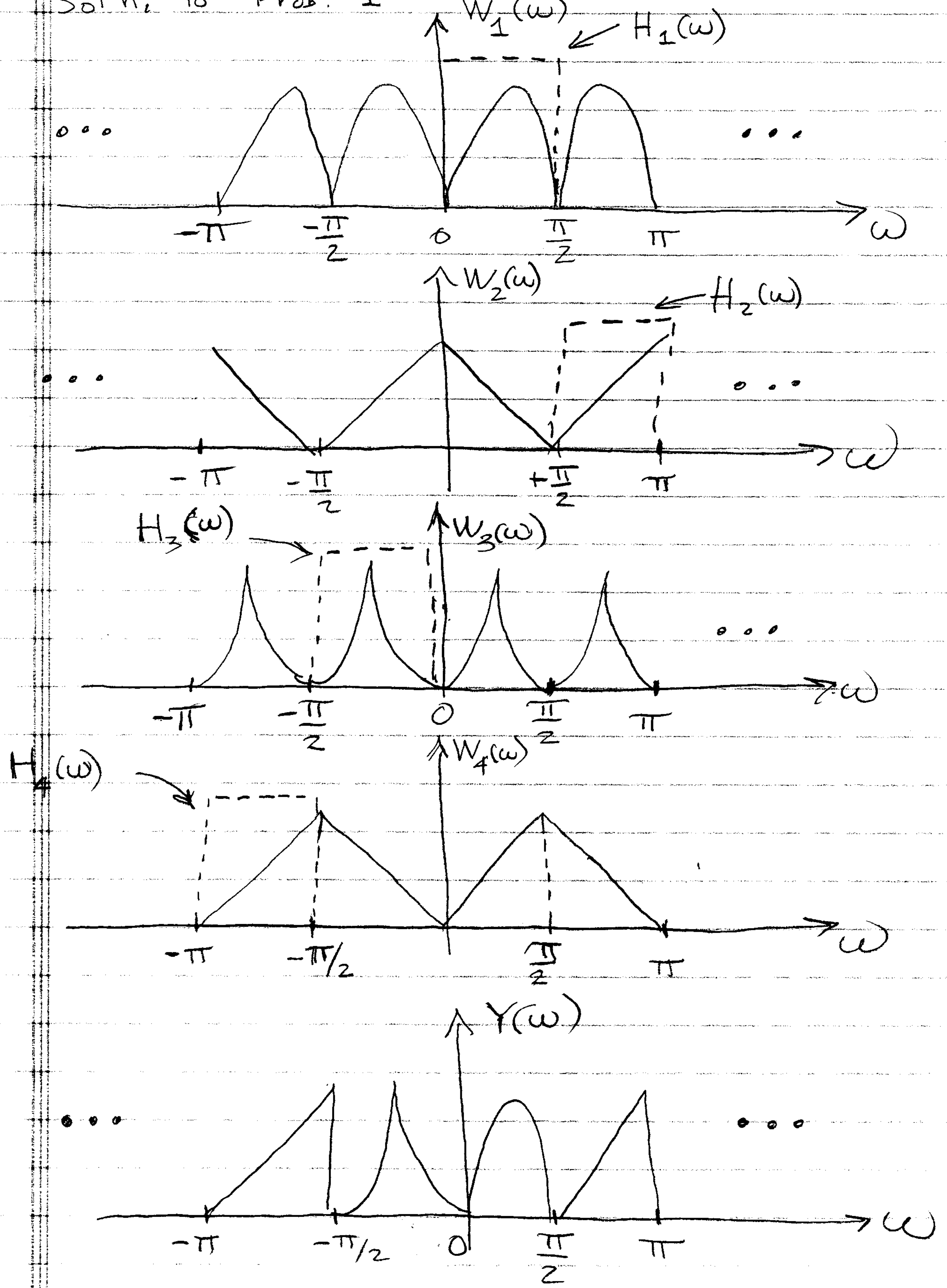
EE538 DSP I Sol'n to Exam 2 F'00

Sol'n. to Prob. 1



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Sol'n to Prob. 1 (cont.)

