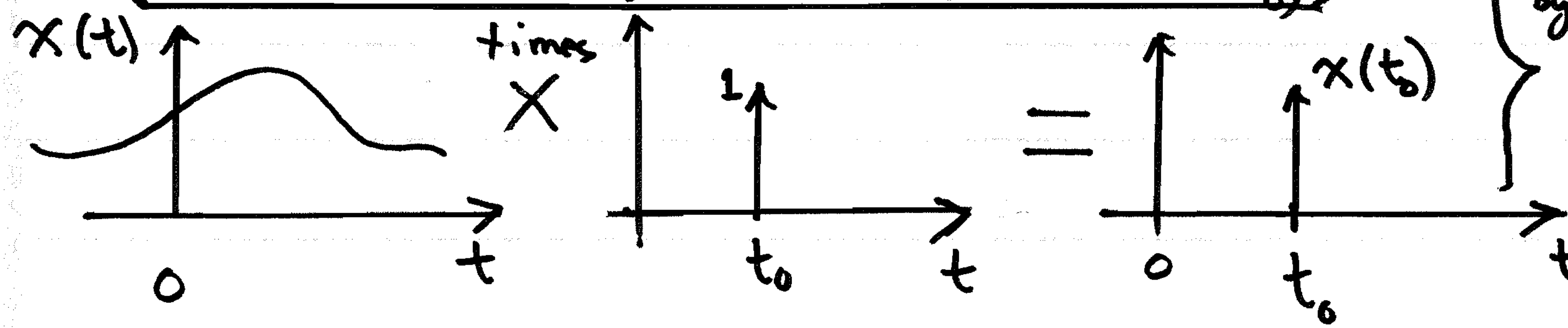
- Recall properties of (Dirac) Delta Function:

Area: $$\int_{-\infty}^{\infty} \delta(t)\, dt = 1 \qquad \int_{-\infty}^{\infty} \delta(t-t_0)\, dt = \int_{t_0^-}^{t_0^+} \delta(t-t_0)\, dt = 1$$

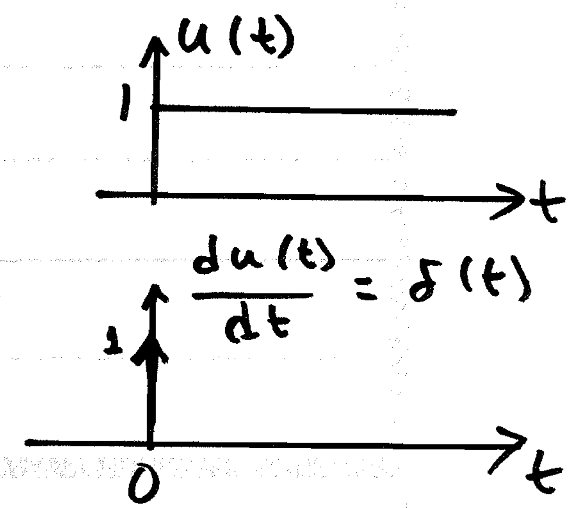
$$\boxed{x(t)\,\delta(t-t_0) = x(t_0)\,\delta(t-t_0)}$$

Multiplication by a Delta Function



Symmetry: $\boxed{\delta(-t) = \delta(t)}$

Derivative at discontinuity: $\boxed{\delta(t) = \dfrac{du(t)}{dt}}$



- Additional Properties of Delta Function:

$$\boxed{x(t) * \delta(t) = x(t)}$$

$$\boxed{x(t) * \delta(t-t_0) = x(t-t_0)}$$

Proof: $x(t) * \delta(t-t_0) = \int_{-\infty}^{\infty} \delta(\tau - t_0)\, x(t-\tau)\, d\tau$

- multiplication property dictates $= \int_{-\infty}^{\infty} \delta(\tau - t_0)\, x(t-t_0)\, d\tau$

- $x(t-t_0)$ does not depend on integration variable τ $= x(t-t_0) \int_{-\infty}^{\infty} \delta(\tau - t_0)\, d\tau$

- area under Delta Function is unity $= x(t-t_0)$

Can easily show: $\delta(t-t_1) * \delta(t-t_2) = \delta(t-(t_1+t_2))$

Convolution with a DT Kronecker Delta

$$y[n] = x[n] * \delta[n-n_0] = x[n-n_0]$$

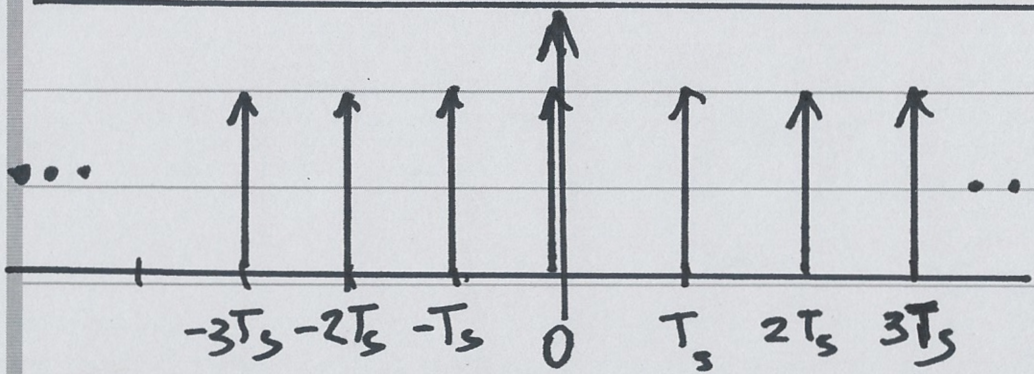
<u>Proof</u>: $$y[n] = \sum_{k=-\infty}^{\infty} \underbrace{\delta[k-n_0]}_{\substack{=1 \quad k=n_0 \\ 0 \quad \text{otherwise}}} x[n-k]$$

$$= x[n-n_0]$$

for all other values of k

so only 1 nonzero term in sum

Fourier-Transform of a Train of Dirac Delta Functions



$$= \sum_{n=-\infty}^{\infty} \delta(t - nT_s) = x_s(t)$$

Periodic Signal can be Expressed as a Fourier Series:

$$x_s(t) = \sum_{k=-\infty}^{\infty} a_k e^{j 2\pi \frac{k}{T_s} t} \quad \Rightarrow a_k = \frac{1}{T_s} \ \forall k \text{ (for all)}$$

for a train of Delta Functions

· Since Fourier Transform is a Linear Operator:

$$x_s(t) = \sum_{k=-\infty}^{\infty} \frac{1}{T_s} e^{j 2\pi \frac{k}{T_s} t} \xleftrightarrow{\mathcal{F}} X_s(f) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} \delta\left(f - \frac{k}{T_s}\right)$$

$$\left(a_k = \frac{1}{T_s} \int_{-T_s/2}^{T_s/2} x_s(t) e^{-j 2\pi \frac{k}{T_s} t} dt = \frac{1}{T_s} \int_{-T_s/2}^{T_s/2} \delta(t) e^{-j 2\pi \frac{k}{T_s} t} dt = \frac{1}{T_s} \right.$$