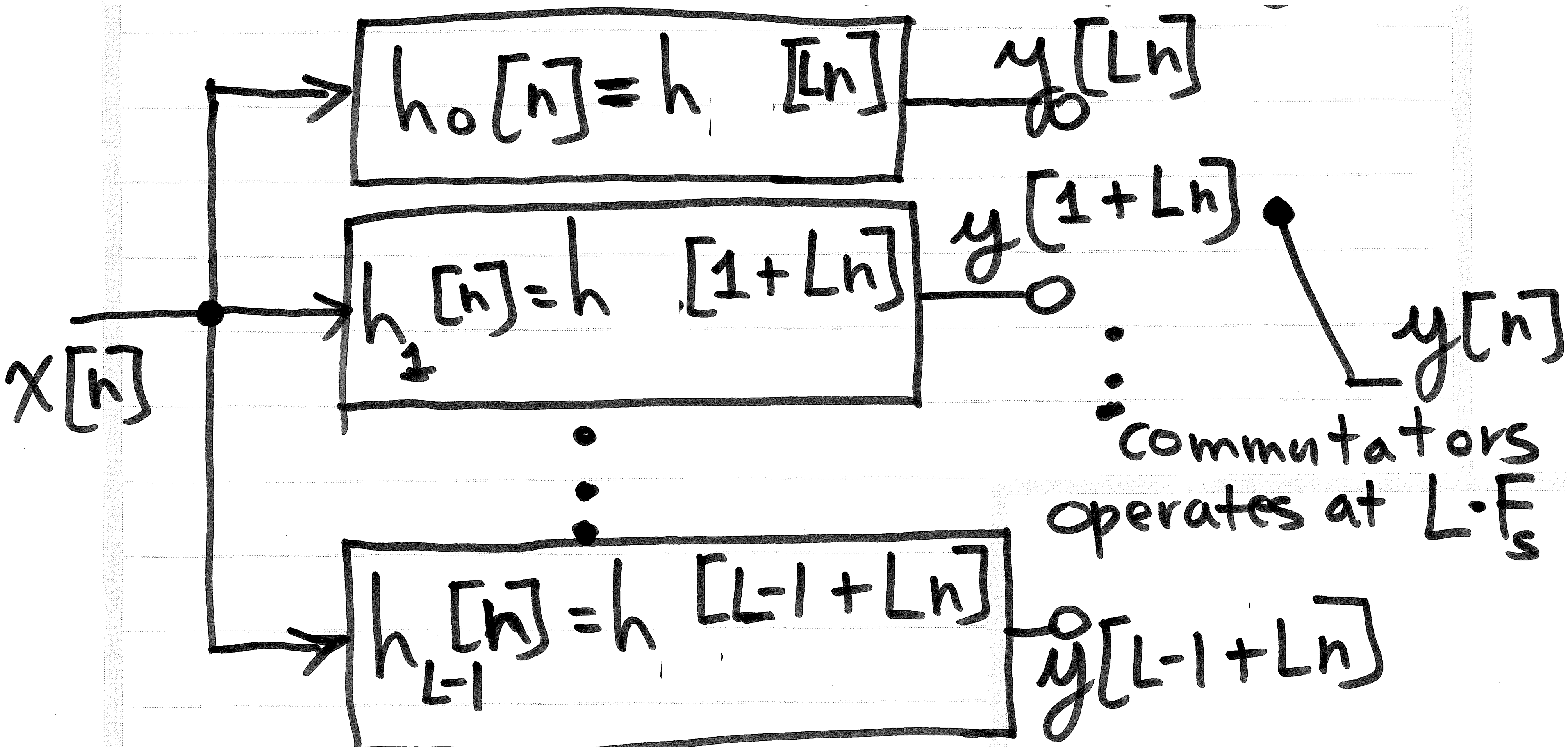
- Addendum Notes on Efficient Upsampling and Efficient "Filtering Followed by Downsampling"
- Efficient Upsampling was about how we read out the upsampled output values
- "Efficient Downsampling" was about how we counted through the summation variable in a summation
- These same concepts will lead to the FFT: Fast Fourier Transform
- We will develop different FFT schemes as a combination of how we read out the DFT values AND how we count thru the summation variable

