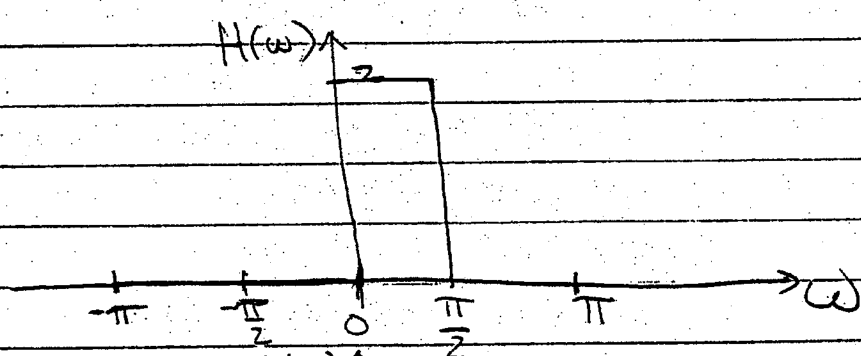
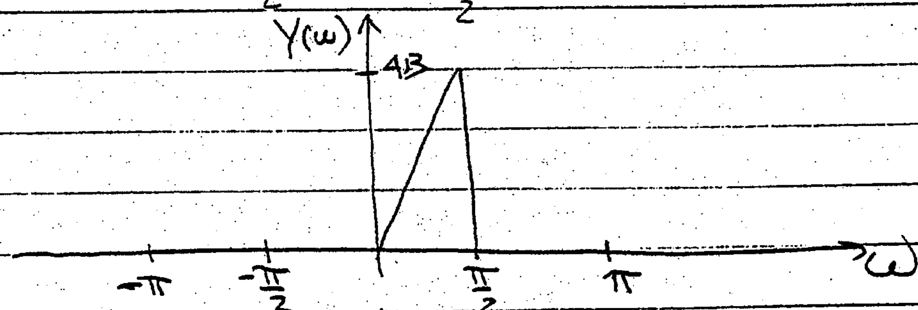
Sol'n. to Prob. 1

