• Addendum Notes on Efficient Upsampling and Efficient "Filtering Followed by Downsampling"

• Efficient Upsampling was about how we read out the upsampled output values

• "Efficient Downsampling" was about how we counted through the summation variable in a summation

• These same concepts will lead to the FFT: Fast Fourier Transform

• We will develop different FFT schemes as a combination of how we read out the DFT values AND how we count thru the summation variable
**Alternative "Efficient Downsampling" Scheme**

\[ x[n] \xrightarrow{h[n]} y[n] \]

\[ y[n] = \sum_{k=-\infty}^{\infty} x[k] h[Dn-k] \]

\[ = \sum_{k'=-\infty}^{\infty} \sum_{l=0}^{D-1} x[DK'+l] h[Dn-(DK'+l)] \]

\[ = \sum_{k'=-\infty}^{\infty} \sum_{l=0}^{D-1} x[DK'+l] h[D(n-k')-l] \]

**Define:**

\[ h'_x[n] = h[Dn-l], \quad l = 0, 1, \ldots, D-1 \]

\[ x'_x[n] = x[Dn+l], \quad l = 0, 1, \ldots, D-1 \]
- Leads to: \[ y[n] = \sum_{l=0}^{D-1} x[DN+l] * h_l[n] \]

In our previous efficient implementation, the + and - signs were reversed.

\[ y[n] = \sum_{l=0}^{D-1} x[DN-l] * h[DN+l] \]
Consider again where we have a bank of filters each followed by decimation, with each filter related as:

\[ f_k[n] = e^{j\frac{2\pi}{L} n} h_{LP}[n] \]

and the decimation is by a factor of \( D = L \) Form:

\[ f_{\frac{L}{k}}[n] = e^{j\frac{2\pi}{L}(Ln-l)} h_{LP}[Ln-l] \]

\[ = e^{j2\pi n} e^{-j\frac{2\pi}{L}k} h_{LP}[n] \]

\[ \frac{1}{\text{conjugate of what we had previously}} \]
The scalars on the polyphase components of the LP filter turn out to comprise either an L-pt DFT or an L-D+ Inverse DFT.

In ideal case where $h_{LP}(n) = L \frac{\sin(\frac{\pi n}{M})}{\pi n}$

$H_2(w) = e^{-j \frac{\theta}{2} w}$ for $-\pi < w < \pi$

See Multiplex3SigsAlt.m
Previous page is efficient implementation of:

\[ x_0[n] \rightarrow f_1 \rightarrow h_0[n] = h[n] \]

\[ x_1[n] \rightarrow f_1 \rightarrow h_1[n] = e^{j \frac{2\pi}{2} n} h[n] \]

\[ \vdots \]

\[ x_{L-1}[n] \rightarrow f_1 \rightarrow h_{L-1}[n] = e^{j \frac{2\pi}{L} n} h[n] \]

\[ \sum \]

\[ \downarrow L \]

\[ h_0[n] \rightarrow \downarrow L \rightarrow x_0[n] \]

\[ h_1[n] \rightarrow \downarrow L \rightarrow x_1[n] \]

\[ \vdots \]

\[ h_{L-1}[n] \rightarrow \downarrow L \rightarrow x_{L-1}[n] \]