EE538 DSP I Exam 1 Fall 2000

Sol'n. to Prob. 1 : 1

- The fact that $c_1[n]$ is orthogonal to $c_2[n]$ implies $r_{c_1c_2}[0] = r_{c_2c_1}[0] = 0$
- In addition, $r_{c_1c_1}[\ell]$, $r_{c_2c_2}[\ell]$, $r_{c_1c_2}[\ell]$, $r_{c_2c_1}[\ell]$ are all 0 for $|\ell| > 3$
- Since $x[n] = \sum_{k=1}^{2} \sum_{m=0}^{1} b_k[m]\, c_k[n-4m]$

$$r_{xc_1}[\ell] = \sum_{k=1}^{2} \sum_{m=0}^{1} b_k[m]\, r_{c_kc_1}[\ell-4m]$$

- due to observations above and $r_{c_1c_1}[0] = r_{c_2c_2}[0] = 4$

$$r_{xc_1}[0] = 4\, b_1[0] \quad \text{and} \quad r_{xc_1}[4] = 4\, b_1[1]$$

- $r_{xc_1}[0] = \{0(1) + 0\cdot(-1) + 2(1) - 2(-1)\} = 4$

$$\Rightarrow \boxed{b_1[0] = 1}$$

- $r_{xc_1}[4] = \{2(1) + -2(-1) + 0(1) + 0(-1)\} = 4$

$$\Rightarrow \boxed{b_1[1] = 1}$$

- Similarly,

$$r_{xc_2}[0] = 4 b_2[0] \quad \text{and} \quad r_{xc_2}[4] = 4 b_2[1]$$

- $r_{xc_2}[0] = \{0(1) + 0(-1) + 2(-1) + -2(1)\} = -4$

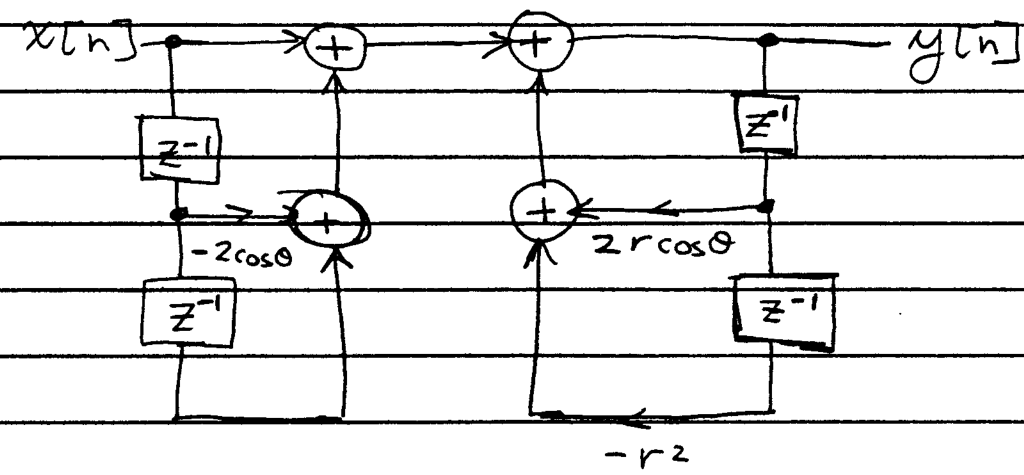
$$\Rightarrow \boxed{b_2[0] = -1}$$

- $r_{xc_2}[4] = \{2(1) + -2(-1) + 0(-1) + 0(1)\} = 4$

$$\Rightarrow \boxed{b_2[1] = 1}$$

Sol'n. to Prob. 2

- This Direct Form 2 realization can be alternatively realized in Direct Form 1 as



$$y[n] = 2r\cos\theta\, y[n-1] - r^2 y[n-2] + x[n] - 2\cos\theta\, x[n-1] + x[n-2]$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{z^2 - 2\cos\theta\, z + 1}{z^2 - 2r\cos\theta\, z + r^2}$$

$$= \frac{(z - e^{j\theta})(z - e^{-j\theta})}{(z - re^{j\theta})(z - re^{-j\theta})}$$

