EE 538

Exam 1 Sol

(1)

(a) $h(n) = \delta(n) - \delta(n-2) = \{\underset{\uparrow}{1}, 0, -1\}$

$x(n) = \{\underset{\uparrow}{1}, 0, -1\} * \{\underset{\uparrow}{1}, 2, 1, 2, 1, 2, 1, 2, 1\}$

$= \{\underset{\uparrow}{1}, 2, 0, 0, 0, 0, 0, 0, 0, -2, -1\}$

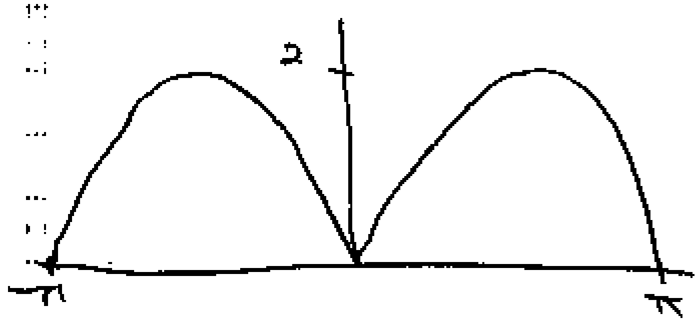
(b) $H(\omega) = 1 - e^{-j2\omega} = 1 - (\cos 2\omega - j\sin 2\omega)$

$|H(\omega)| = \sqrt{(1-\cos 2\omega)^2 + \sin^2 2\omega}$

$= \sqrt{1 - 2\cos 2\omega + \cos^2 2\omega + \sin^2 2\omega}$

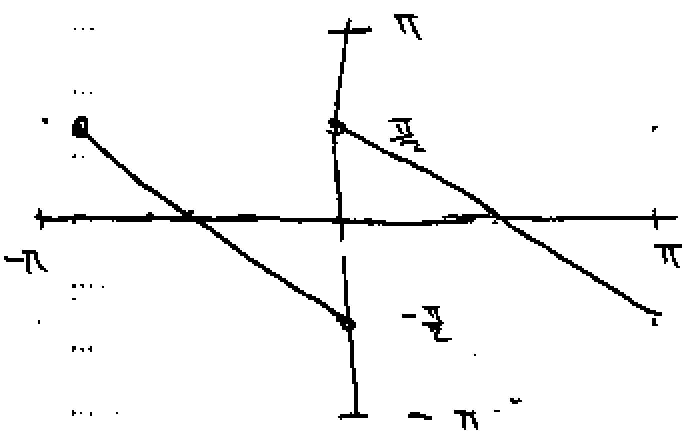
$= \sqrt{2 - 2\cos 2\omega} = \sqrt{4 - 4\cos^2 \omega} = \sqrt{4\sin^2 \omega}$

$= 2\,|\sin \omega|$



$H(\omega) = 1 - e^{-j2\omega} = e^{-j\omega}(e^{j\omega} - e^{-j\omega}) = 2j\, e^{-j\omega} \sin(\omega)$

$= e^{j(\frac{\pi}{2} - \omega)} \cdot 2\sin(\omega)$



$\angle H(\omega) = (\frac{\pi}{2} - \omega) \cdot \mathrm{sgn}(\sin(\omega))$

$= \begin{cases} \frac{\pi}{2} - \omega & \omega > 0 \\ -\frac{\pi}{2} + \omega + 2\pi & \omega < 0 \end{cases}$

$= \begin{cases} \frac{\pi}{2} - \omega & \omega > 0 \\ \frac{3\pi}{2} + \omega & \omega < 0 \end{cases}$

(c) $y(n) = x(n) - x(n-2)$

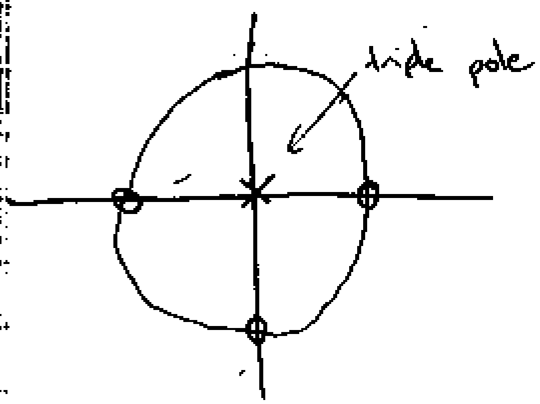
(a) $y(n) = j\, y(n-1) + x(n) - x(n-4)$

$$Y(z) = j z^{-1} Y(z) + X(z) + z^{-4} X(z)$$
$$Y(z)\left[1 - j z^{-1}\right] = X(z)\left[1 - z^{-4}\right]$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 - z^{-4}}{1 - j z^{-1}} = \frac{z^4 - 1}{z^4 - j z^3} = \frac{z^4 - 1}{z^3 (z - j)}$$