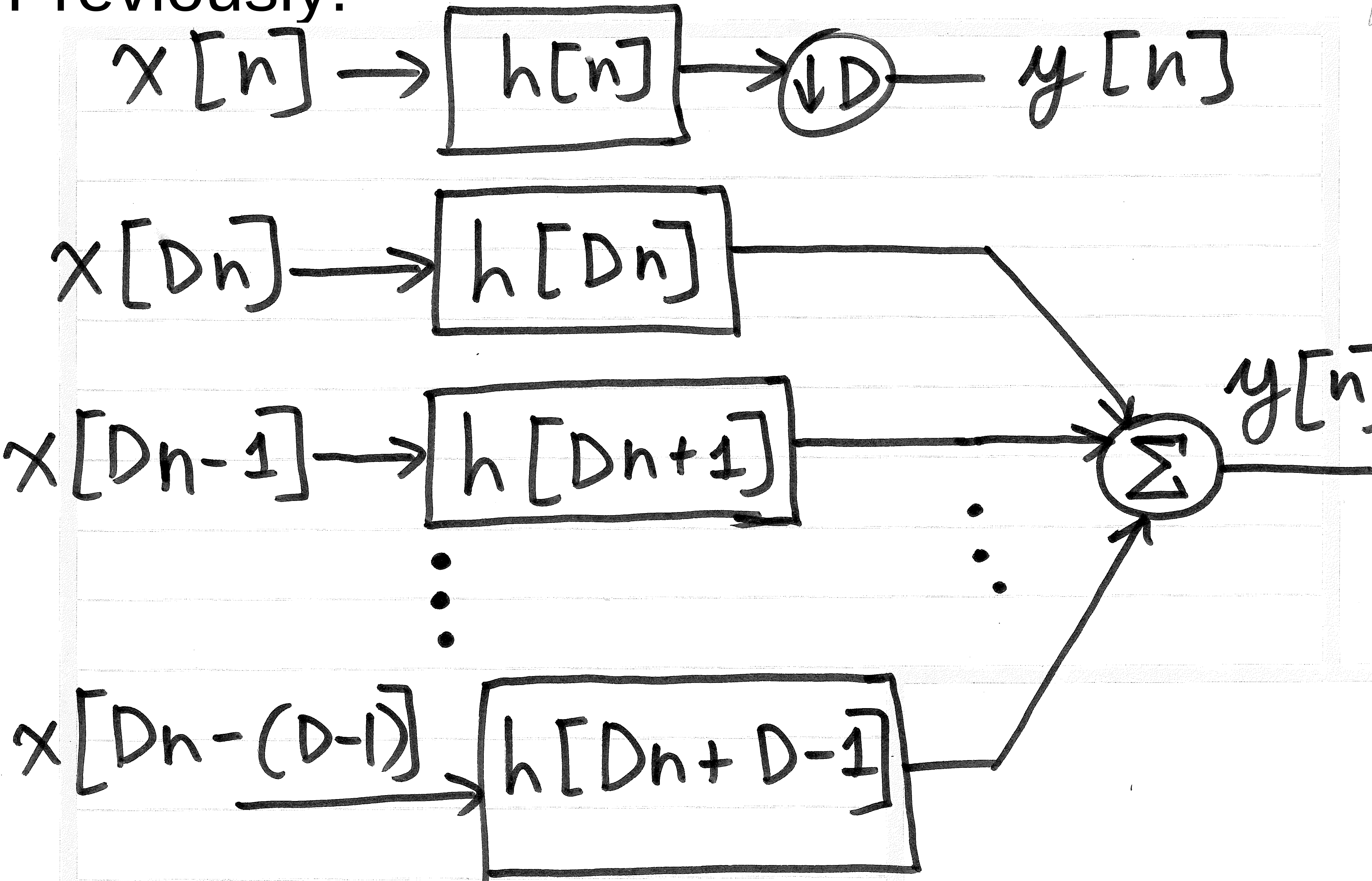
- efficient implementation for

for zero-inserts followed by filtering with h[n]