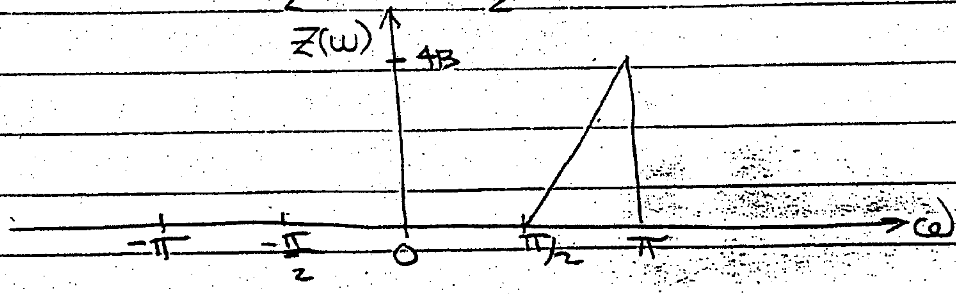


$= 2\pi \frac{B}{2B} = \pi$





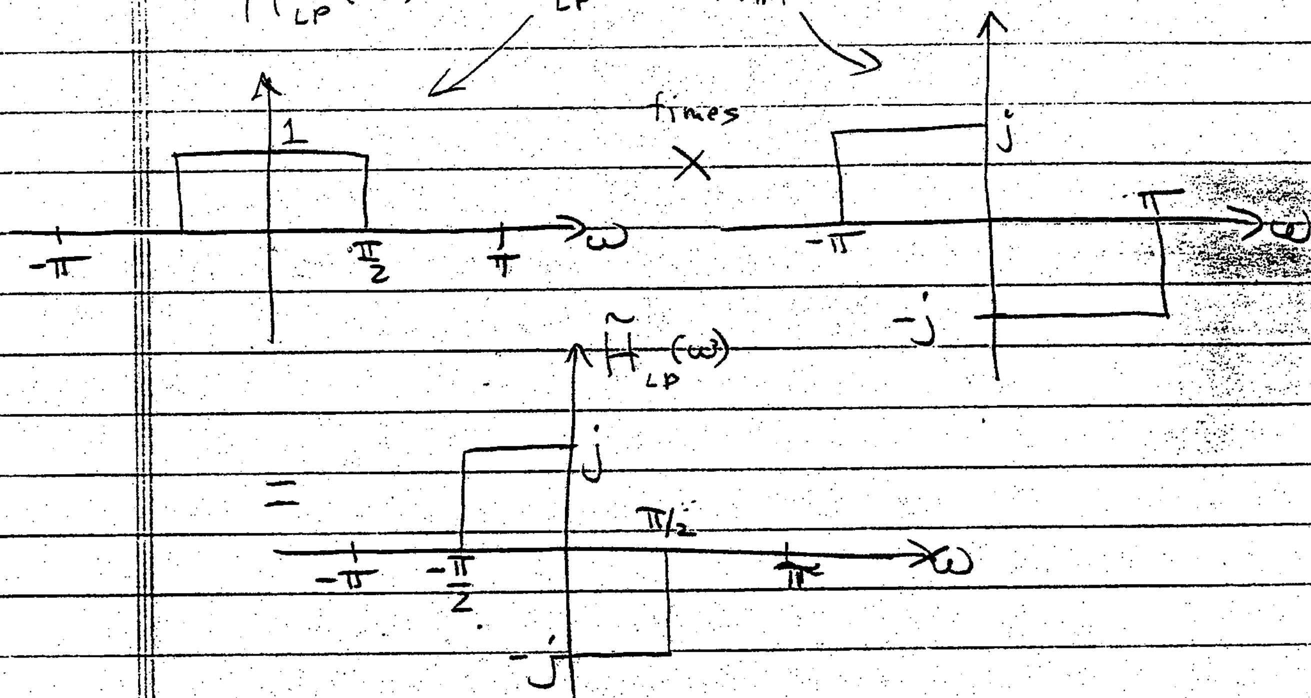




Sol'n. to Problem 1 (b)

$$\tilde{h}_{LP}(n) = h_{LP}(n) * h_{HT}(n)$$

$$\tilde{H}_{LP}(\omega) = H_{LP}(\omega) \cdot H_{HT}(\omega)$$



using the sine modulation property:

$$x(n)\sin(\omega_0 n) \xleftrightarrow{\text{DTFT}} \frac{1}{2j}X(\omega-\omega_0) - \frac{1}{2j}X(\omega+\omega_0)$$

$$\left.\begin{aligned} \tilde{h}_{LP}(n) &= 2\frac{\sin\left(\frac{\pi}{4}n\right)}{\pi n}\sin\left(\frac{\pi}{4}n\right) \\ &= 2\frac{\sin^2\left(\frac{\pi}{4}n\right)}{\pi n} \end{aligned}\right\} \text{Answer}$$

Sol'n. to Prob. 2

(a) $x(n) = \cos\left(\underbrace{\tfrac{\pi}{2}}_{\frac{2\pi(2)}{8}} n\right)\{u(n) - u(n-8)\}$

$$\cos\left(\frac{2\pi(2)}{8}n\right) \xleftrightarrow[8]{\text{DFT}} 4\delta(k-2) + 4\delta(k-6) = X_8(k)$$

$k = 0, 1, \ldots, 8$

N= 8 pt. DFT of $h(n)$:

$$H_8(k) = \sum_{n=0}^{7} \left(\tfrac{1}{2}\right)^n e^{-j\frac{2\pi kn}{8}} = \sum_{n=0}^{7} \left(\tfrac{1}{2} e^{-j\frac{2\pi k}{8}}\right)^n$$

$$= \frac{1 - \frac{1}{2^8} e^{-j2\pi k}}{1 - \frac{1}{2} e^{-j\frac{2\pi k}{8}}} = \frac{1}{1 - \frac{1}{2} e^{-j\frac{\pi k}{4}}}$$

