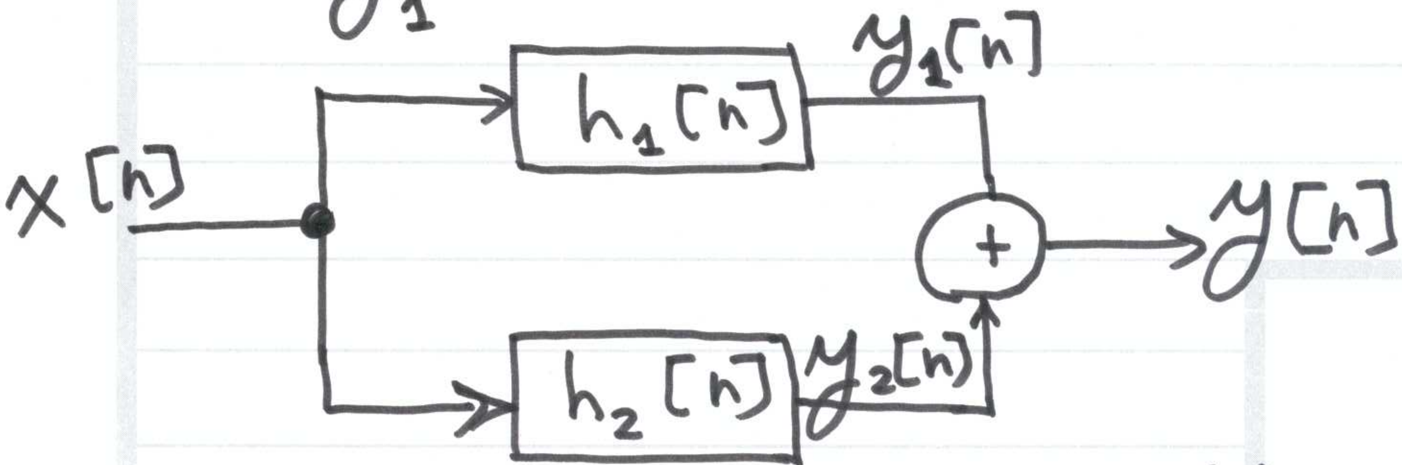
# Sol'n to Prob. 2 of Exam 1 :



$$y[n] = -.95\,y[n-1] - .9025\,y[n-2] + x[n] + x[n-1] + x[n-2]$$

- implement as

$$y_1[n] = a_1^{(1)} y_1[n-1] + b_0^{(1)} x[n] + b_1^{(1)} x[n-1]$$




$$y_2[n] = a_1^{(2)} y_2[n-1] + b_0^{(2)} x[n] + b_1^{(2)} x[n-1]$$

$$(z - |p| e^{j\theta})(z - |p| e^{-j\theta})$$

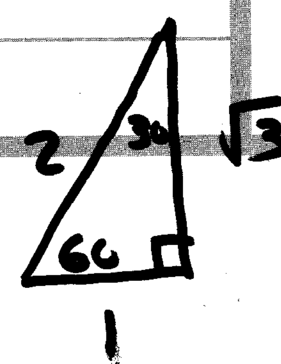
$$z^2 - 2|p|\cos\theta \, z + |p|^2$$

$$= z^2 + .95\, z + (.95)^2$$

$$\Rightarrow -2\cos\theta = 1$$

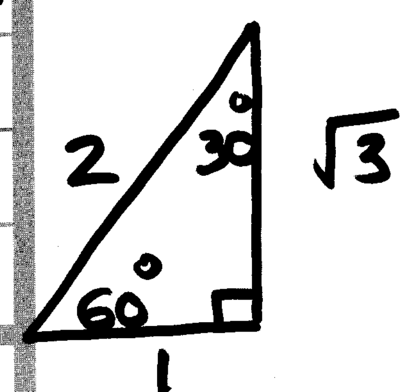
$$\cos\theta = -\frac{1}{2}$$

$$\theta = 120^\circ = \frac{2\pi}{3}$$




poles:

$$.95 e^{j\frac{2\pi}{3}}, \quad .95 e^{-j\frac{2\pi}{3}}$$



$$H(z) = \frac{z^2 + z + 1}{\left(z - .95e^{j\frac{2\pi}{3}}\right)\left(z - .95e^{-j\frac{2\pi}{3}}\right)}$$

$$= \frac{b_0 z + b_1}{z - P_1} + \frac{b_0^* z + b_1^*}{z - P_2}$$

$$\boxed{z_1 = -\frac{b_1}{b_0}}$$

$b_0$ real-valued

$$= \frac{b_0 (z - z_1)}{z - P_1} + \frac{b_0 (z - z_1^*)}{z - P_1^*}$$

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$$b_0(z - z_1)(z - P_1^*) + b_0(z - z_1^*)(z - P_1)$$

$$= z^2 + z + 1$$

choose $b_0 = \frac{1}{2}$

- equating coeff's of $z$ on both sides

$$-\frac{1}{2}\left\{z_1 + p_1^* + z_1^* + p_1\right\} = 1 \qquad \text{A}$$

- equating coeff's of $z^0$ on both sides:

$$\frac{1}{2}\left\{z_1 p_1^* + z_1^* p_1\right\} = 1 \qquad \text{B}$$

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$$\left.\begin{array}{l} \text{A} \quad \text{Real}\left\{z_1^* + p_1\right\} = -1 \\ \text{B} \quad \text{Real}\left\{z_1^* p_1\right\} = 1 \\ p_1 = .95e^{j\frac{2\pi}{3}} \quad \text{try: } z_1 = \frac{1}{.95}e^{j\frac{2\pi}{3}} \end{array}\right\}$$

satisfies B
check A

A $\text{Real}\left\{\frac{1}{.95}e^{-j\frac{2\pi}{3}} + .95e^{j\frac{2\pi}{3}}\right\} =$

$= \frac{1}{.95}\cos\left(\frac{2\pi}{3}\right) + .95\cos\left(\frac{2\pi}{3}\right) =$

$= -\frac{1}{2}\left\{\frac{1}{.95} + .95\right\} =$

$= -\frac{1}{2}\left\{1 + .05 + .95\right\}$

$= -1$

$\frac{1}{1-x} \approx 1 + x$

$\frac{1}{1-.05} \approx 1 + .05$

$$\frac{\frac{1}{2}\left(z - \frac{1}{.95}e^{j\frac{2\pi}{3}}\right)}{z - .95e^{j\frac{2\pi}{3}}} \Rightarrow H_1(z)$$

$$+ \frac{\frac{1}{2}\left(z - \frac{1}{.95}e^{-j\frac{2\pi}{3}}\right)}{z - .95e^{-j\frac{2\pi}{3}}} \Rightarrow H_2(z)$$

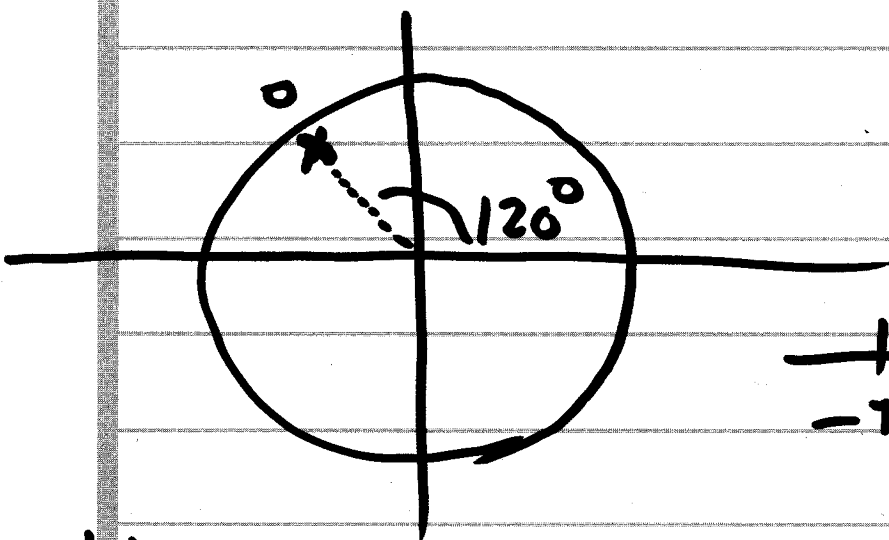
$$= \frac{z^2 + z + 1}{z^2 + .95z + .9025}$$