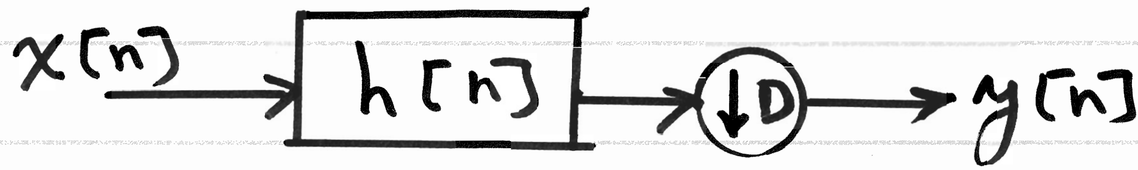


Previously:

L5



- Alternative "Efficient Downsampling" Scheme



$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[Dn-k]$$

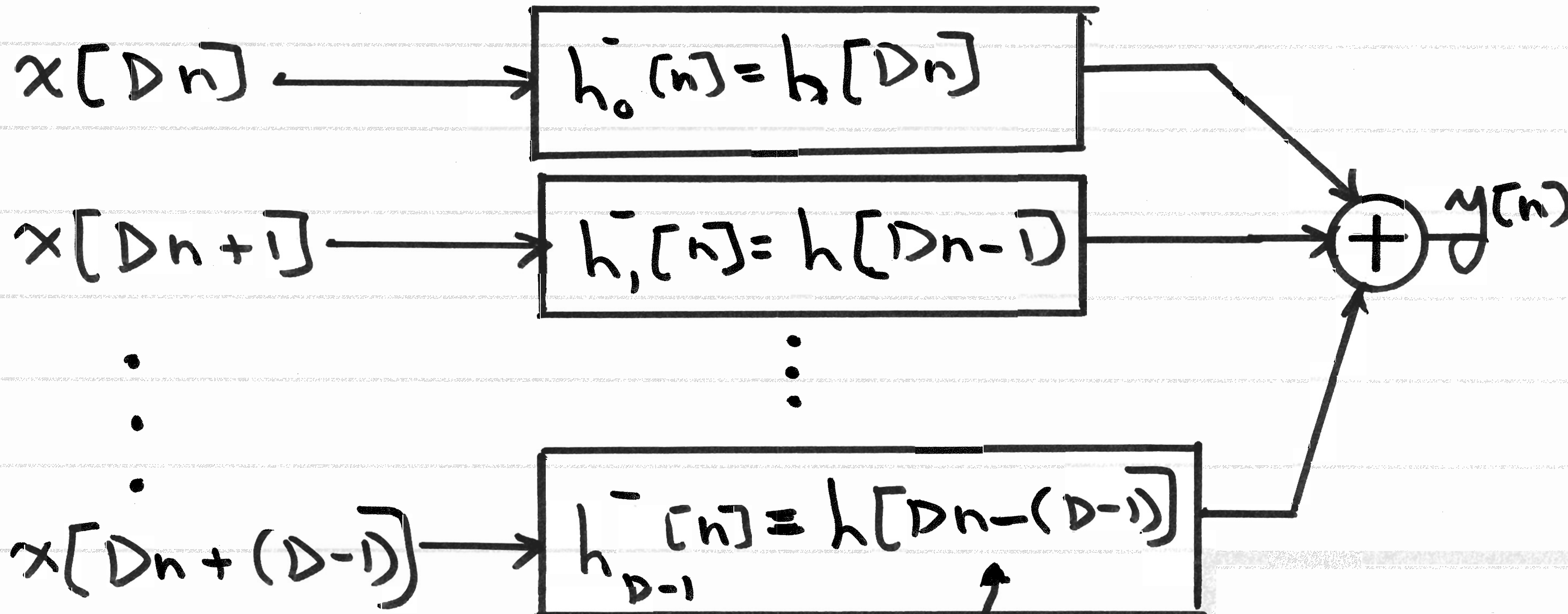
$$= \sum_{\ell=0}^{D-1} \sum_{k'=-\infty}^{\infty} x[Dk'+\ell]\, h[Dn-(Dk'+\ell)]$$