$k = 0, 1, \ldots, 7$

where: $1 - \frac{1}{2^8} \approx 1$

(b)

$$Y_8(k) = H_8(k) X_8(k)$$

$$= 4 \frac{1}{1 - \frac{1}{2} e^{-j\frac{\pi}{2}}} \delta(k-2) + 4 \frac{1}{1 - \frac{1}{2} e^{-j\frac{\pi 6}{4}}} \delta(k-6)$$

$$= \frac{8}{2+j} \delta(k-2) + \frac{8}{2-j} \delta(k-6)$$

$$= \frac{8}{\sqrt{5}} e^{-j53.1^\circ} \delta(k-2) + \frac{8}{\sqrt{5}} e^{+j53.1^\circ} \delta(k-6)$$

$$y_8(n) = \frac{2}{\sqrt{5}} \cos\left(\frac{\pi}{2} n - 53.1^\circ\right)\{u(n) - u(n-8)\}$$

Sol'n. to Prob. 2 (cont.)

(c) $x(n)$ is of length $L=8$
$h(n)$ is of length $M=8$

} linear convolution $y(n) = x(n) * h(n)$ is of length $M+L-1 = 15$

- from class, we know: $y_8(n) = \sum_{\ell=-\infty}^{\infty} y(n-\ell 8)\{u(n)-u(n-8)\}$

$$= y(n) + y(n+8) \quad n=0,1,\ldots,7$$

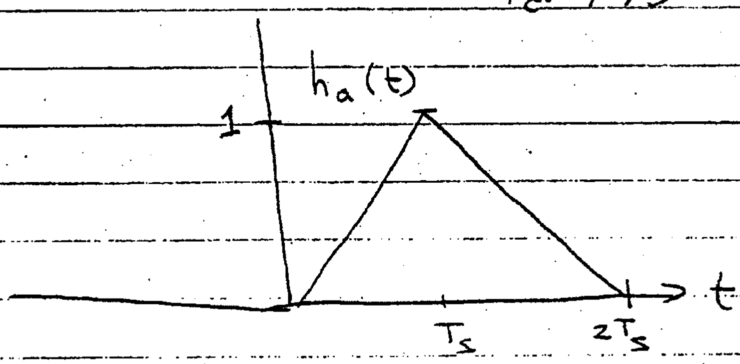
- since $y(n)$ is only nonzero for $n=0,1,\ldots,14$

thus: $y_8(7) = y(7) + y(15) = y(7)$ } only unaliased point

all other seven (7) points are aliased: $y_8(n) \neq y(n)$ for $n=0,1,\ldots,6$

Sol'n. to Problem 3

- Answer can be found on pg. 773 of Textbook



Sol'n. to Prob. 4

$$x(n) = e^{-anT_s} = \left(e^{-aT_s}\right)^n u(n)$$

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n} = \sum_{n=0}^{\infty} \left(e^{-aT_s}e^{-j\omega}\right)^n$$

$$\left.= \frac{1}{1 - e^{-(aT_s + j\omega)}}\right\} \text{answer}$$

$$= F_s \sum_{k=-\infty}^{\infty} \frac{1}{a + jF_s(\omega + k2\pi)}$$

Sol'n. to Prob. 5

- only feasible divide and conquer approach

$$N = 7 \cdot 3$$

- total no. of complex multiplications:

$$\underbrace{3(7)^2}_{3\ \text{7pt. DFT's}} + \underbrace{21}_{\text{intermediate mults}} + \underbrace{7(3)^2}_{7\ \text{3pt. DFT's}}$$

$$= 21(7+1+3) = 21 \cdot 11 = 231$$

Answer: 231 complex mults.