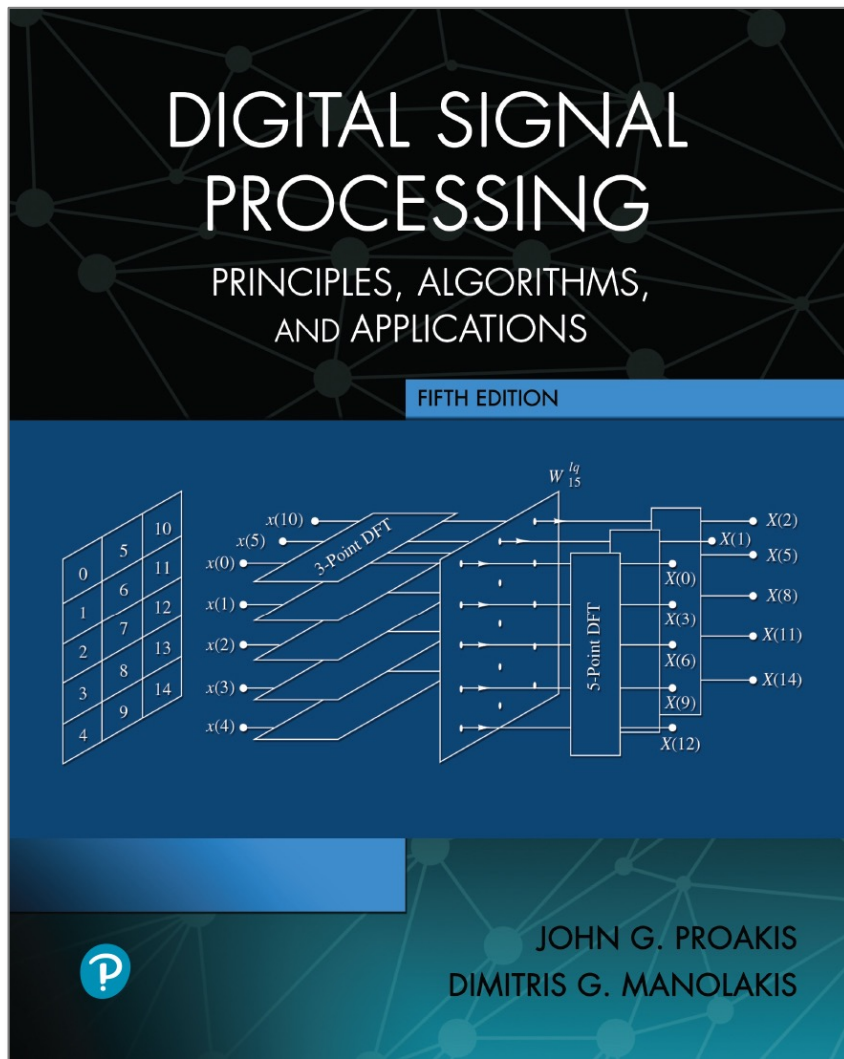


Digital Signal Processing

Fifth Edition



Chapter 7

The Discrete Fourier Transform: Its Properties and Applications

Figure 7.1.1 Frequency-domain sampling of the Fourier transform.

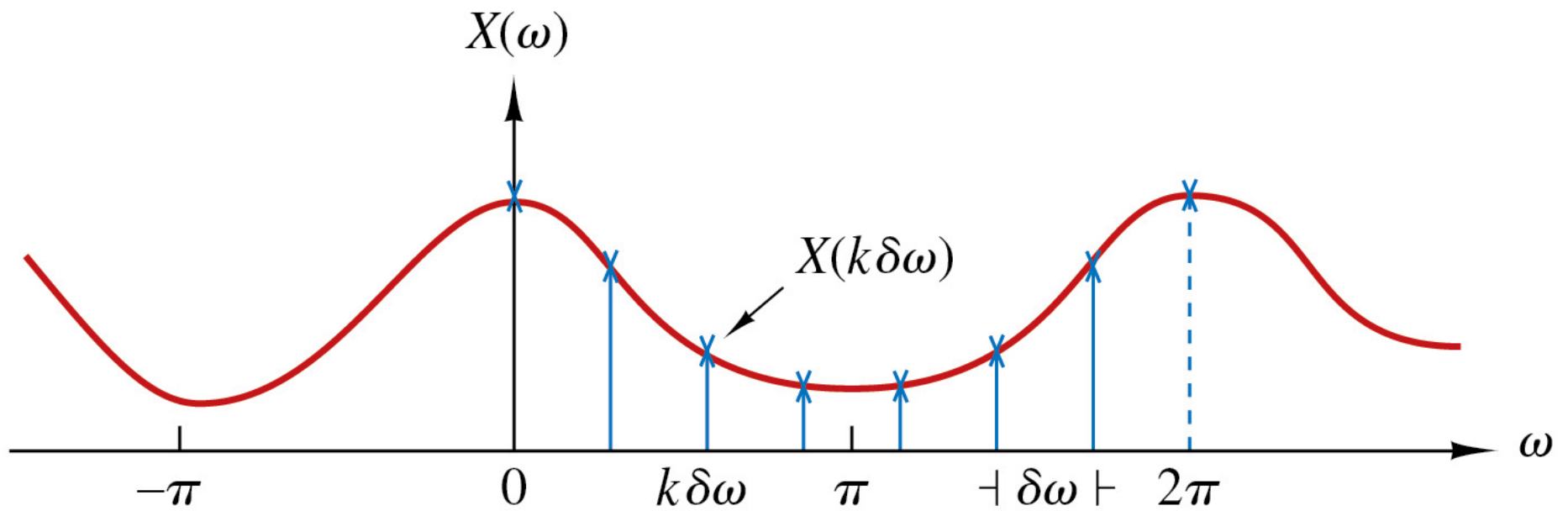


Figure 7.1.2 Aperiodic sequence $x(n)$ of length L and its periodic extension for $N \geq L$ (no aliasing) and $N < L$ (aliasing).

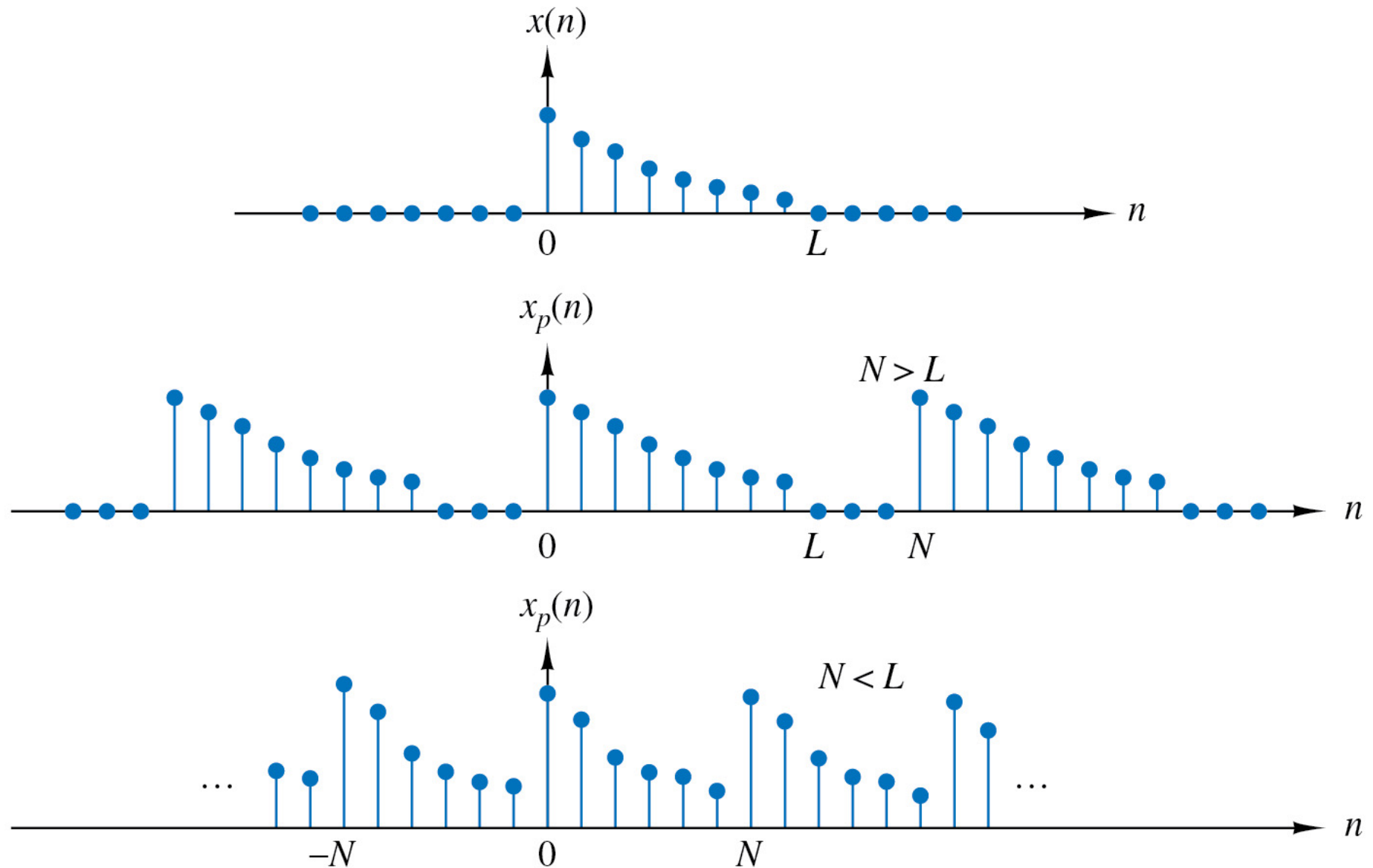


Figure 7.1.3 Plot of the function $[\sin(\omega N/2)]/[N \sin(\omega/2)]$.