$$= \sum_{\ell=0}^{D-1} \sum_{k'=-\infty}^{\infty} x[Dk'+\ell]\, h[D(n-k')-\ell]$$

<u>Define:</u> $\bar{h}_\ell[n] = h[Dn-\ell], \quad \ell = 0, 1, \ldots, D-1$

$$x_\ell[n] = x[Dn+\ell], \quad \ell = 0, 1, \ldots, D-1$$

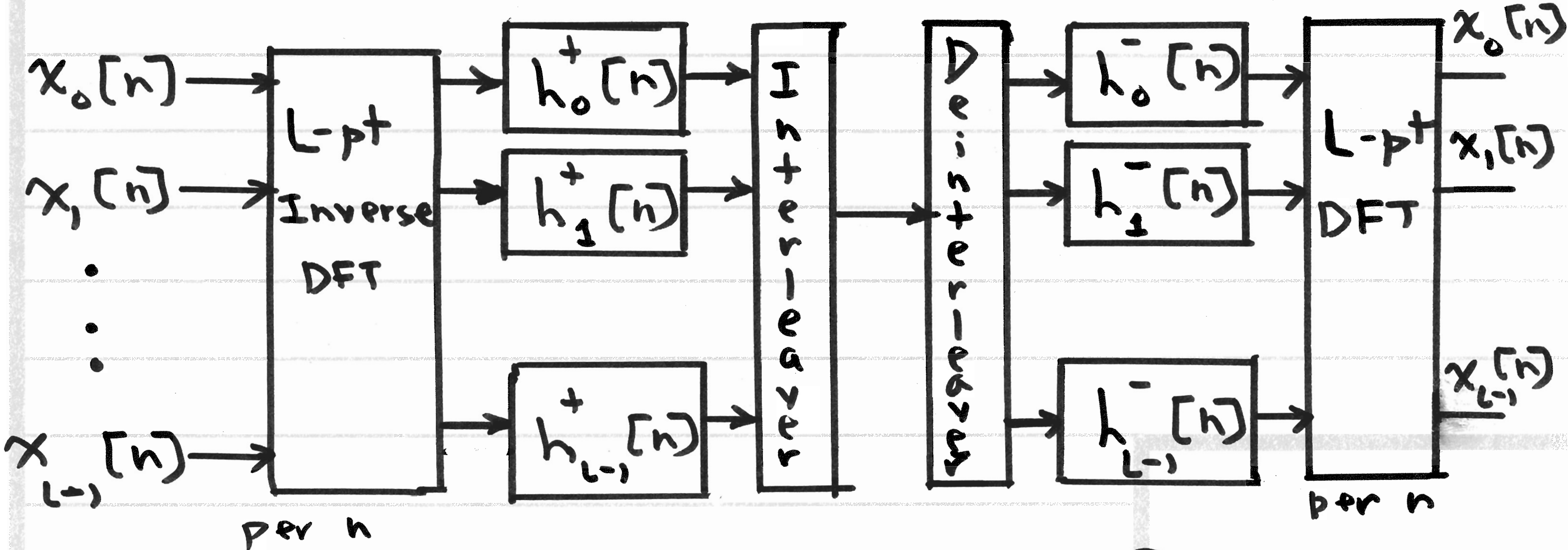
• Leads to: $y[n] = \sum_{\ell=0}^{D-1} x[Dn+\ell] * \bar{h}_\ell[n]$



in our previous efficient implementation the + and − signs were reversed

$$y[n] = \sum_{\ell=0}^{D-1} x[Dn-\ell] * h[Dn+\ell]$$

- The scalars on the polyphase components of the LP filter turn out to comprise either an L-pt DFT or an L-pt Inverse DFT

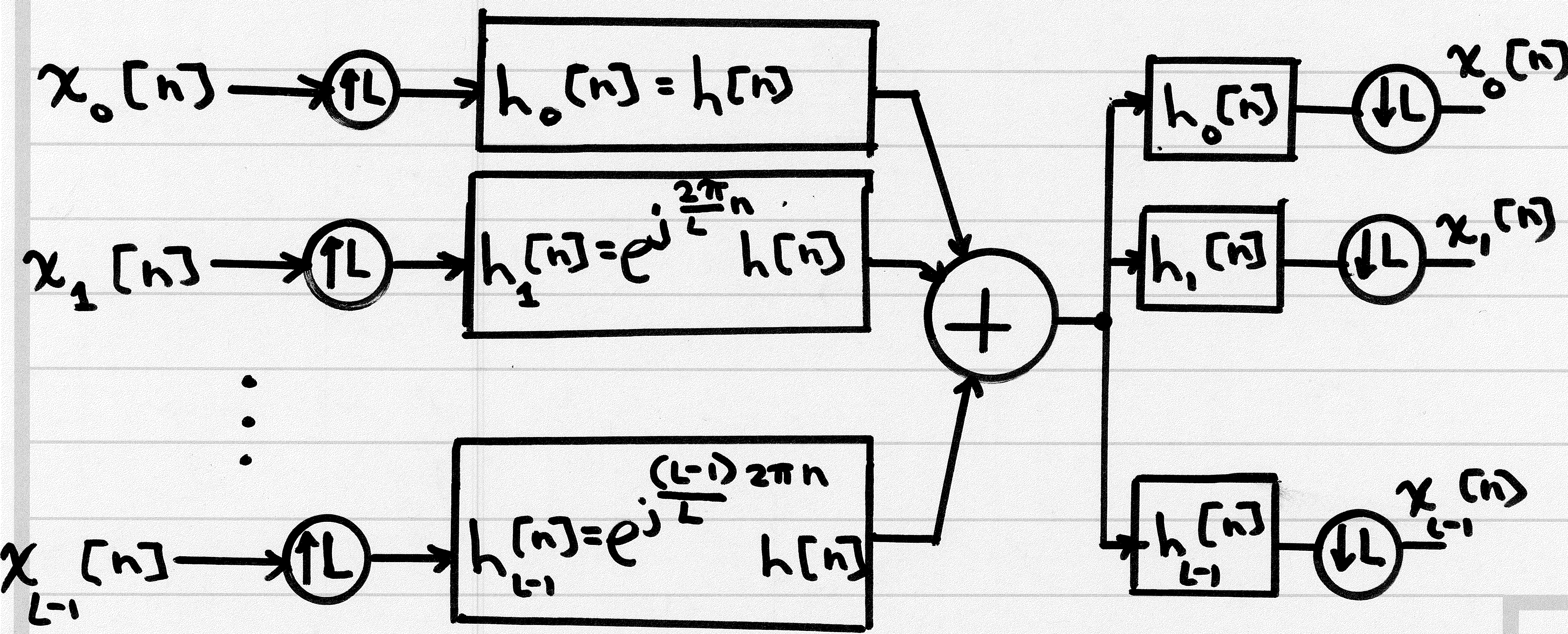


- In ideal case where $h_{LP}[n] = L\frac{\sin(\frac{\pi}{L}n)}{\pi n}$

$$H_\ell^-(\omega) = e^{-j\frac{\ell}{L}\omega} \quad \text{for } -\pi < \omega < \pi$$

