

• DFT of finite length sinusoid

$$\cdot X[n] = e^{j\omega_0 n}, \quad n=0, 1, \dots, L-1$$

$= 0$ otherwise

$$= e^{j\omega_0 n} \{u[n] - u[n-L]\}$$

• First, compute DTFT of the rectangular window: $W[\omega] = u[\omega] - u[\omega-L]$

• then use modulation property of DTFT:

$$X(\omega) = W(\omega - \omega_0)$$

$$X_N(k) = X(\omega) \Big|_{\omega = \frac{2\pi k}{N}}$$

$$k = 0, 1, \dots, N-1$$

• I. DTFT of $w[n] = u[n] - u[n-L]$

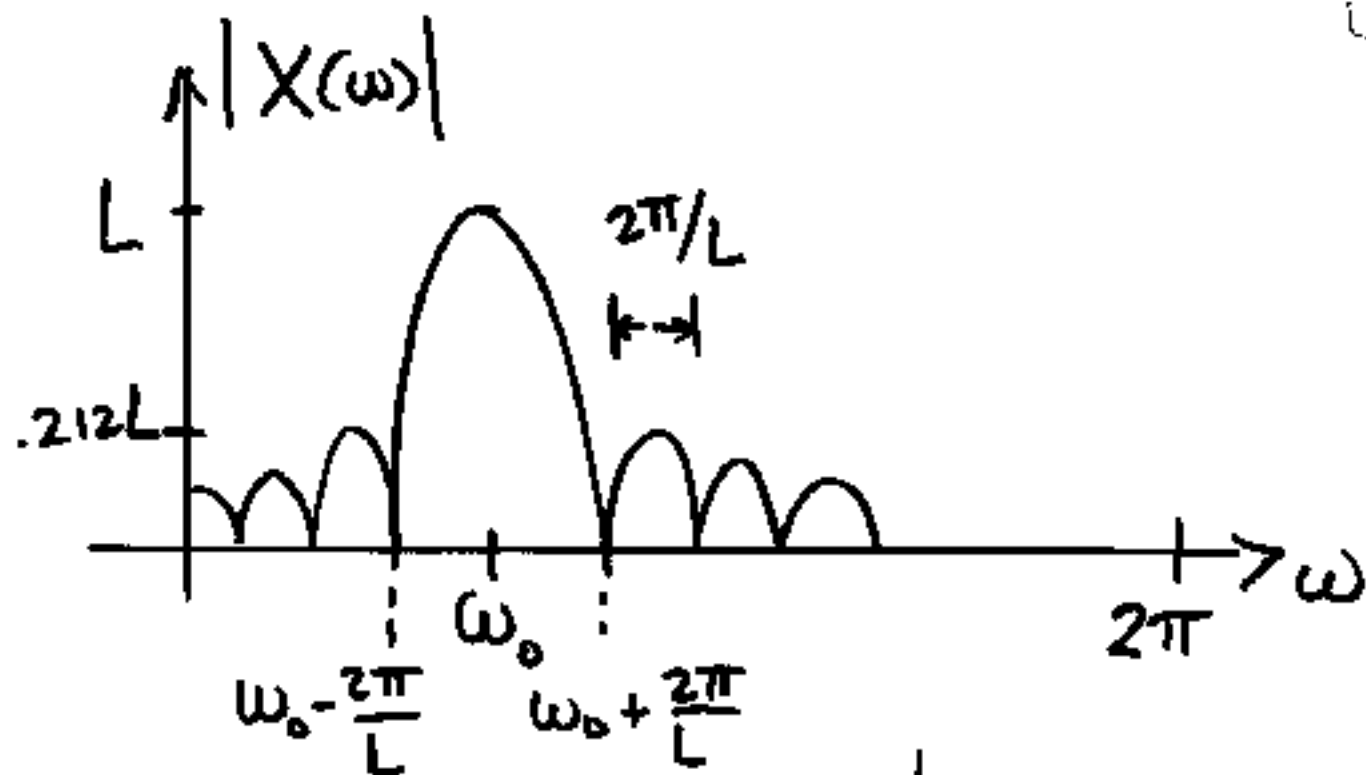
$$W(\omega) = \sum_{n=0}^{L-1} e^{-j\omega n} = \frac{1 - e^{-j\omega L}}{1 - e^{-j\omega}}$$

$$= \frac{e^{-j\frac{L}{2}\omega}}{e^{-j\frac{\omega}{2}}} \cdot \frac{(e^{j\frac{\omega}{2}} - e^{-j\frac{\omega}{2}})}{e^{j\frac{\omega}{2}} - e^{-j\frac{\omega}{2}}} \cdot \frac{1 - e^{-j\omega}}{1 - e^{-j\omega}}$$

$$W(\omega) = e^{-j \frac{(L-1)}{2} \omega} \frac{\sin\left(\frac{L}{2} \omega\right)}{\sin\left(\frac{1}{2} \omega\right)}$$

• thus: DTFT of finite length sinewave:

$$X(\omega) = e^{-j \frac{(L-1)}{2} (\omega - \omega_0)} \frac{\sin\left(\frac{L}{2} (\omega - \omega_0)\right)}{\sin\left(\frac{1}{2} (\omega - \omega_0)\right)}$$



• Finally: $X_N(k) = X(\omega) \Big|_{\omega = \frac{2\pi k}{N}}$
 $k = 0, 1, \dots, N-1$

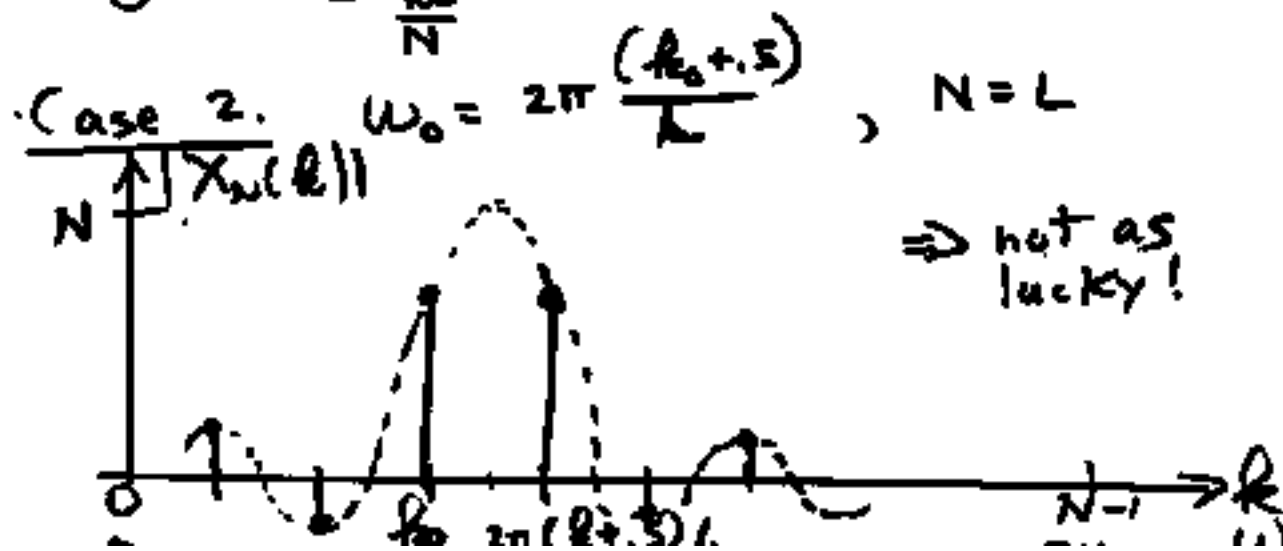
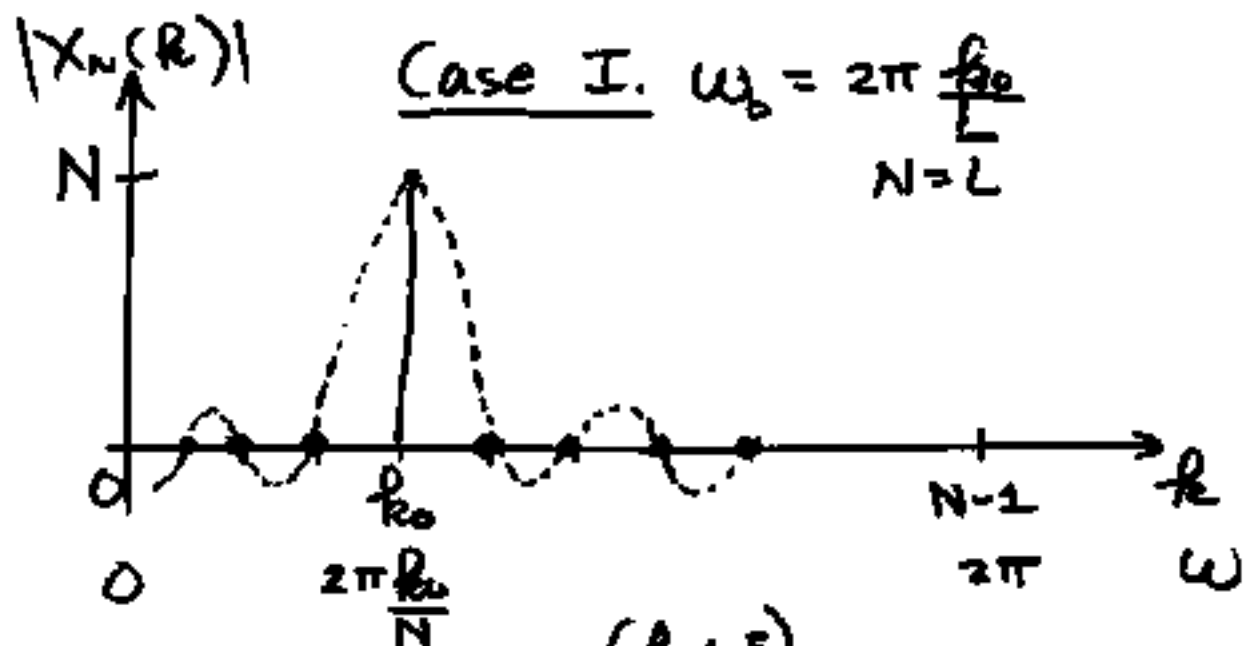
Consider three cases:

1.) $\omega_0 = 2\pi \frac{k_0}{L}$ and $N=L$

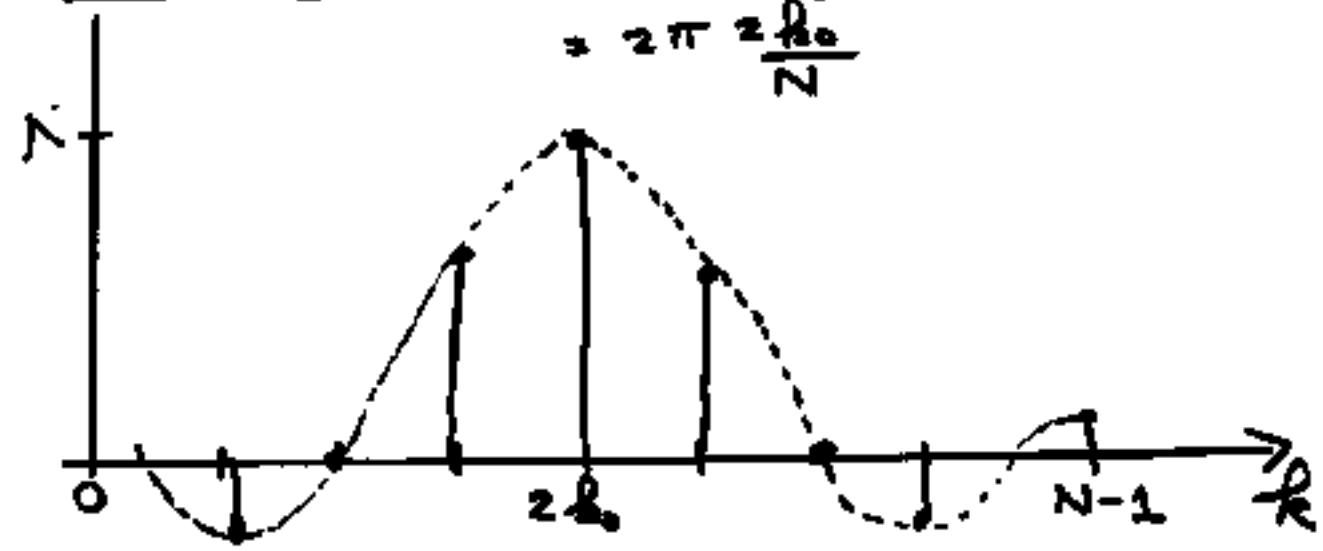
2.) $\omega_0 = 2\pi \frac{k_0}{L}$ and $N=2L$

3.) $\omega_0 = \frac{2\pi(k_0 + 0.5)}{L}$ and $N=L$

• See sine DFT eg 1. m } at course
sine DFT eg 2. m } web site
sine DFT eg 3. m }



• Case 3. $\omega_0 = 2\pi \frac{f_0}{L}$ $N = 2L$
 $= 2\pi \frac{2f_0}{L}$



• as you increase N over L , get a better and better "picture" of DTFT $X(\omega)$ which has both mainlobe and sidelobes \Rightarrow only get Dirac Delta function with $L \rightarrow \infty$ infinite length sine wave