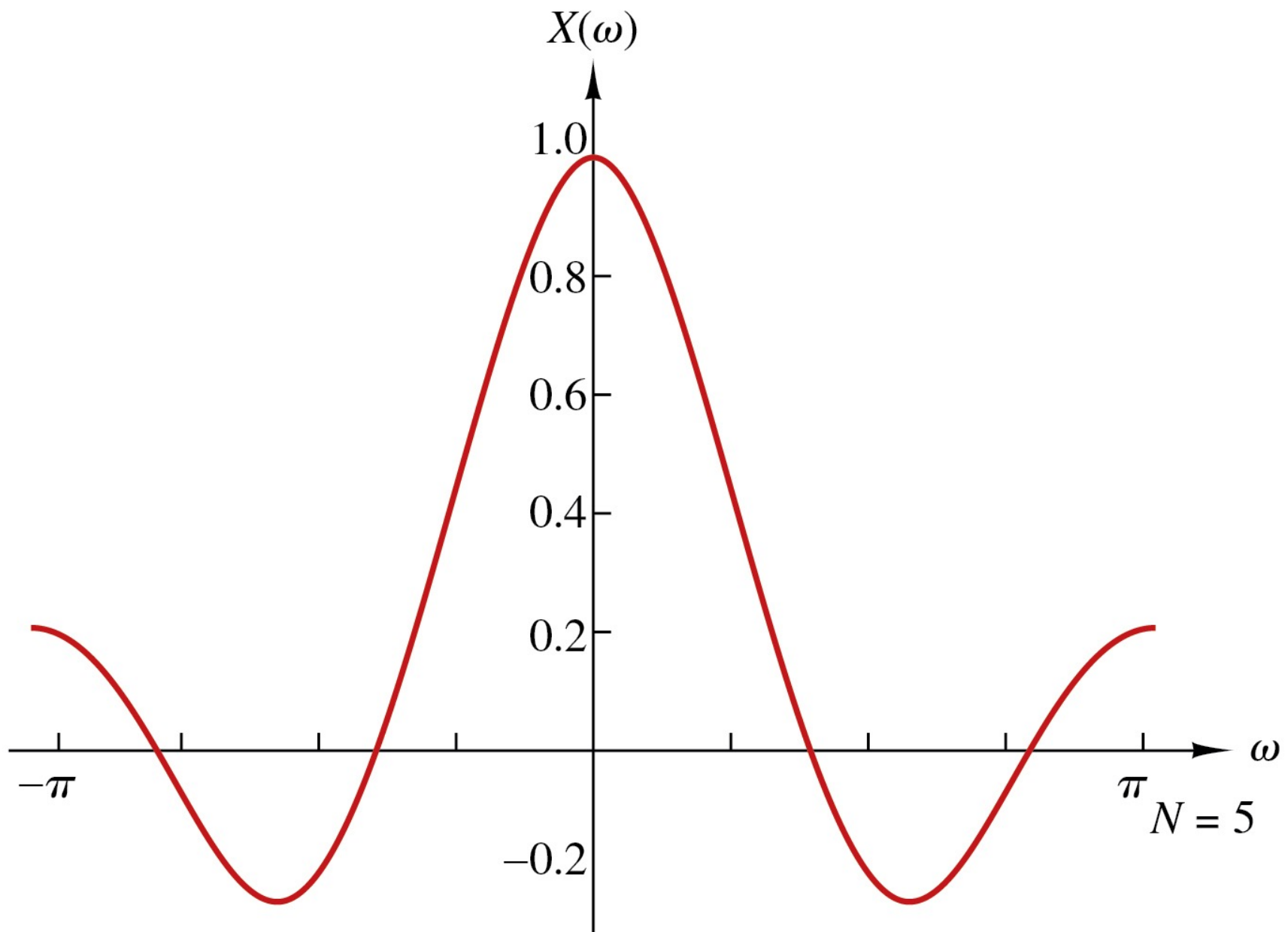


Figure 7.1.4 (a) Plot of sequence $x(n) = (0.8)^n u(n)$; (b) its Fourier transform (magnitude only); (c) effect of aliasing with $N = 5$; (d) reduced effect of aliasing with $N = 50$.

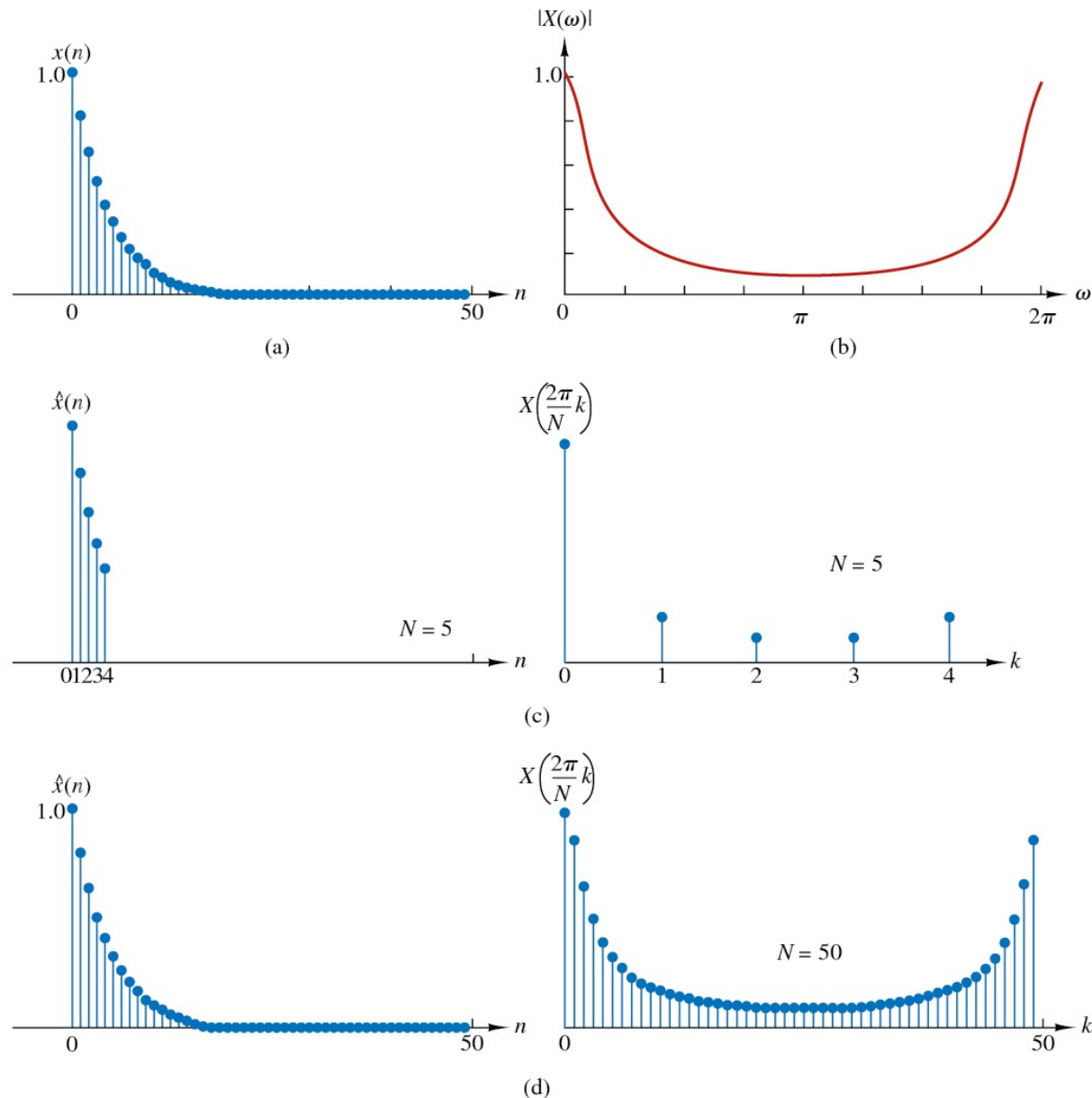


Figure 7.1.5 Magnitude and phase characteristics of the Fourier transform for the signal in Example 7.1.2.

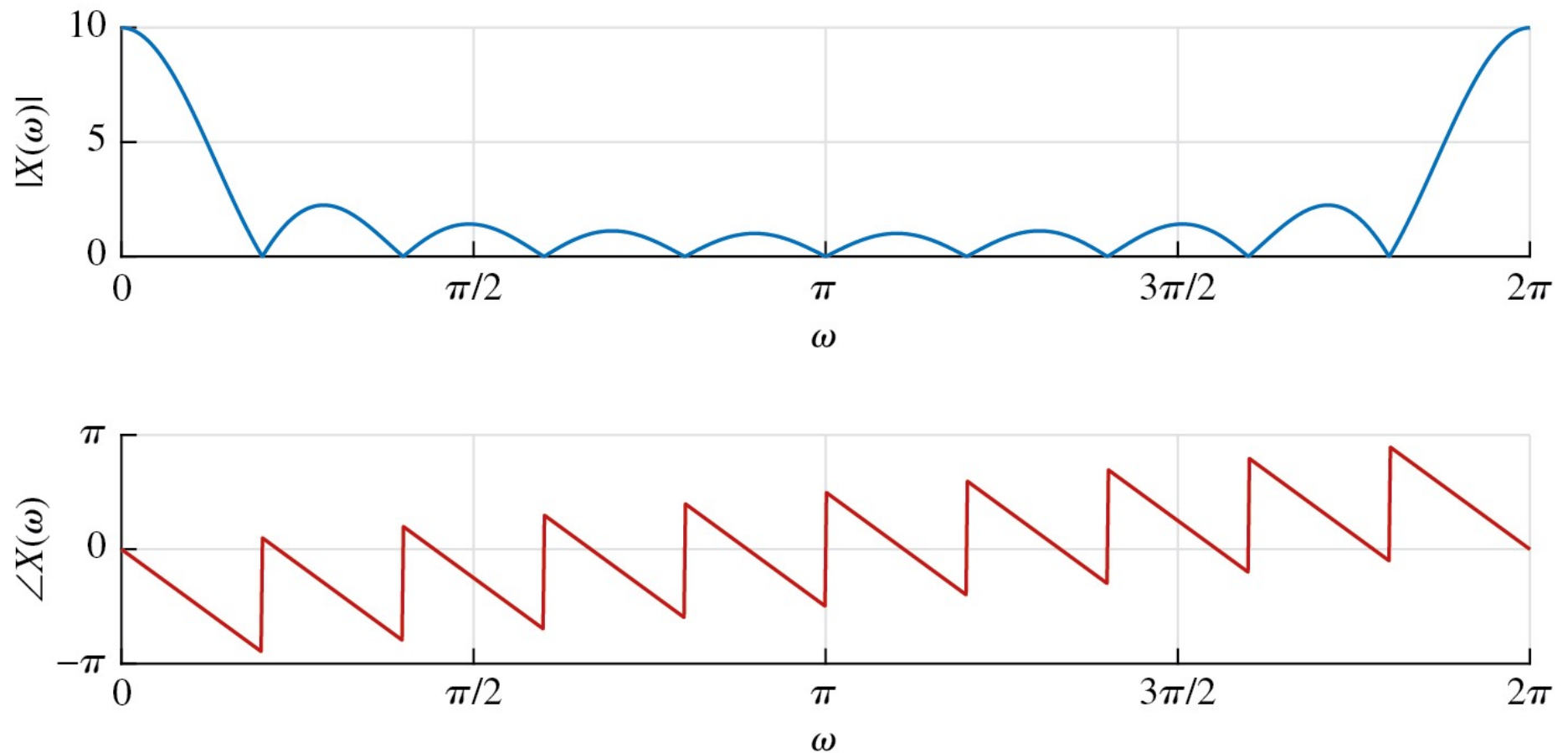


Figure 7.1.6 Magnitude and phase of an N -point DFT in Example 7.1.2;
 (a) $L = 10$, $N = 50$; (b) $L = 10$, $N = 100$.