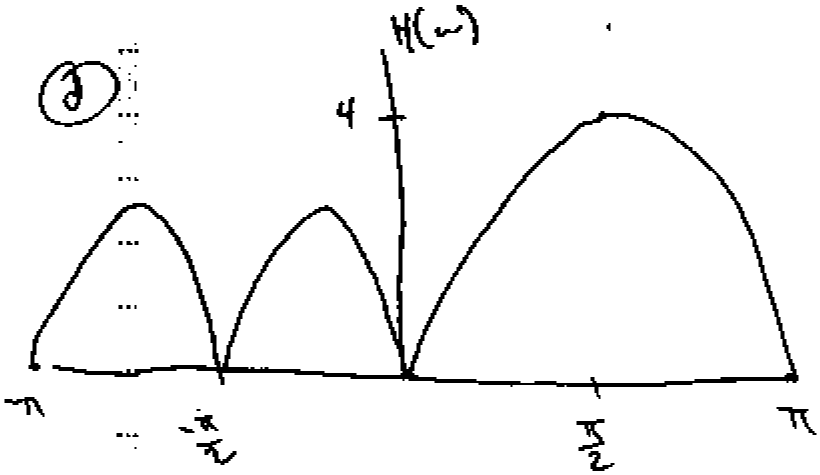
$$= \frac{(z+j)(z-1)(z+1)}{z^3}$$

(b)



(c) region of convergence $|z| > 0$
since ROC includes unit circle, yes — is BIBO stabl

(d)



② cont. ⓔ find $y(n)$ for $x(n) = 4 + \sin(\pi n)$ $\sin \pi n$ always $= 0$

Ans: DC component goes away, $H(0) = 0$
$\sin(\pi n) = 0$ for all n, so

$y(n) = 0$

ⓕ $H(z) = \frac{(z+j)(z+1)(z-1)}{z^3} = \frac{z^3 + jz^2 - z - j}{z^3}$

$= \boxed{1 + jz^{-1} - z^{-2} - jz^{-3}}$

Since the impulse response has finite length, system is $\boxed{\text{FIR}}$.

3 Let $H(z)$ be in form (notch filter)

$$H(z) = \frac{1 - 2\cos w_0 z^{-1} + z^{-2}}{1 - 2r\cos w_0 z^{-1} + r^2 z^{-2}}$$

where $f_0 = \frac{3}{9} = \frac{1}{3}$ & letting $r = .95$ for sharp notch

$w_0 = 2\pi f_0 = \frac{2}{3}\pi$

$$H(z) = \frac{1 + z^{-1} + z^{-2}}{1 + .95 z^{-1} + .9025 z^{-2}}$$

$$Y(z)\left[1 + .95 z^{-1} + .9025 z^{-2}\right] = X(z)\left[1 + z^{-1} + z^{-2}\right]$$

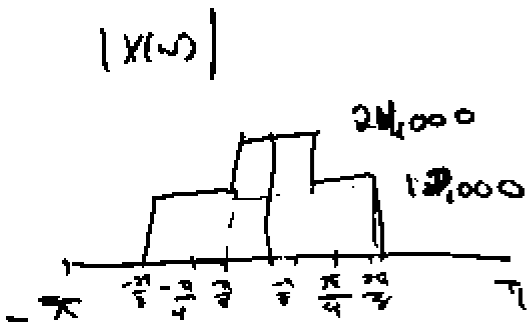
$$Y(n) = -.95\, y(n-1) - .9025\, y(n-2) + x(n) + x(n-1) + x(n-2)$$

$\therefore$

$$\boxed{\begin{aligned} a_1 &= .95 \\ a_2 &= .9025 \\ b_1 &= 1 \\ b_2 &= 1 \end{aligned}}$$

4. (a) $x(n) = x_a\left(\frac{n}{F_s}\right) = \frac{\sin\left(2\pi \cdot \frac{3}{24} n\right)}{\pi \frac{n}{24000}} \cos\left(2\pi \frac{1.5}{24} n\right)$

$$= 24{,}000 \frac{\sin(\frac{\pi}{4} n)}{\pi n} \cos\left(\frac{\pi}{8} n\right)$$



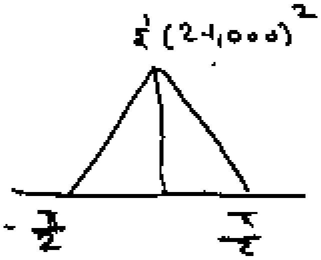
(b) $\frac{24{,}000}{\pi} = \frac{F_s}{\frac{3\pi}{8}}$ $\qquad F_s = 24{,}000 \cdot \frac{\frac{3\pi}{8}}{\pi}$

$$= \boxed{9{,}000 \text{ Hz}}$$

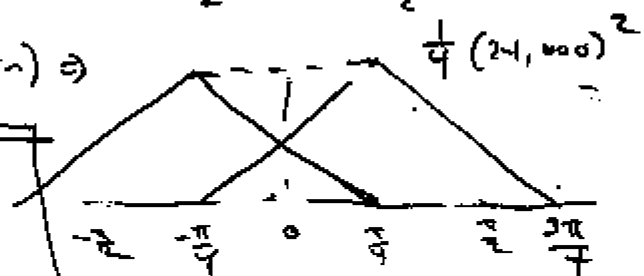
(c) $y(n) = x^2(n)$

$$= (24{,}000)^2 \frac{\sin^2(\frac{\pi}{4} n)}{(\pi n)^2} \left[\frac{1}{2} + \frac{1}{2} \cos \frac{\pi}{4} n\right]$$

$\frac{1}{2}(24{,}000)^2 \left(\frac{\sin(\frac{\pi}{4} n)}{\pi n}\right)^2 \Rightarrow$



$+ \quad \frac{1}{2}(24{,}000)^2 \left(\frac{\sin(\frac{\pi}{4} n)}{\pi n}\right)^2 \cos\left(\frac{\pi}{4} n\right) \Rightarrow$



Ans: