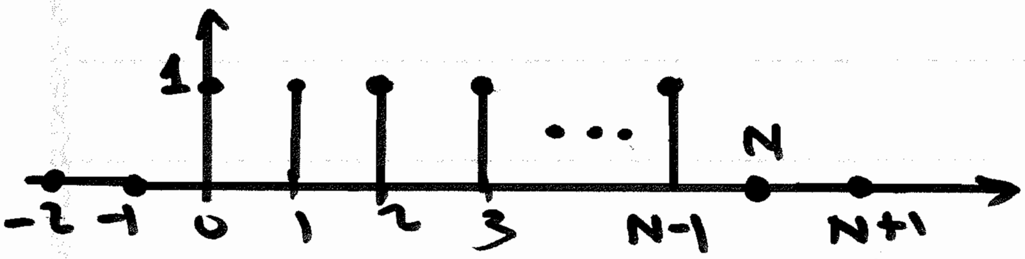
- DT rectangle:

$$x[n] = u[n] - u[n-N] \xleftrightarrow{\text{DTFT}} X(\omega) = \frac{\sin\left(\frac{N}{2}\omega\right)}{\sin\left(\frac{1}{2}\omega\right)} e^{-j\frac{(N-1)}{2}\omega}$$



Proof: $$X(\omega) = \sum_{n=0}^{N-1} (1) e^{-j\omega n} = \frac{1 - e^{-j\omega N}}{1 - e^{-j\omega}}$$

since $$\left(e^{-j\omega}\right)^n = e^{-j\omega n}$$

- Use "half-angle trick" to simplify:

$$X(\omega) = \frac{\left(e^{j\omega\frac{N}{2}} - e^{-j\omega\frac{N}{2}}\right)}{\left(e^{j\frac{\omega}{2}} - e^{-j\frac{\omega}{2}}\right)} \frac{e^{-j\omega\frac{N}{2}}}{e^{-j\omega\frac{1}{2}}} \frac{\left(\frac{1}{2j}\right)}{\left(\frac{1}{2j}\right)}$$

6b

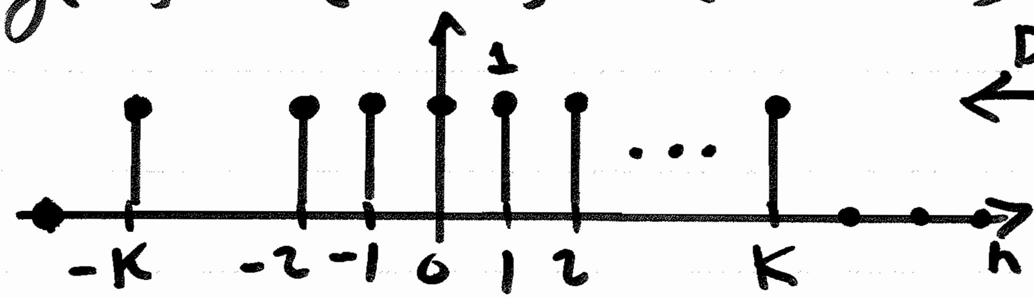
$$X(\omega) = \frac{\sin\left(\frac{N}{2}\omega\right)}{\sin\left(\frac{1}{2}\omega\right)} e^{-j\frac{(N-1)}{2}\omega}$$

- almost in polar form, but $\dfrac{\sin\left(\frac{N}{2}\omega\right)}{\sin\left(\frac{1}{2}\omega\right)}$ can go negative for certain frequency bands
- Suppose N is odd such that $N = 2K+1$ and $K = \frac{N-1}{2}$ is an integer
- Form $y[n] = x[n+K]$ $\Rightarrow$ shift to left by K so that DT rectangle is centered at $n=0$
- Time-Shift Property yields:

$$Y(\omega) = e^{jK\omega} X(\omega) = \underbrace{e^{j\frac{(N-1)}{2}\omega} e^{-j\frac{(N-1)}{2}\omega}}_{\text{cancel}} \frac{\sin\left(\frac{N}{2}\omega\right)}{\sin\left(\frac{1}{2}\omega\right)}$$

This yields DTFT pair:

$$y[n] = u[n+K] - u[n-(K+1)] \quad \xleftrightarrow{\text{DTFT}} \quad Y(\omega) = \frac{\sin\left(\frac{(2K+1)}{2}\omega\right)}{\sin\left(\frac{1}{2}\omega\right)}$$



let $N = 2K+1$.

since $\sin(\theta) = 0$ for $\theta = m\pi$, m integer

$\sin\left(\frac{N}{2}\omega\right) = 0$ for $\omega = m\frac{2\pi}{N}$, m integer