See Multiplex3SigsAlt.m

Previous page is efficient implementation of :



- Consider again where have a bank of filters, each followed by decimation, with each filter related as:

$$f_k[n] = e^{jk\frac{2\pi}{L}n}\, h_{LP}[n]$$

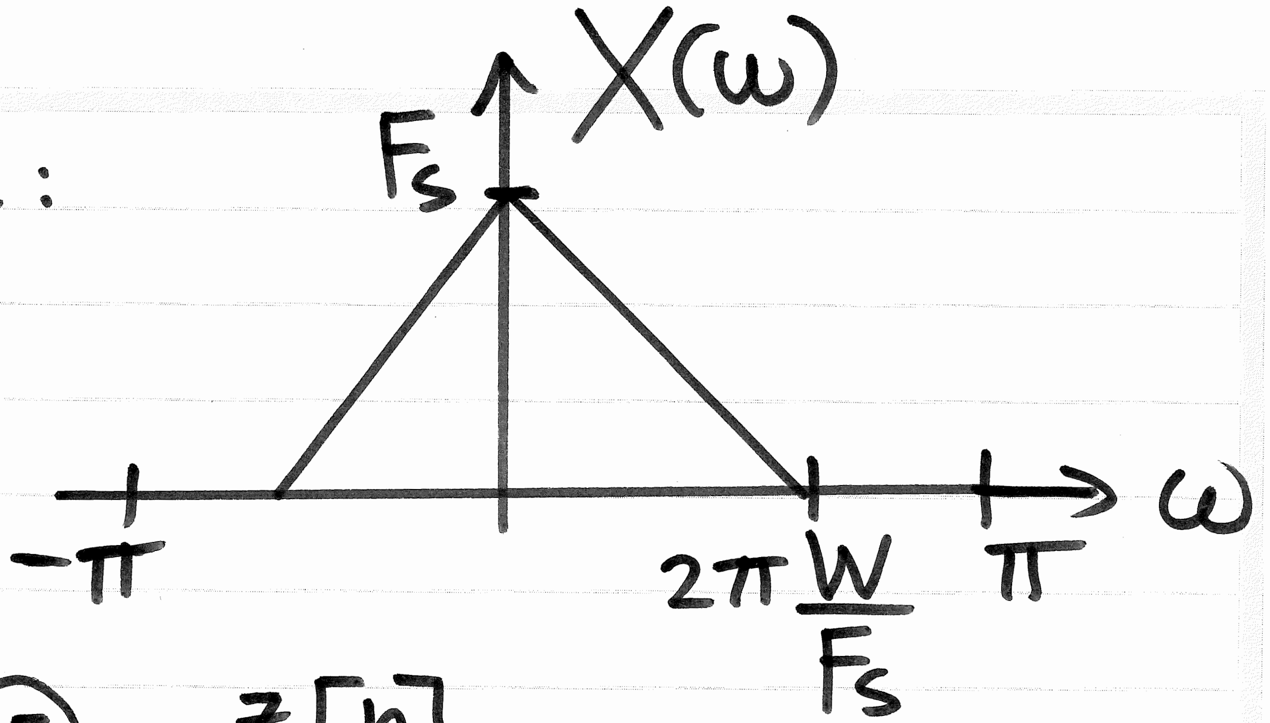
and the decimation is by a factor of $D = L$

Form:

$$\bar{f}_{k_\ell}[n] = e^{jk\frac{2\pi}{L}(Ln-\ell)}\, h_{LP}[Ln-\ell]$$

$$= \underbrace{e^{jk2\pi n}}_{1}\ \underbrace{e^{-j\frac{2\pi}{L}k\ell}}_{\substack{\text{conjugate} \\ \text{of what we} \\ \text{had previously.}}}\ \bar{h}_\ell[n]$$

• Example:



$x[n] \rightarrow \uparrow 5 \rightarrow z[n]$