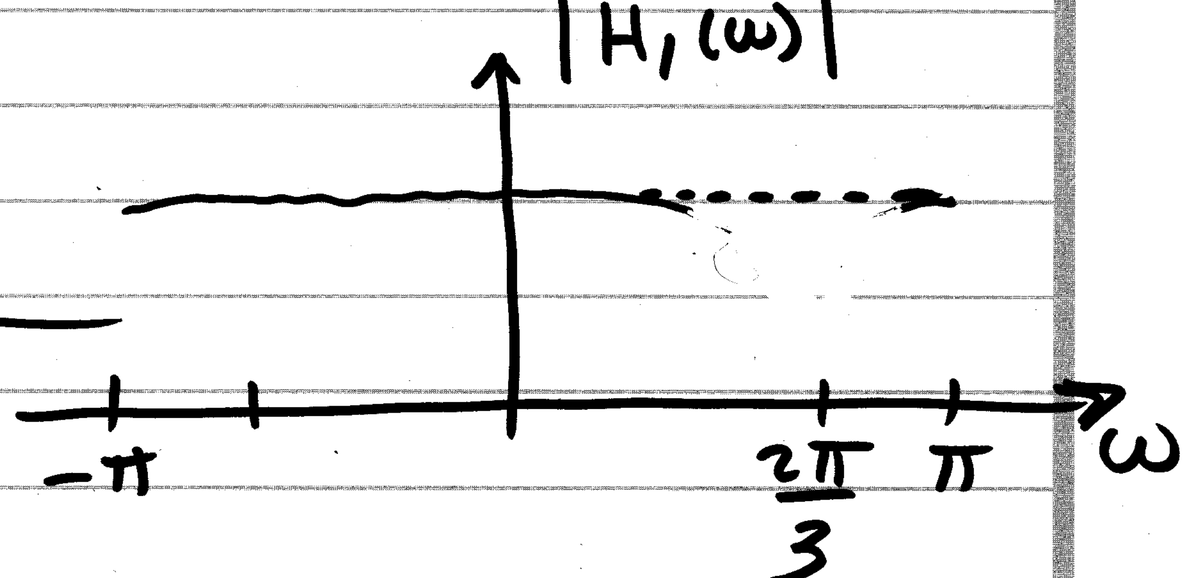
$$b_0^{(1)} = b_0^{(2)} = \frac{1}{2}$$

$$b_1^{(1)} = b_1^{(2)*} = -\frac{1}{2}\,\frac{1}{.95}e^{j\frac{2\pi}{3}}$$

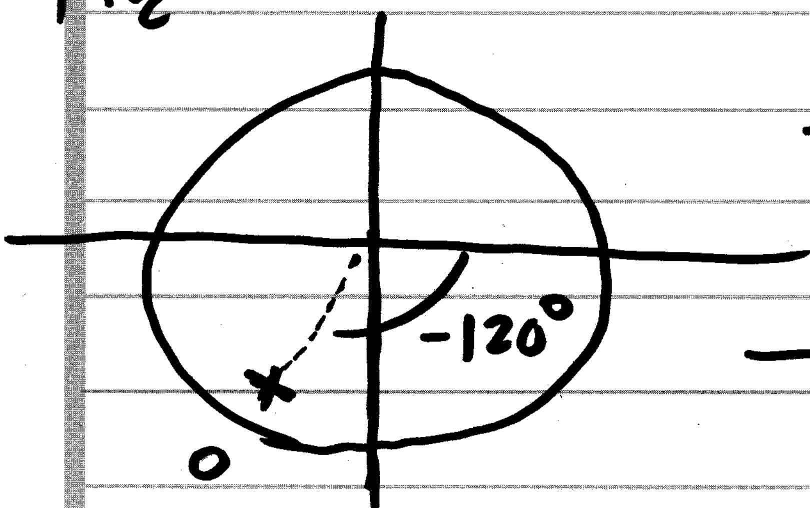
$$a_1^{(1)} = a_1^{(2)*} = .95e^{j2\pi/3}$$