$$y[n] = \sum_{i=1}^{4} y_i[n]$$

where:

$$y_1[n] = x_{a1}\left(\frac{n}{4B}\right) + j\,\tilde{x}_{a1}\left(\frac{n}{4B}\right)$$

$$y_2[n] = \left\{x_{a2}\left(\frac{n}{4B}\right) - j\,\tilde{x}_{a2}\left(\frac{n}{4B}\right)\right\} e^{j\pi n}$$

$$y_3[n] = x_{a3}\left(\frac{n}{4B}\right) - j\,\tilde{x}_{a3}\left(\frac{n}{4B}\right)$$

$$y_4[n] = \left\{x_{a4}\left(\frac{n}{4B}\right) + j\,\tilde{x}_{a4}\left(\frac{n}{4B}\right)\right\} e^{-j\pi n}$$

These mathematical representations are not unique

Note:

$$\mathrm{Re}\{y_1[2n]\} = x_{a1}\left(\frac{2n}{4B}\right) = x_{a1}\left(\frac{n}{2B}\right) = x_1[n]$$

$$\mathrm{Re}\{y_2[2n]\} = \mathrm{Re}\left\{\left[x_{a2}\left(\frac{2n}{4B}\right) - j\,\tilde{x}_{a2}\left(\frac{2n}{4B}\right)\right] \underbrace{e^{j2\pi n}}_{=1\ \forall n}\right\}$$

$$= x_{a2}\left(\frac{n}{2B}\right) = x_2[n]$$

Similarly:

$$\mathrm{Re}\{y_3[2n]\} = x_3[n]$$

$$\mathrm{Re}\{y_4[2n]\} = x_4[n]$$

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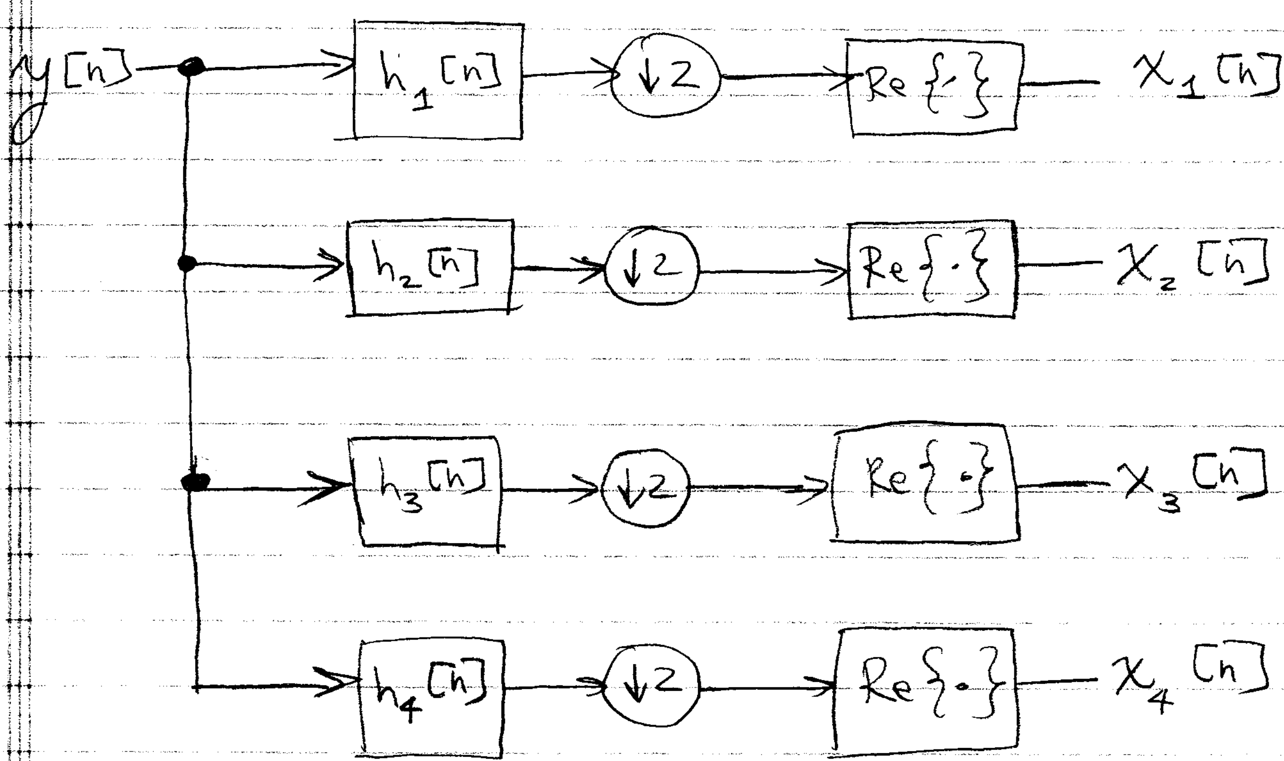
Sol'n to Prob. 1



(Of course, this is not computationally efficient.)

Sol'n. to Prob. 2

$$H_1(\omega) = \begin{cases} 1 + j(-j) = 2, & 0 < \omega < \pi \\ 1 + j(+j) = 0, & -\pi < \omega < 0 \end{cases}$$

$$H_2(\omega) = \begin{cases} 1 - j(-j) = 0, & 0 < \omega < \pi \\ 1 - j(j) = 2, & -\pi < \omega < 0 \end{cases}$$

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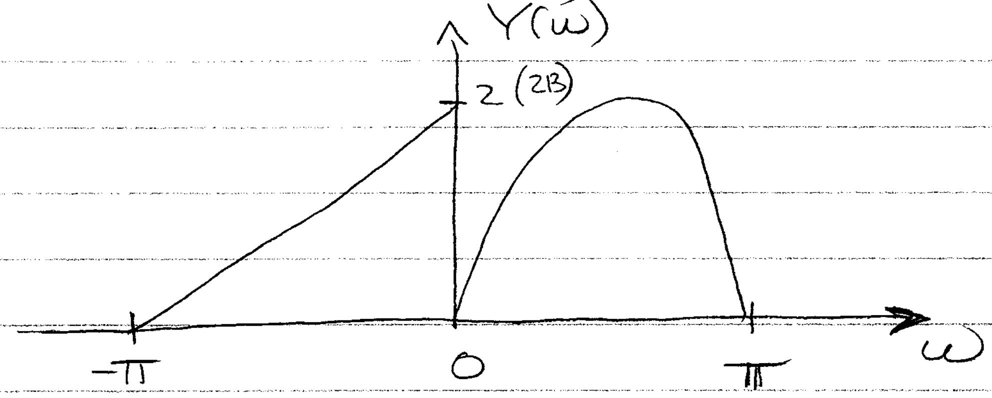
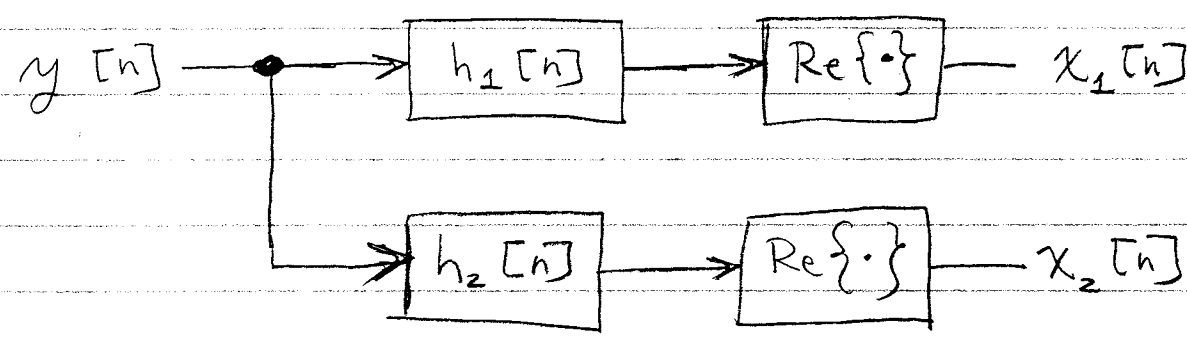
Sol'n. to Prob. 2 (cont.)

(a)



$$y[n] = x_1[n] + j\,\tilde{x}_1[n]$$
$$+ x_2[n] - j\,\tilde{x}_2[n]$$

(b)



- Alternatively, note: $h_{HT}[n] * h_{HT}[n] = -\delta[n]$
- Thus, if $\tilde{x}[n] = x[n] * h_{HT}[n]$

then $\tilde{x}[n] * h_{HT}[n] = x[n] * h_{HT}[n] * h_{HT}[n]$

$= -x[n]$

$$y_R[n] = \mathrm{Re}\{y[n]\} = x_1[n] + x_2[n]$$

$$y_I[n] = \mathrm{Im}\{y[n]\} = \tilde{x}_1[n] - \tilde{x}_2[n]$$
$$= (x_1[n] - x_2[n]) * h_{HT}[n]$$

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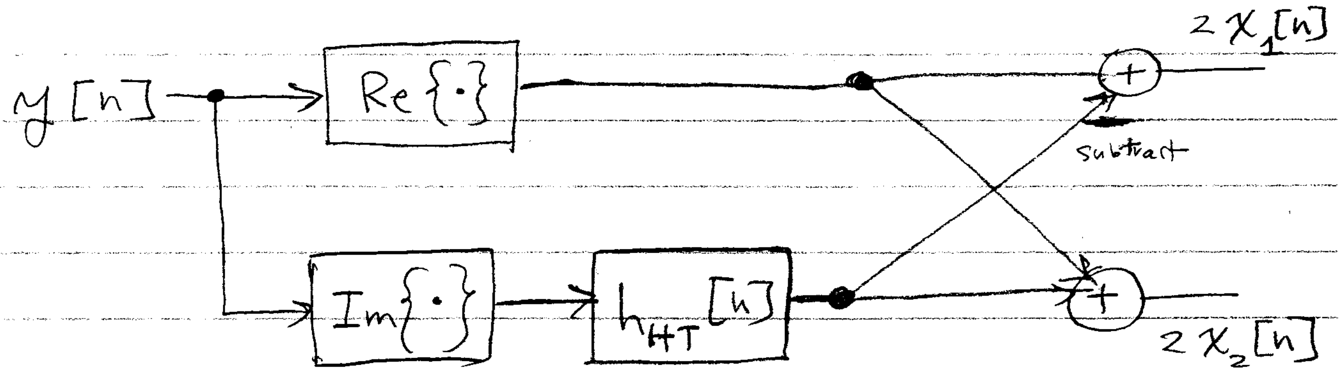
Sol'n to Prob. 2

- Alternative solution:

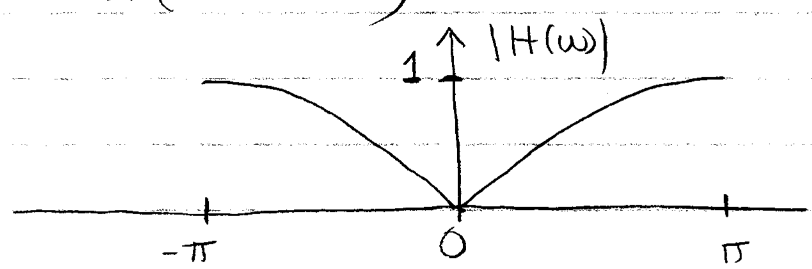


Sol'n to Prob. 3

(a) $$H(z) = \frac{1}{\frac{z+1}{z-1} + 1} = \frac{z-1}{2z}$$

$$= \frac{1}{2}\left\{1 - z^{-1}\right\}$$

$$H(\omega) = \frac{1}{2}\left\{1 - e^{-j\omega}\right\} = j \sin\left(\frac{\omega}{2}\right)$$



Highpass Filter

(b) Yes $\Rightarrow$ only 1 pole at $z=0$ $\Rightarrow$ stable!

(c) $$y[n] = \frac{1}{2}\left\{x[n] - x[n-1]\right\}$$