Discrete-Time Signals (DT signals)

- typically obtained by sampling an analog signal at equi-spaced instants in time

\[ x[n] = x(t) \bigg|_{t=nT_s} = x(nT_s) \quad -\infty < n < \infty \]

\[ \vdots \]

\[ x[-1] = x(-T_s) \]
\[ x[0] = x(0) \]
\[ x[1] = x(T_s) \]
\[ x[2] = x(2T_s) \]
\[ x[3] = x(3T_s) \]

The samples of the CT signal are stored in an array.

Thus:

\[ x[n] \]

\( n \) must be an integer (an index location)
- $X[n + \frac{1}{2}]$ is meaningless $\Rightarrow$ doesn't make sense

- Corresponds to $X((n+\frac{1}{2})T_s) = X(nT_s + \frac{T_s}{2})$

- We have a sample at $nT_s$ and $(n+1)T_s$ but not at halfway in between

Also, how can you have an index location for an array (vector) that is not an integer?

$\frac{1}{T_s} = \text{sampling rate} = \text{no. of samples per sec.}$
**DT sinewaves:**

\[
x[n] = e^{j \omega_0 n} \quad \forall \ n \text{ integer}
\]

- Suppose DT sinewave was obtained by sampling a CT sinewave.

\[
x[n] = e^{j \frac{2\pi f_0}{T_s} t} \bigg|_{t=nT_s} = e^{j(2\pi f_0 T_s)n}
\]

- Because \( n \) is an integer, DT sinewaves are very different from CT sinewaves. Two major differences.
1. CT sinus waves are unique as long as frequencies are different
   \[ \Rightarrow \text{not true for DT sinus waves} \]
   \[ e^{j(\omega_0 + l2\pi)n} \]
   • consider: \( e^{j\theta} \)
   • where \( l \) is an integer
   • recall: \( e^{j\theta} = \cos \theta + j \sin \theta \)
   \[ e^{j2\pi} = \cos (2\pi) + j \sin (2\pi) = 1 \]
   \[ \frac{1}{1} \]
   • thus:
   \[ e^{j(\omega_0 + l2\pi)n} = e^{j\omega_0 n} e^{j2\pi ln} = e^{j\omega_0 n} (e^{j\pi})^l n = e^{j\omega_0 n} \]
Any two DT sinewaves whose frequencies are separated by an integer multiple of $2\pi$ are the SAME sinewave.

2. CT sinewaves are always periodic

$\Rightarrow$ not true for DT sinewaves always

- the period for a DT sinewave has to be an integer $N \Rightarrow x[n] = x[n+N]$

$$e^{j\omega_0(n+N)} = e^{j\omega_0 n} e^{j\omega_0 N} \quad + n$$

$$= e^{j\omega_0 n} e^{j m 2\pi} \Rightarrow \omega_0 N = m 2\pi$$

$\Rightarrow \frac{\omega_0}{2\pi} = \frac{m}{N}$ \quad $\{ \text{must be rational for DT sinewave to be periodic} \}$
If this condition holds:

\[ e^{jωon} = e^{j \frac{2π}{N} (n+N)} = e^{j \frac{2π}{N} mn} \]

- Divide out any common divisor between \( m \) and \( N \) \( \Rightarrow \) resulting period is called the fundamental period \( N_0 = \frac{N}{\gcd(m,N)} \)

- Table 1.1 in text summarizes differences between \( e^{jωt} \) and \( e^{jωon} \)

- See also Hmwk. Prob. 1.36

- See Fig. 1.27 on pg. 27
• Basic DT signals
  • unit step
    \[ x[n] = u[n] = \begin{cases} 
    1, & n \geq 0 \\
    0, & n < 0 
    \end{cases} \]
  • Kronecker Delta Function (DT impulse)
    \[ x[n] = \delta[n] = \begin{cases} 
    1, & n = 0 \\
    0, & n \neq 0 
    \end{cases} \]
  • DT rectangle
    \[ x[n] = u[n] - u[n-N] \]
    "turned on" for N units of DT from \( h=0 \) to \( h=N-1 \)
CT Exponential Signals and
DT Geometric Signals/Sequences

CT: \( x(t) = e^{-at} u(t) \)
where \( a \) can be complex-valued, in general.

• If \( a \) is real-valued and \( a > 0 \)

Consider sampling \( x(t) \) at equi-spaced instants in time \( \Rightarrow \) every \( T_s \) seconds
\( \Rightarrow \) replace \( t \) by \( nT_s \), where \( n = \text{integer} \)
\[ x[n] = x(t) \bigg|_{t = nT_s} = x(nT_s) = e^{-anT_s} u(nT_s) = (e^{-aT_s})^n u[n] \]

\[ \Rightarrow x[n] = \alpha^n u[n] \]

where \( \alpha = e^{-aT_s} \)

- Sampling a CT exponential signal yields a DT geometric signal/sequence.