zeroes: $e^{j\theta}$, $e^{-j\theta}$

poles: $re^{j\theta}$, $re^{-j\theta}$

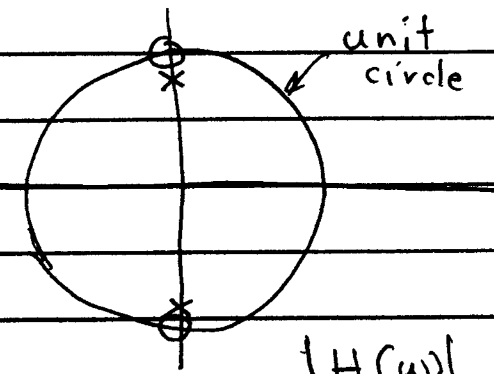
- for stability, require $r < 1$

(b) $r = .95$, $\theta = \frac{\pi}{2}$ $\Rightarrow$ zeroes: $j, -j$

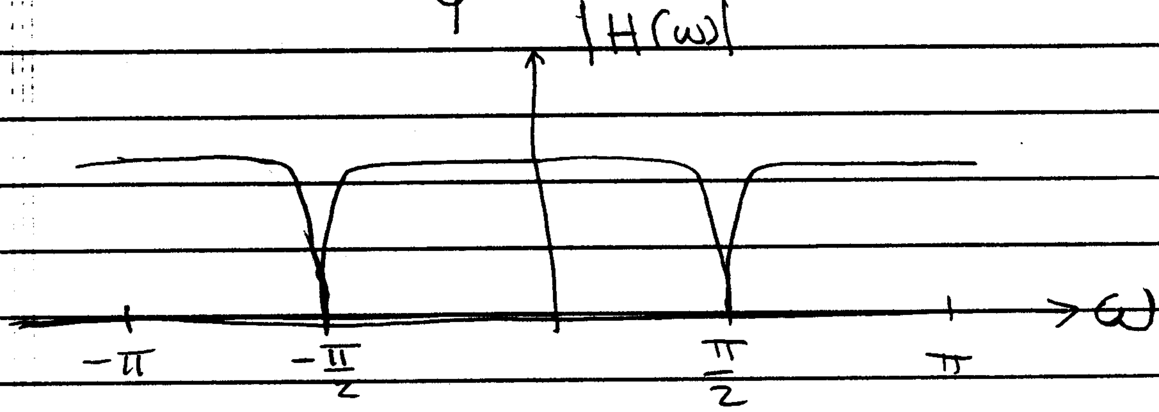
poles: $.95j, -.95j$

Sol'n. to Prob. 2 (cont.)

(c)



notch filter $\Rightarrow$ notches out $\frac{\pi}{2}$



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Sol'n. to Prob. 3 4

$$y(t) = x(t) * g(t)$$

$$= \sum_k b[k]\, p(t-kT_0) * \{\delta(t) + e^{j\theta}\delta(t-T_0)\}$$

$$= \sum_k b[k]\{p(t-kT_0) + e^{j\theta} p(t-(k+1)T_0)\}$$

$$y[n] = y(nT_0) = \sum_k b[k]\{p[(n-k)T_0] + e^{j\theta} p[(n-k)T_0 - T_0]\}$$

define:

$$p[n] = p(nT_0) = \tilde{p}[2n] = \frac{\sin(\pi n)}{\pi n} \frac{\cos(\frac{\pi}{2}n)}{1-n^2}$$

$$= \delta[n]$$

$$y[n] = \sum_k b[k]\{\delta[n-k] + e^{j\theta}\delta[n-k-1]\}$$

$$= \sum_k b[k]\, h[n-k]$$

where: $h[n] = \delta[n] + e^{j\theta}\delta[n-1]$

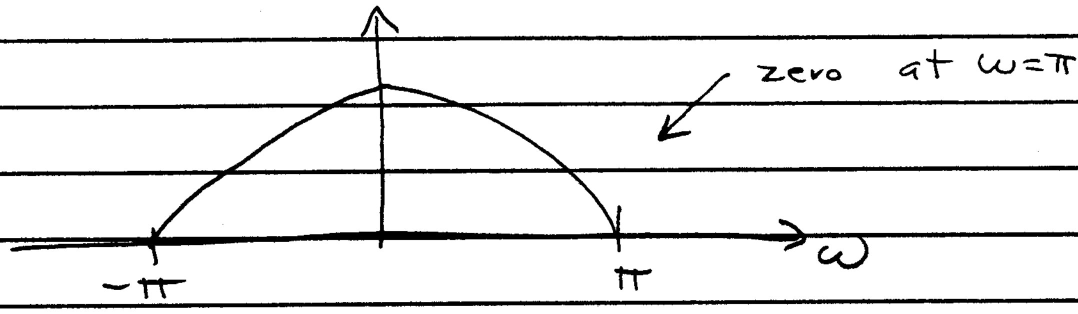
$$H(\omega) = 1 + e^{j(\theta-\omega)}$$

$$= 2\cos\left(\frac{\theta-\omega}{2}\right) e^{j\frac{(\theta-\omega)}{2}}$$

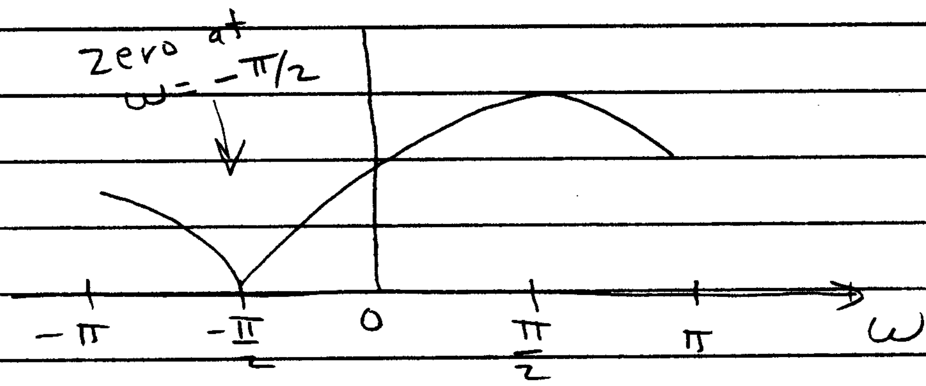
$$|H(\omega)| = 2\left|\cos\left(\frac{\omega-\theta}{2}\right)\right|$$

Sol'n. to Prob. 3

(i) $\theta = 0$ $\quad |H(\omega)| = 2\left|\cos\left(\frac{\omega}{2}\right)\right|$



(ii) $\theta = \frac{\pi}{2}$ $\quad |H(\omega)| = 2\left|\cos\left(\frac{\omega - \pi/2}{2}\right)\right|$



(iii) $\theta = \pi$ $\quad |H(\omega)| = 2\left|\cos\left(\frac{\omega - \pi}{2}\right)\right|$