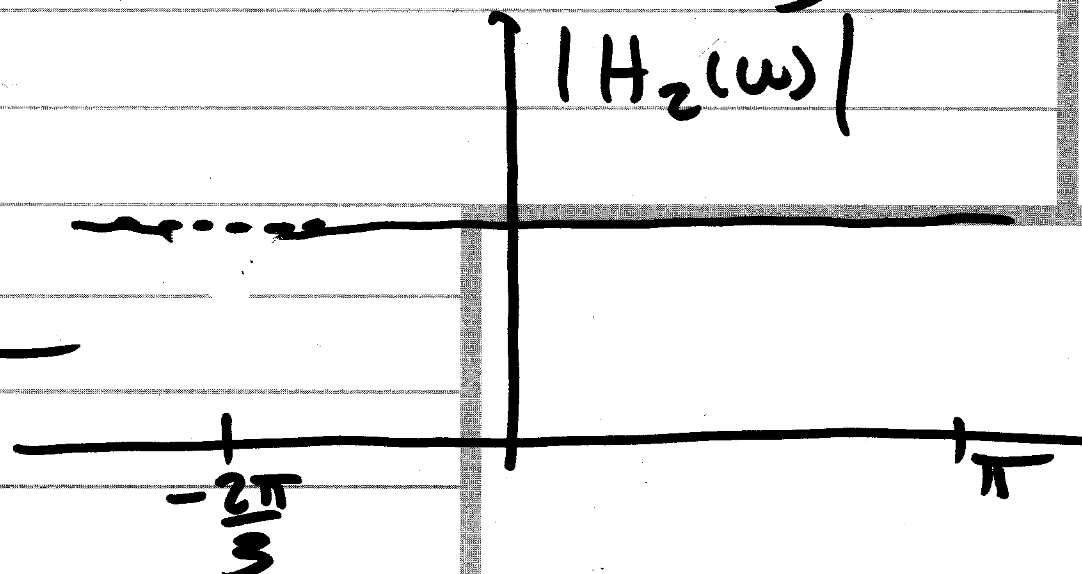
$H_1(z)$: ROC: $|z| > .95$




$H_2(z)$



- Correction to Sol'n to Prob. 2 of Exam 1
- "my" form of sol'n for each of the $1^{st}$-order systems was of the form:

$$H_i(z) = \frac{1}{2} \frac{\left(z - \frac{1}{|p|} e^{\pm j\theta}\right)}{\left(z - |p| e^{\pm j\theta}\right)}$$

- Consider:

$$H_{ap}(\omega) = H_{ap}(z)\Big|_{z=e^{j\omega}} = \left.\frac{\left(z - \frac{1}{|p|} e^{j\theta}\right)}{z - |p| e^{j\theta}}\right|_{z=e^{j\omega}}$$

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$$|H_{ap}(\omega)|^2 = \frac{\left(e^{j\omega} - \frac{1}{|p|}e^{j\theta}\right)}{\left(e^{j\omega} - |p|e^{j\theta}\right)} \cdot \frac{\left(e^{-j\omega} - \frac{1}{|p|}e^{-j\theta}\right)}{\left(e^{-j\omega} - |p|\, e^{-j\theta}\right)}$$

$$= \frac{1 - 2\frac{1}{|p|}\cos(\omega-\theta) + \frac{1}{|p|^2}}{1 - 2|p|\cos(\omega-\theta) + |p|^2}$$

$$= \frac{1}{|p|^2} \cdot \frac{|p|^2 - 2|p|\cos(\omega-\theta) + 1}{|p|^2 - 2|p|\cos(\omega-\theta) + 1}$$

$$\boxed{\begin{array}{l}\text{Sect.}\\ 4.5,6\end{array}}$$

$|H_{ap}(\omega)| = \frac{1}{|p|}$ for all $\omega$

$\Rightarrow$ all-pass filter