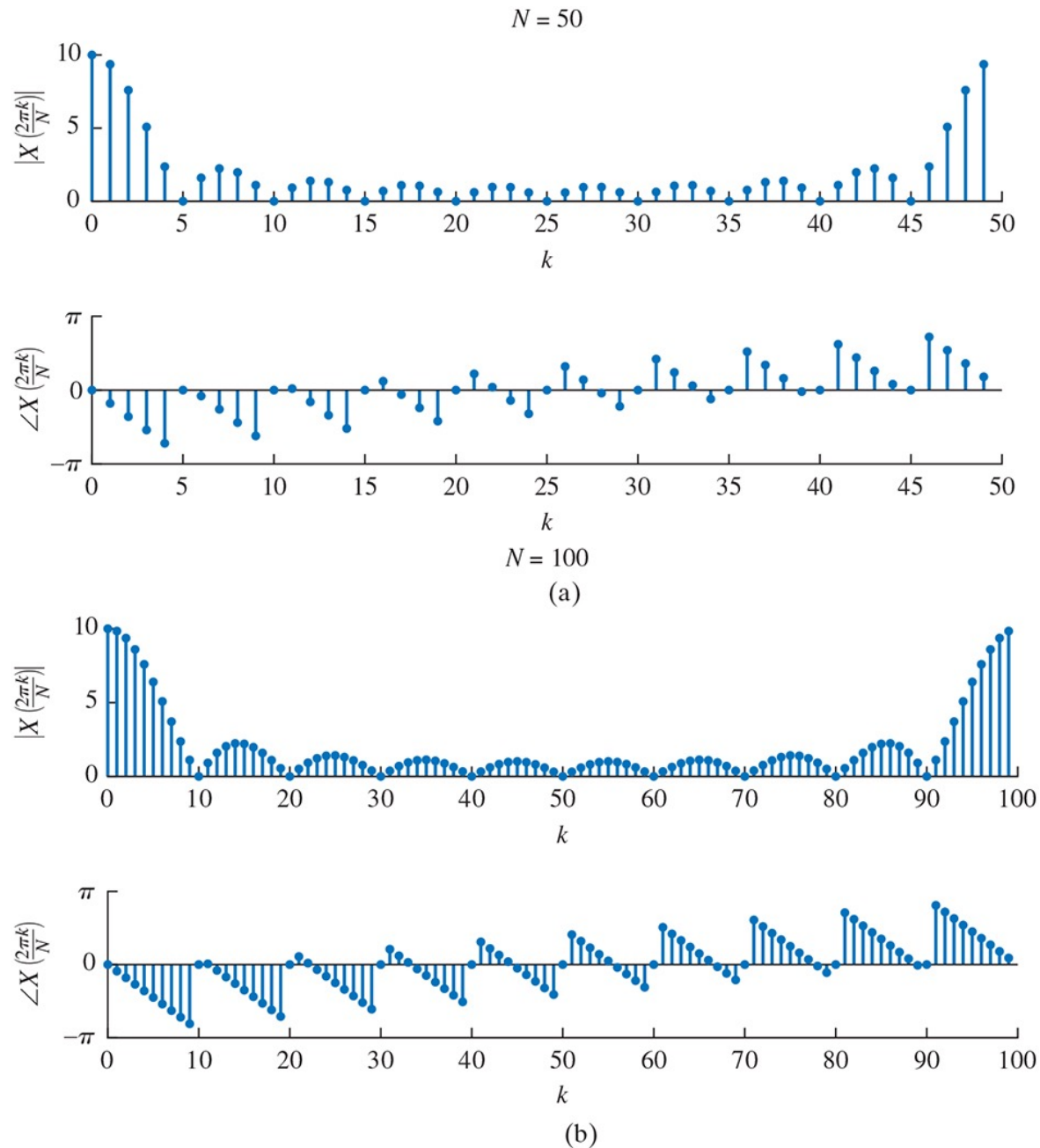
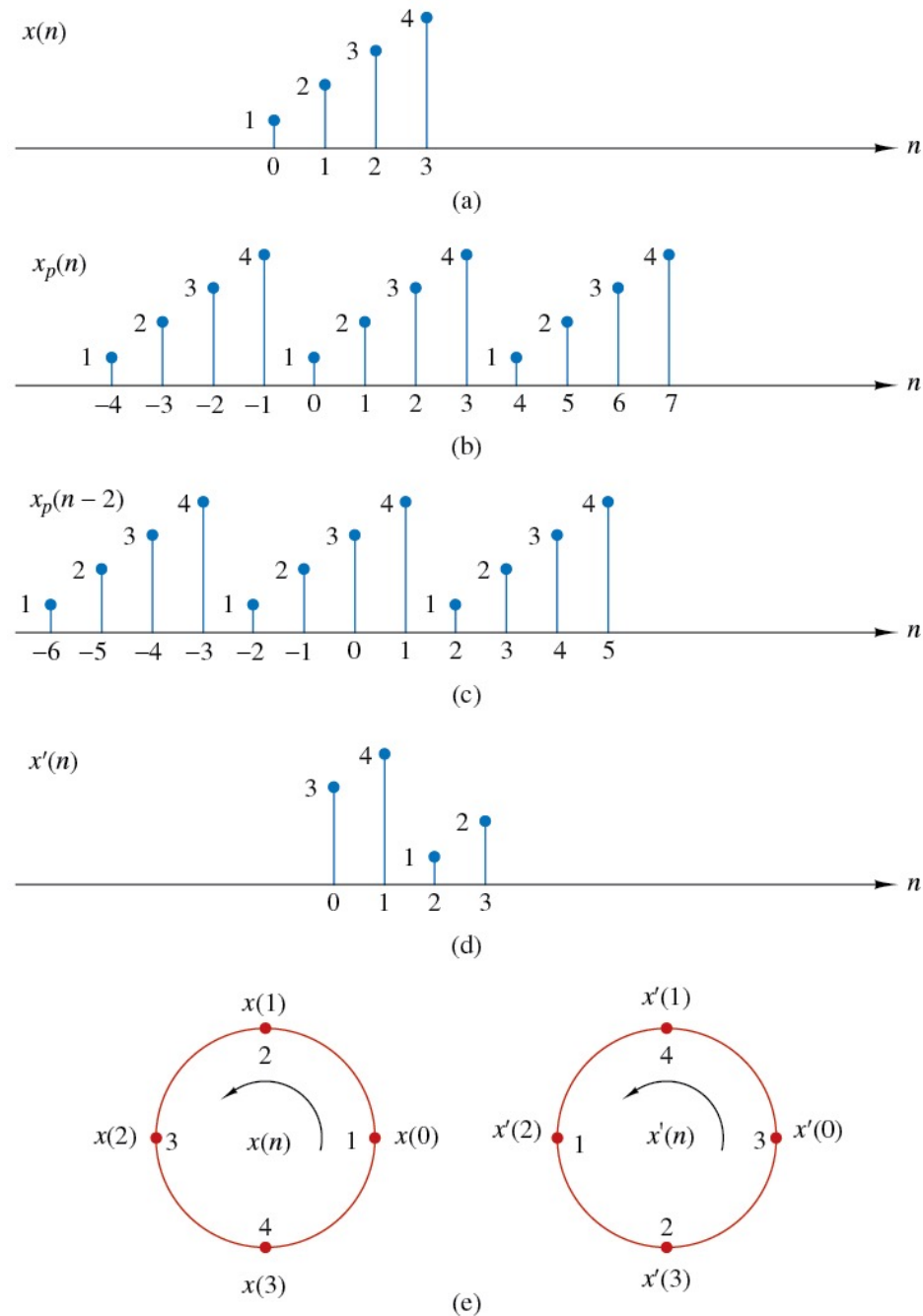


Figure 7.2.1 Circular shift of a sequence.



Equation 7.2.31

$$\begin{aligned} x(n) &= x_R^e(n) + x_R^o(n) + jx_I^e(n) + jx_I^o(n) \\ &\quad \updownarrow \quad \swarrow \quad \updownarrow \quad \searrow \\ X(k) &= X_R^e(k) + X_R^o(k) + jX_I^e(k) + jX_I^o(k) \end{aligned}$$

Table 7.1 Symmetry Properties of the DFT

N -Point Sequence $x(n)$, $0 \leq n \leq N - 1$	N -Point DFT
$x(n)$	$X(k)$
$x^*(n)$	$X^*(N - k)$
$x^*(N - n)$	$X^*(k)$
$x_R(n)$	$X_{ce}(k) = \frac{1}{2}[X(k) + X^*(N - k)]$
$jX_I(n)$	$X_{co}(k) = \frac{1}{2}[X(k) - X^*(N - k)]$
$x_{ce}(n) = \frac{1}{2}[x(n) + x^*(N - n)]$	$X_R(k)$
$x_{co}(n) = \frac{1}{2}[x(n) - x^*(N - n)]$	$jX_I(k)$
Real Signals	
Any real signal	$X(k) = X^*(N - k)$
$x(n)$	$X_R(k) = X_R(N - k)$
	$X_I(k) = -X_I(N - k)$
	$ X(k) = X(N - k) $
	$\angle X(k) = -\angle X(N - k)$
$x_{ce}(n) = \frac{1}{2}[x(n) + x(N - n)]$	$X_R(k)$
$x_{co}(n) = \frac{1}{2}[x(n) - x(N - n)]$	$jX_I(k)$

Figure 7.2.2 Circular convolution of two sequences.

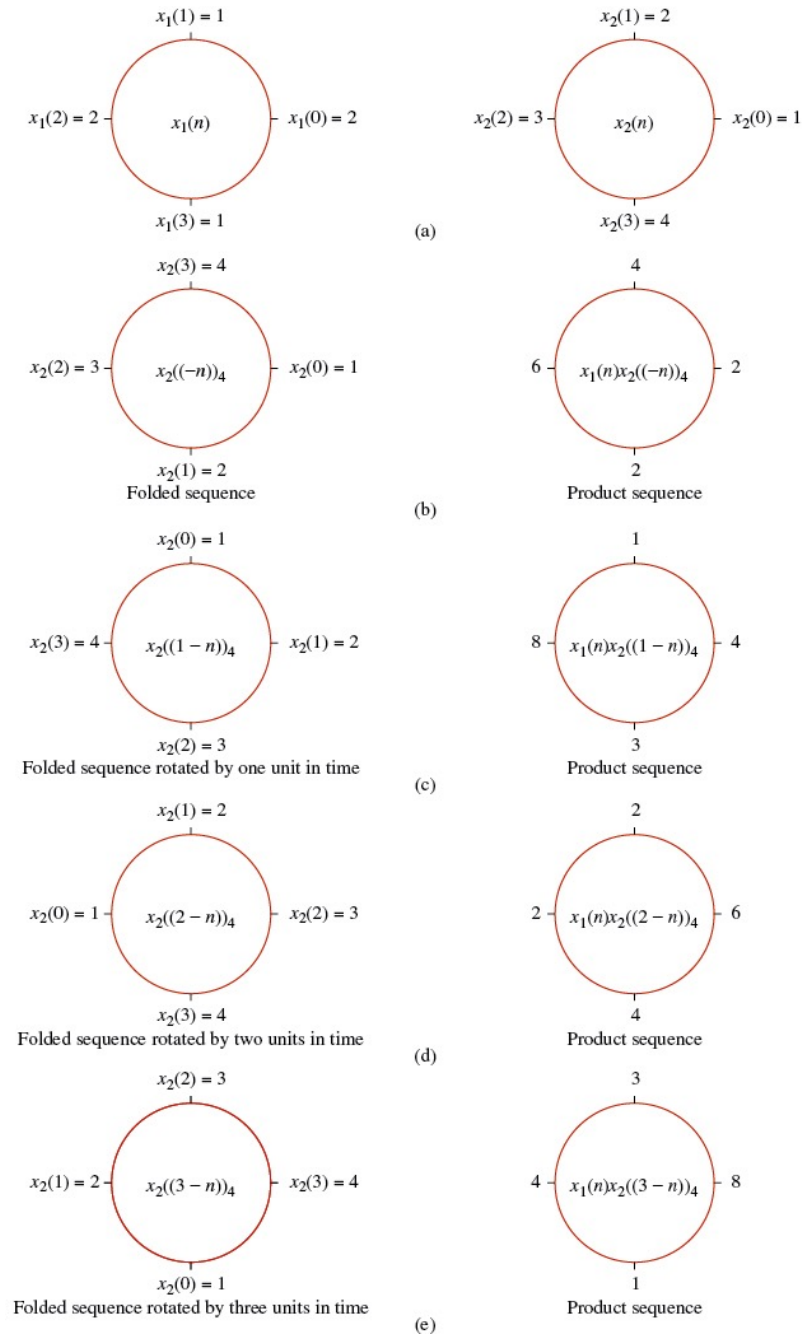


Figure 7.2.3 Time reversal of a sequence.

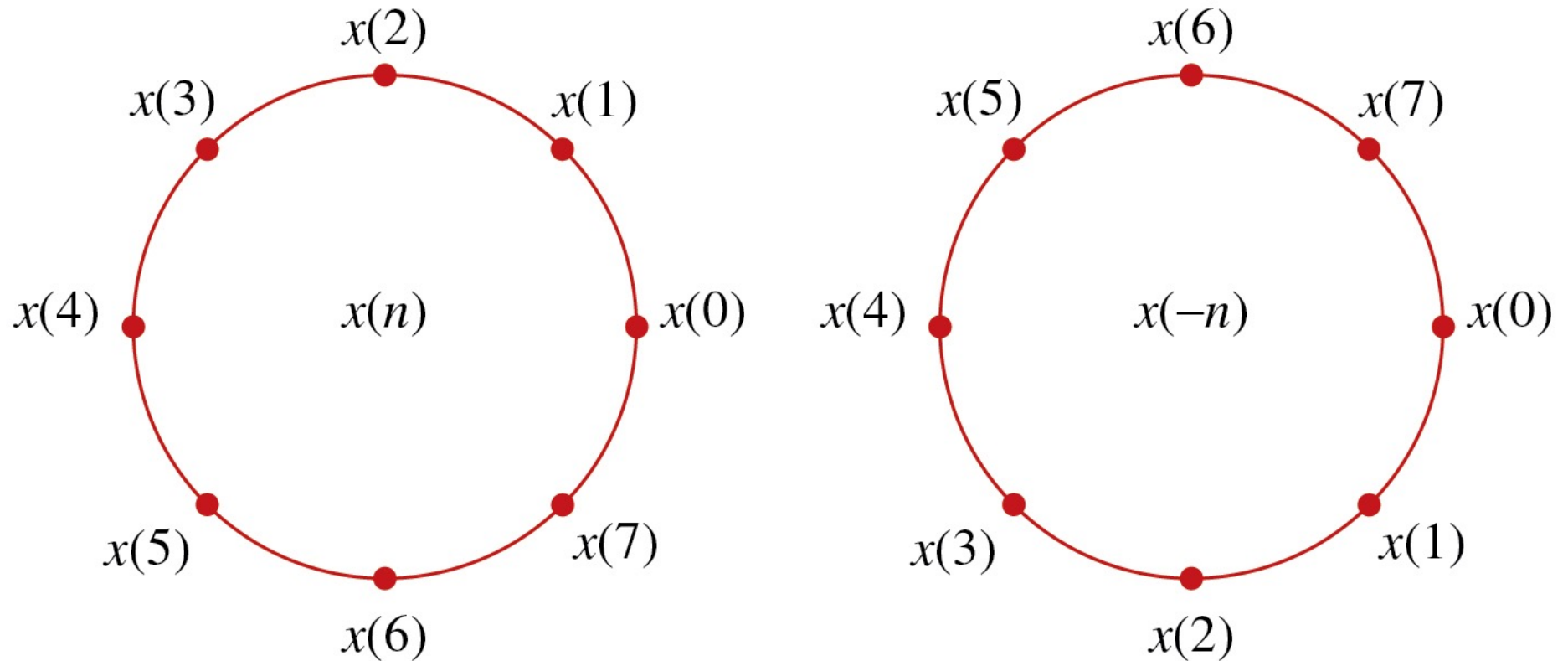


Table 7.2 Properties of the DFT

Property	Time Domain	Frequency Domain
Notation	$x(n), y(n)$	$X(k), Y(k)$
Periodicity	$x(n) = x(n + N)$	$X(k) = X(k + N)$
Linearity	$a_1x_1(n) + a_2x_2(n)$	$a_1X_1(k) + a_2X_2(k)$
Time reversal	$x(N - n)$	$X(N - k)$
Circular time shift	$x((n - l))_N$	$X(k)e^{-j2\pi kl/N}$
Circular frequency shift	$x(n)e^{j2\pi ln/N}$	$X((k - l))_N$
Complex conjugate	$x^*(n)$	$X^*(N - k)$
Circular convolution	$x_1(n) \circledast x_2(n)$	$X_1(k)X_2(k)$
Circular correlation	$x(n) \circledast y^*(-n)$	$X(k)Y^*(k)$
Multiplication of two sequences	$x_1(n)x_2(n)$	$\frac{1}{N}X_1(k) \circledast X_2(k)$
Parseval's theorem	$\sum_{n=0}^{N-1} x(n)y^*(n)$	$\frac{1}{N} \sum_{k=0}^{N-1} X(k)Y^*(k)$

Figure 7.3.1 Linear FIR filtering by the overlap-save method.

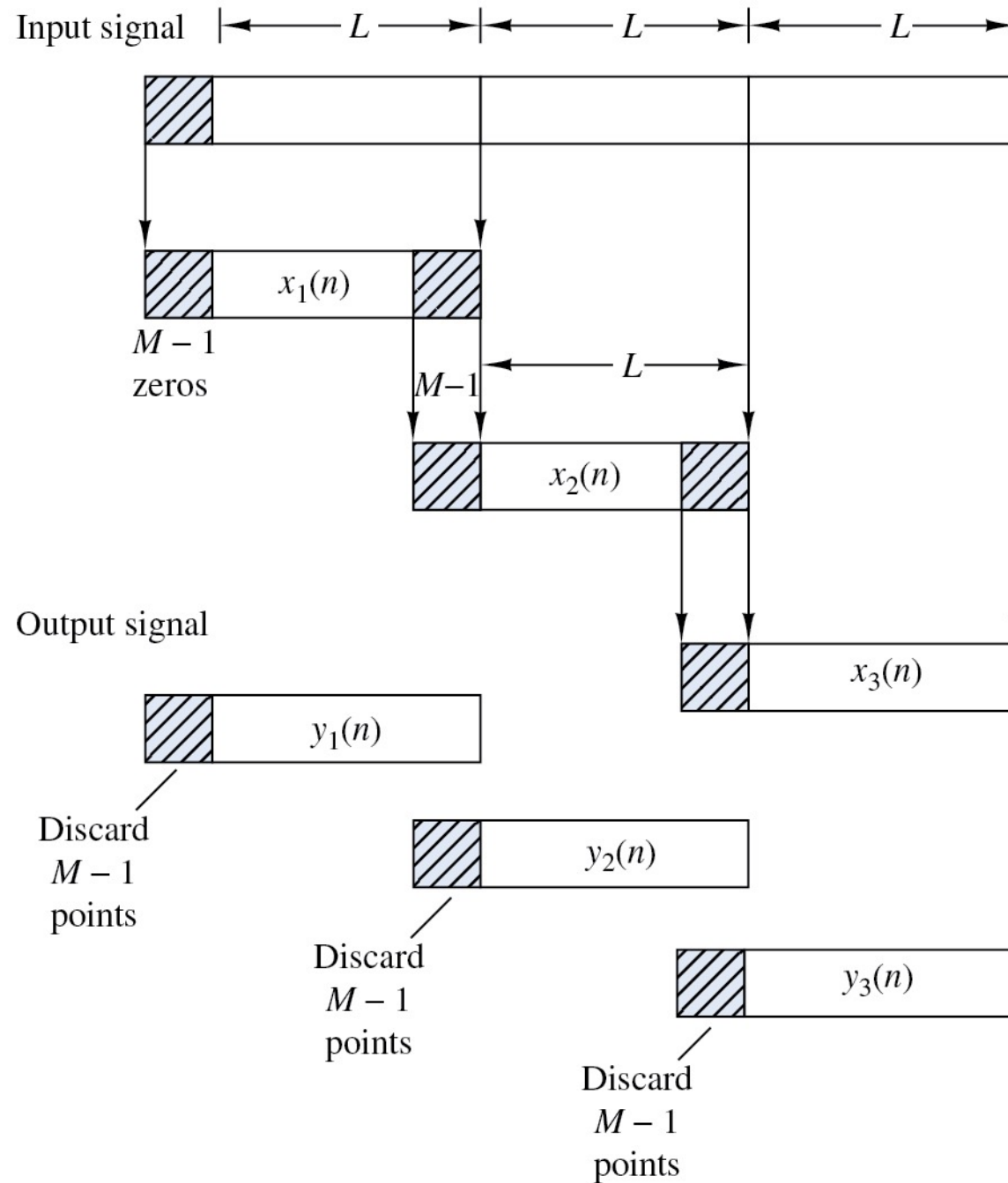


Figure 7.3.2 Linear FIR filtering by the overlap-add method.

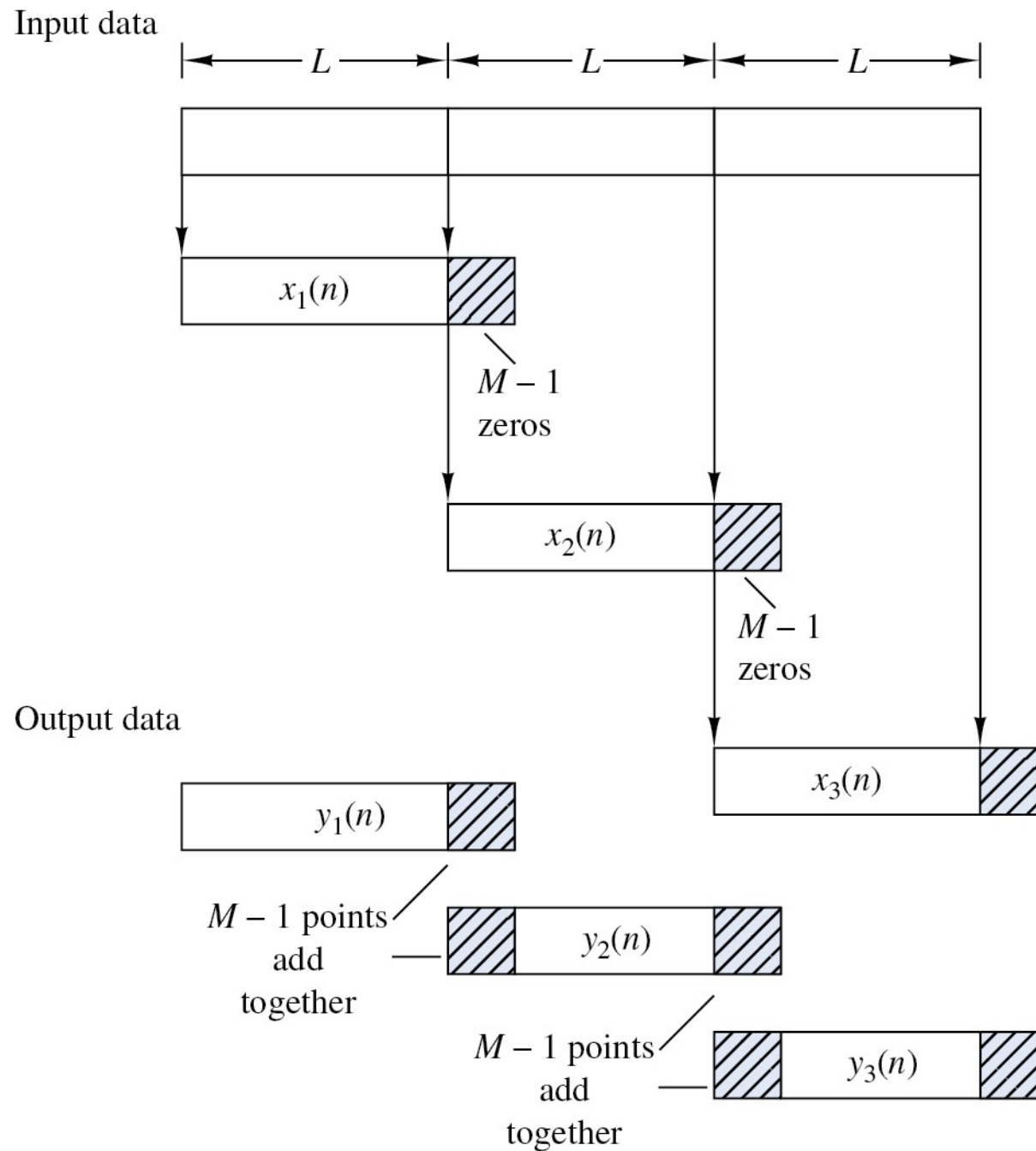


Figure 7.4.1 Magnitude spectrum for $L = 25$ and $N = 2048$, illustrating the occurrence of leakage.

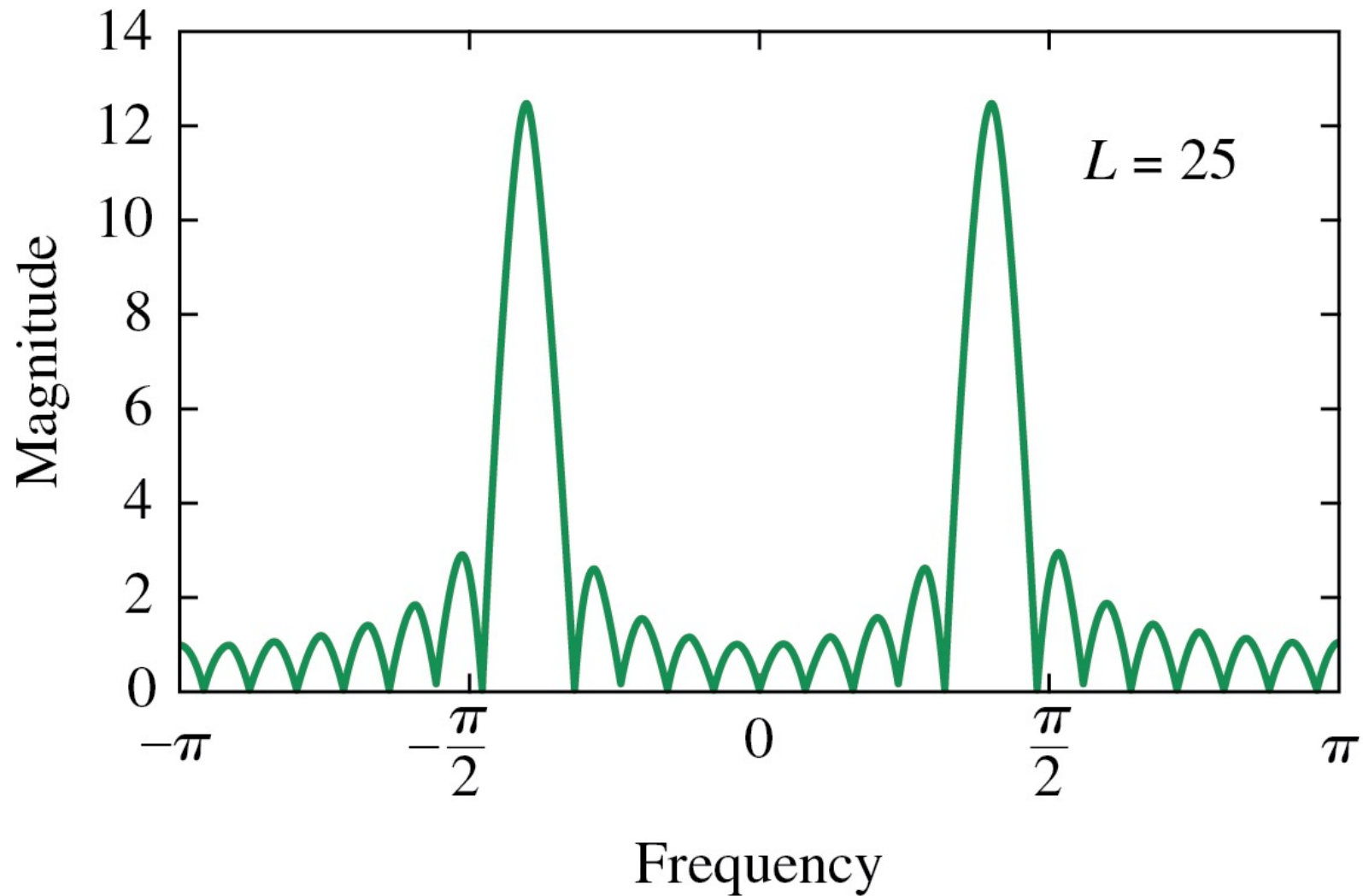
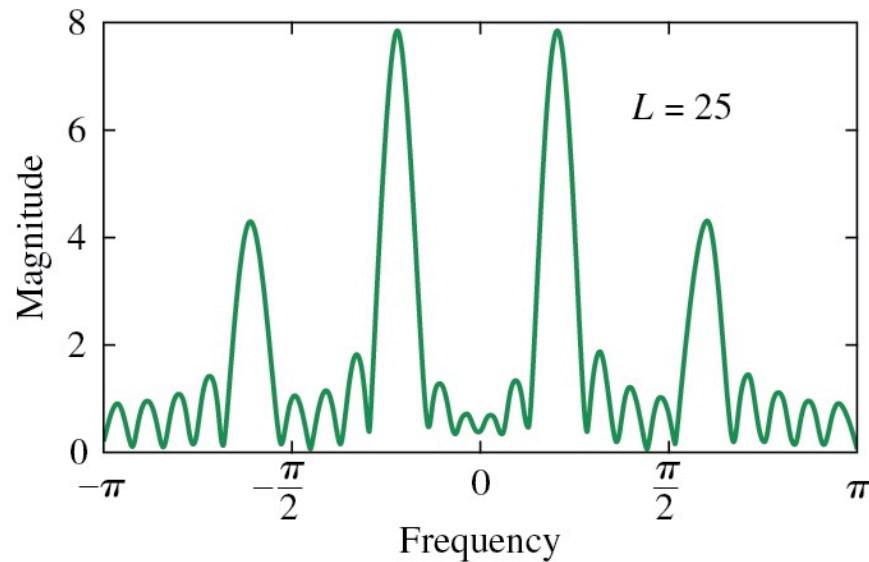
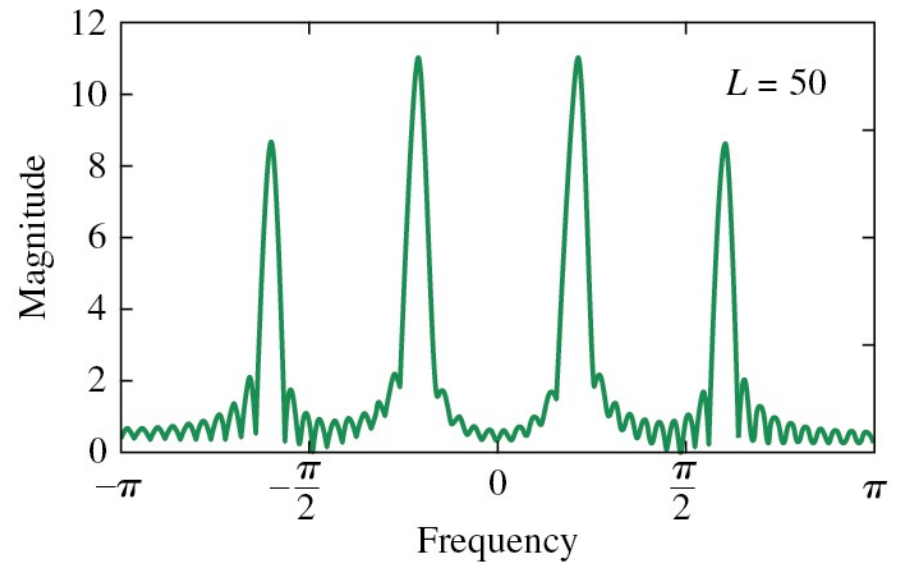


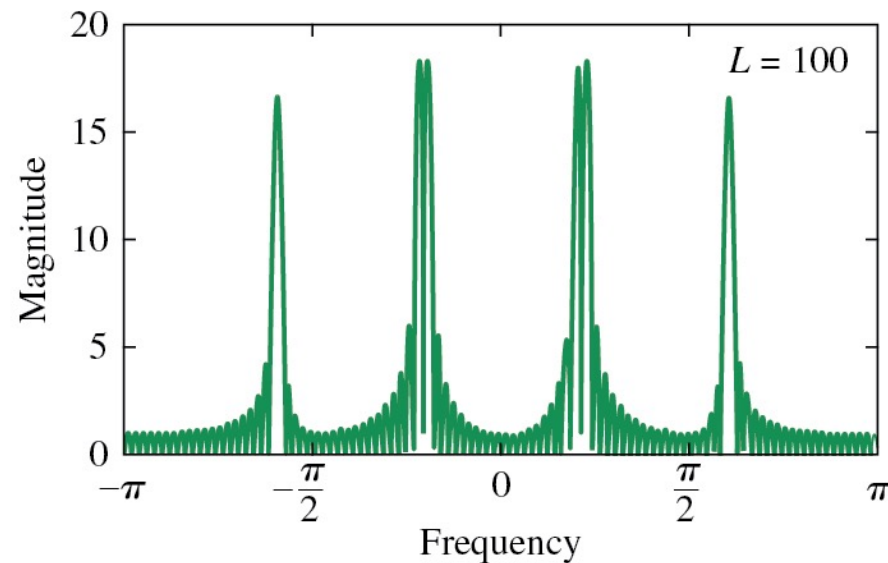
Figure 7.4.2 Magnitude spectrum for the signal given by (7.4.8), as observed through a rectangular window.



(a)



(b)



(c)

Figure 7.4.3 Magnitude spectrum of the Hanning window.

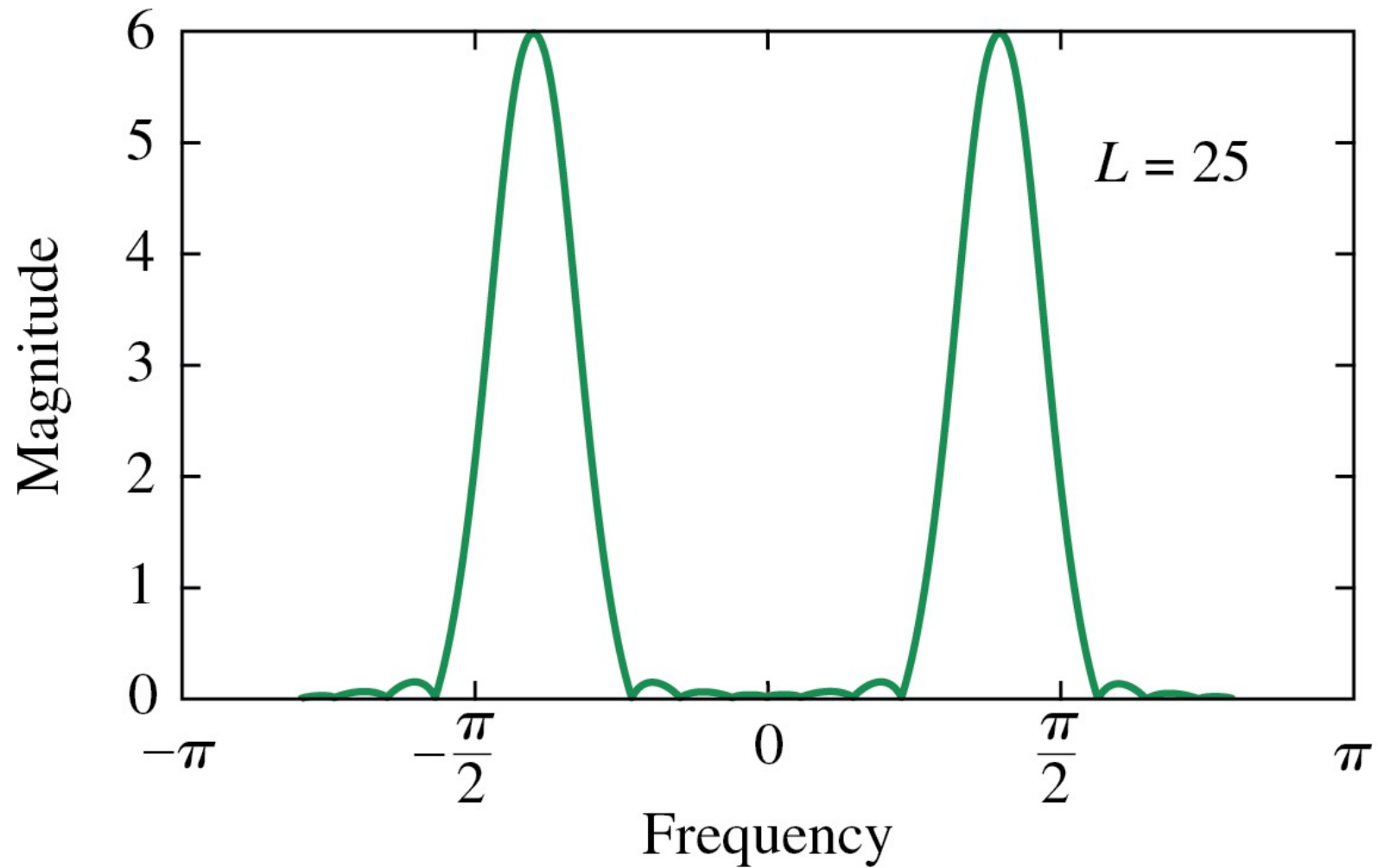


Figure 7.4.4 Magnitude spectrum of the signal in (7.4.8) as observed through a Hanning window.

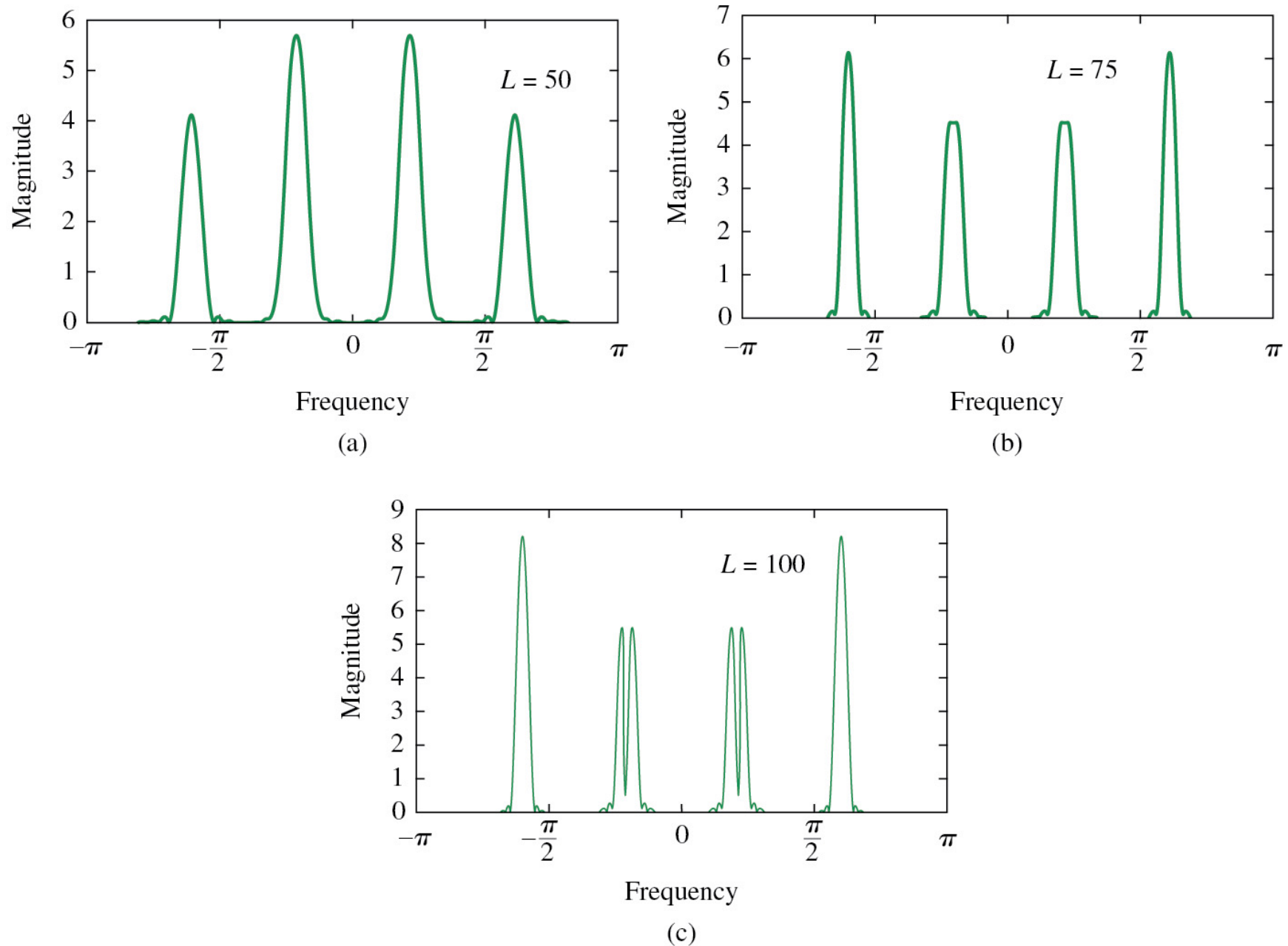


Figure 7.4.5 Effect of windowing (truncating) the sampled version of the analog signal in Example 7.4.1.

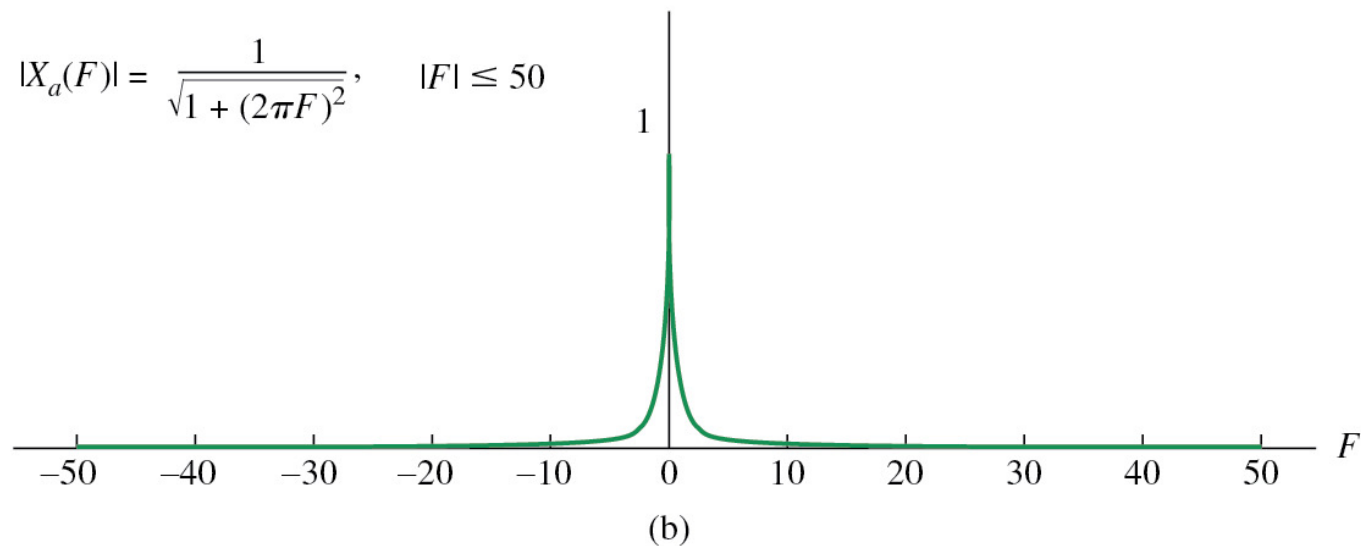
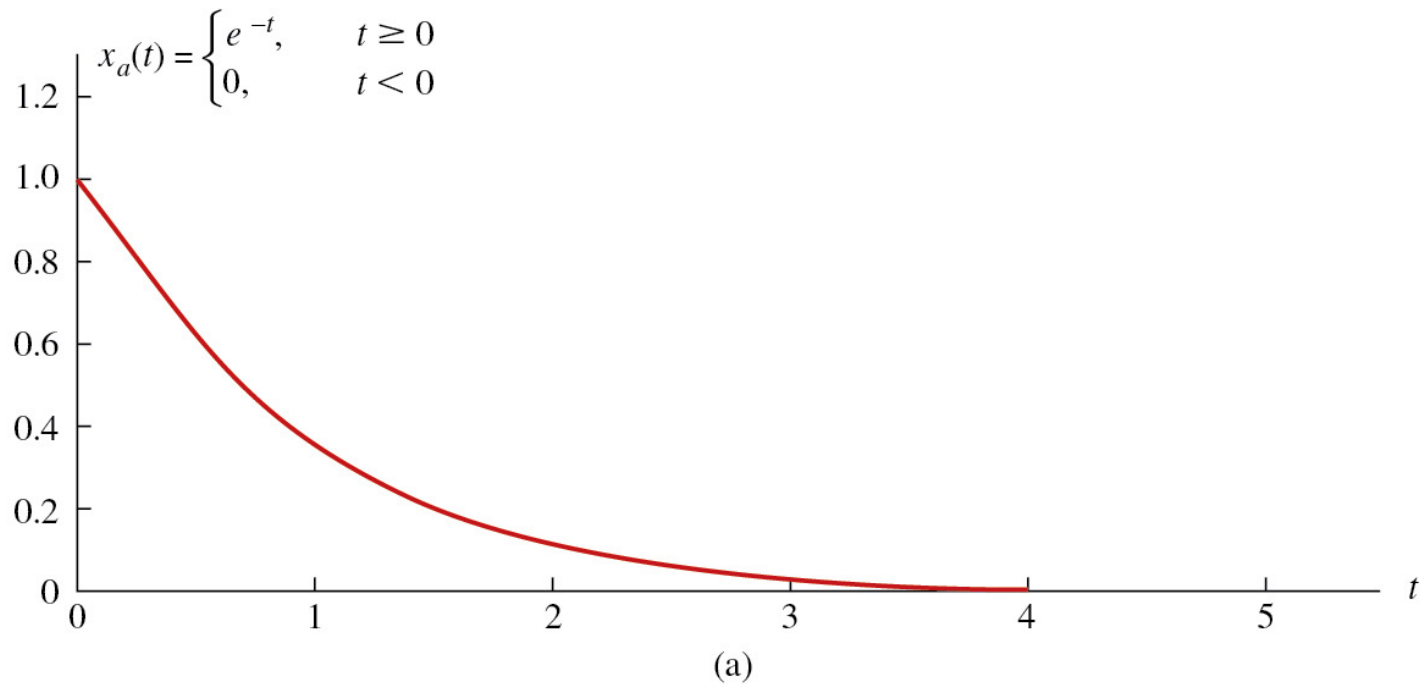
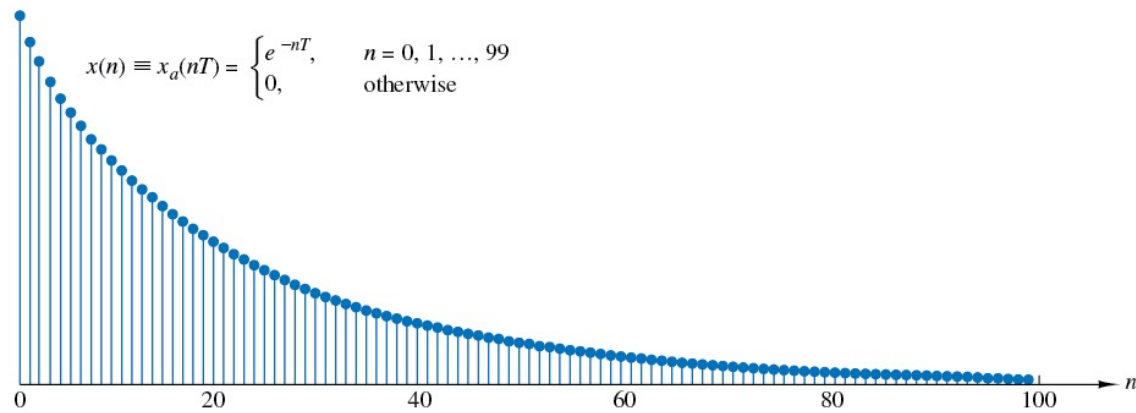
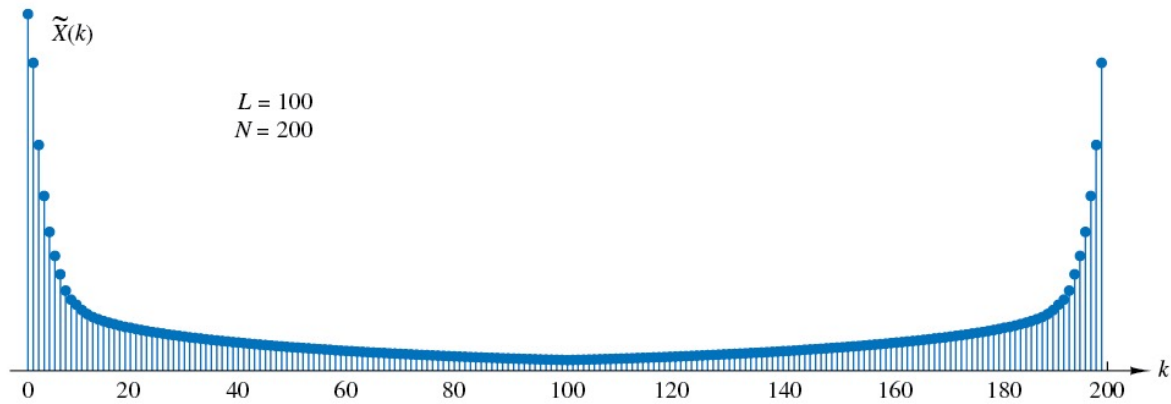


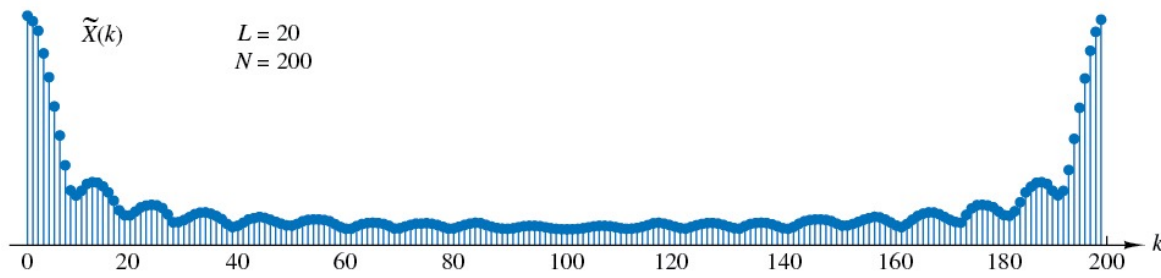
Figure 7.4.5 (Continued)



(c)



(d)



(e)

Figure 7.4.6 Illustration of bandlimited interpolation using the DFT.

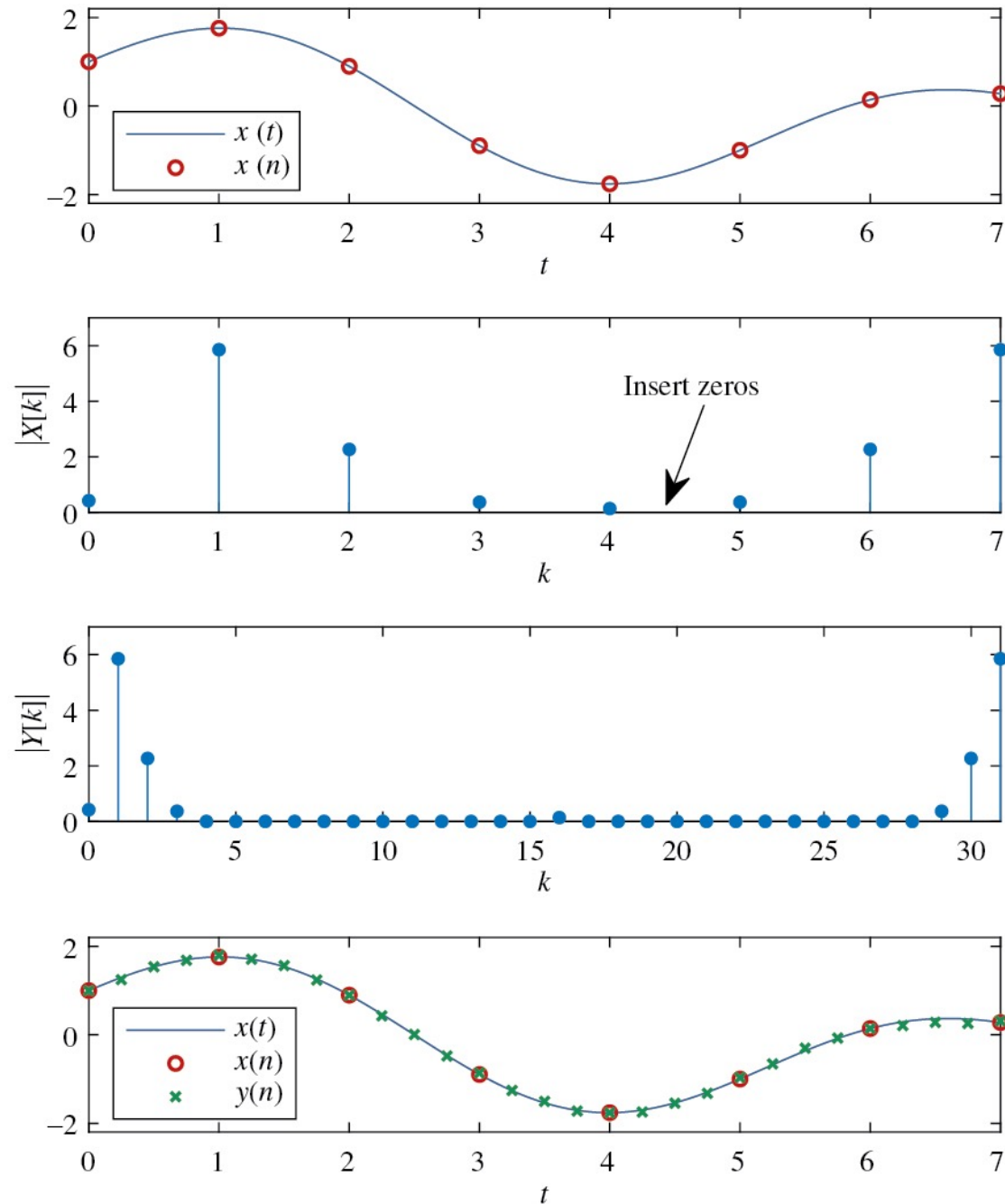


Figure 7.5.1 The STFT for the sinusoidal signal defined by equation (7.5.11).

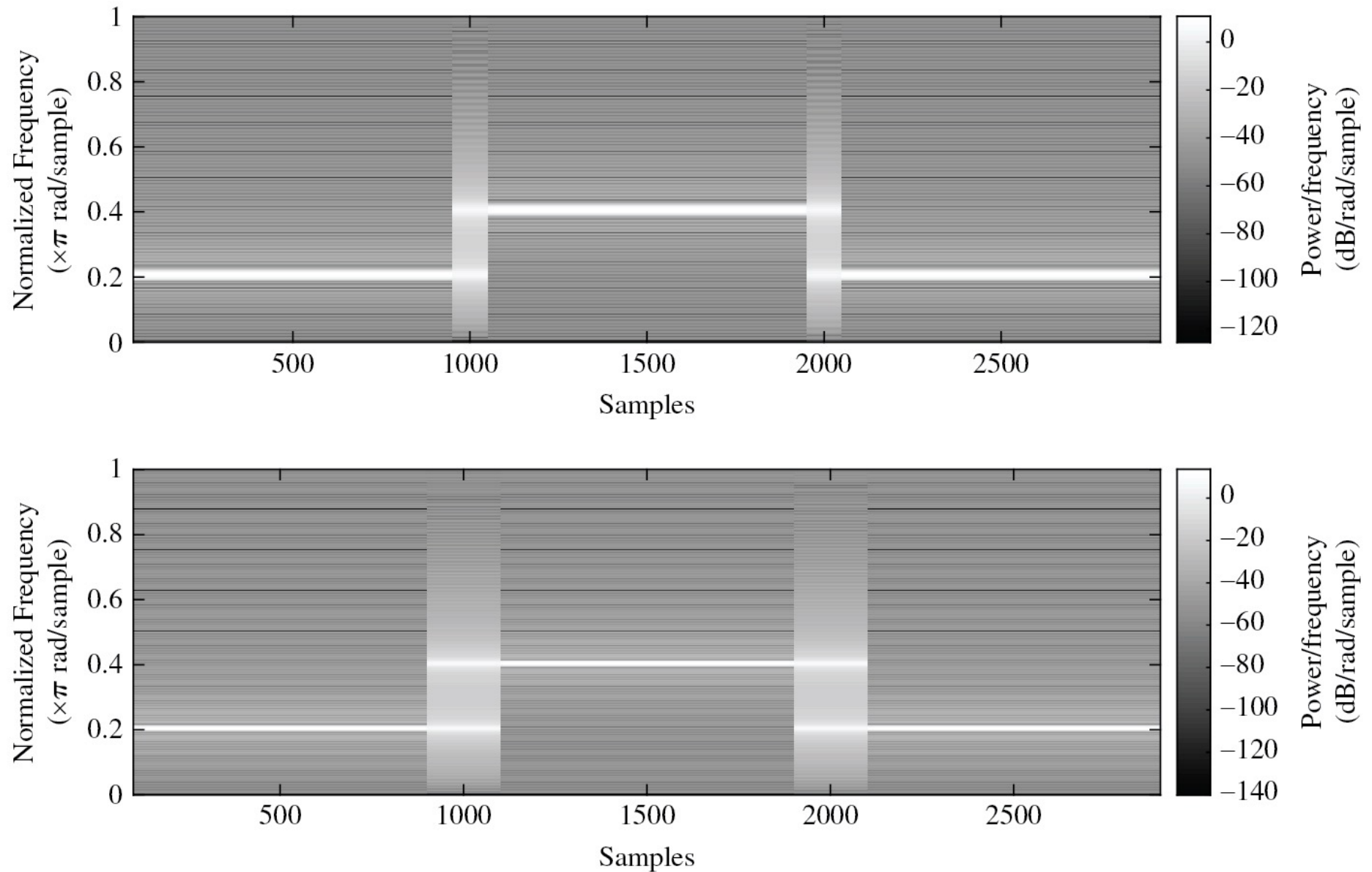
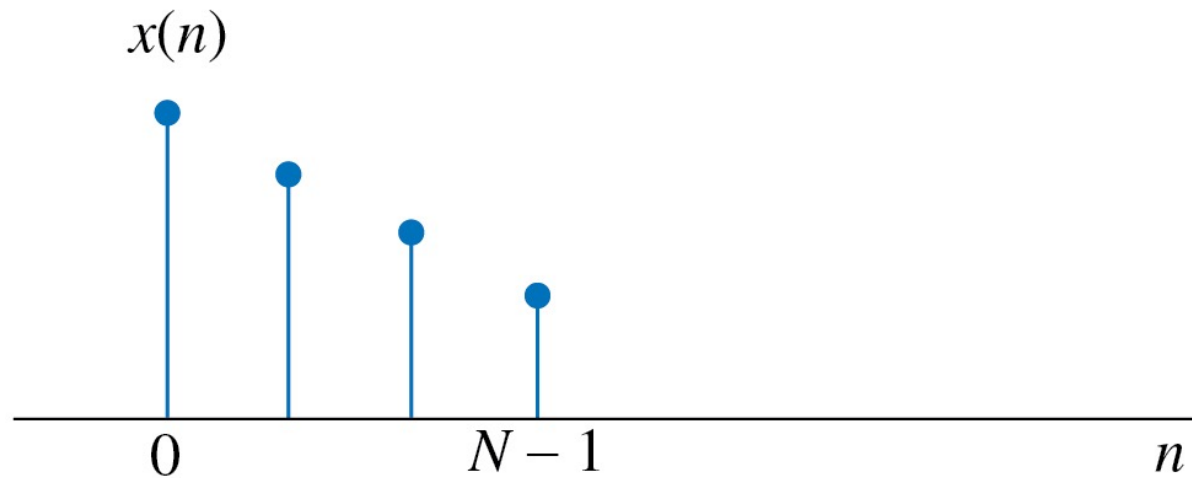
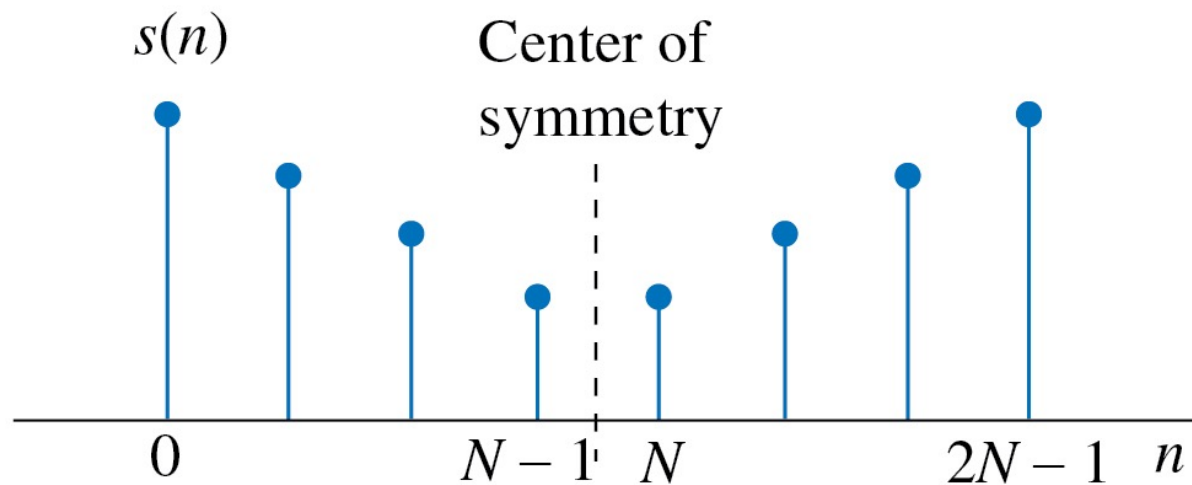


Figure 7.6.1 Original sequence $x(n)$, $0 \leq n \leq N - 1$ and its $2N$ -point even extension $s(n)$, $0 \leq n \leq 2N - 1$.



(a)



(b)

Figure 7.6.2 A discrete-time sinusoidal signal and its DFT and DCT representations.

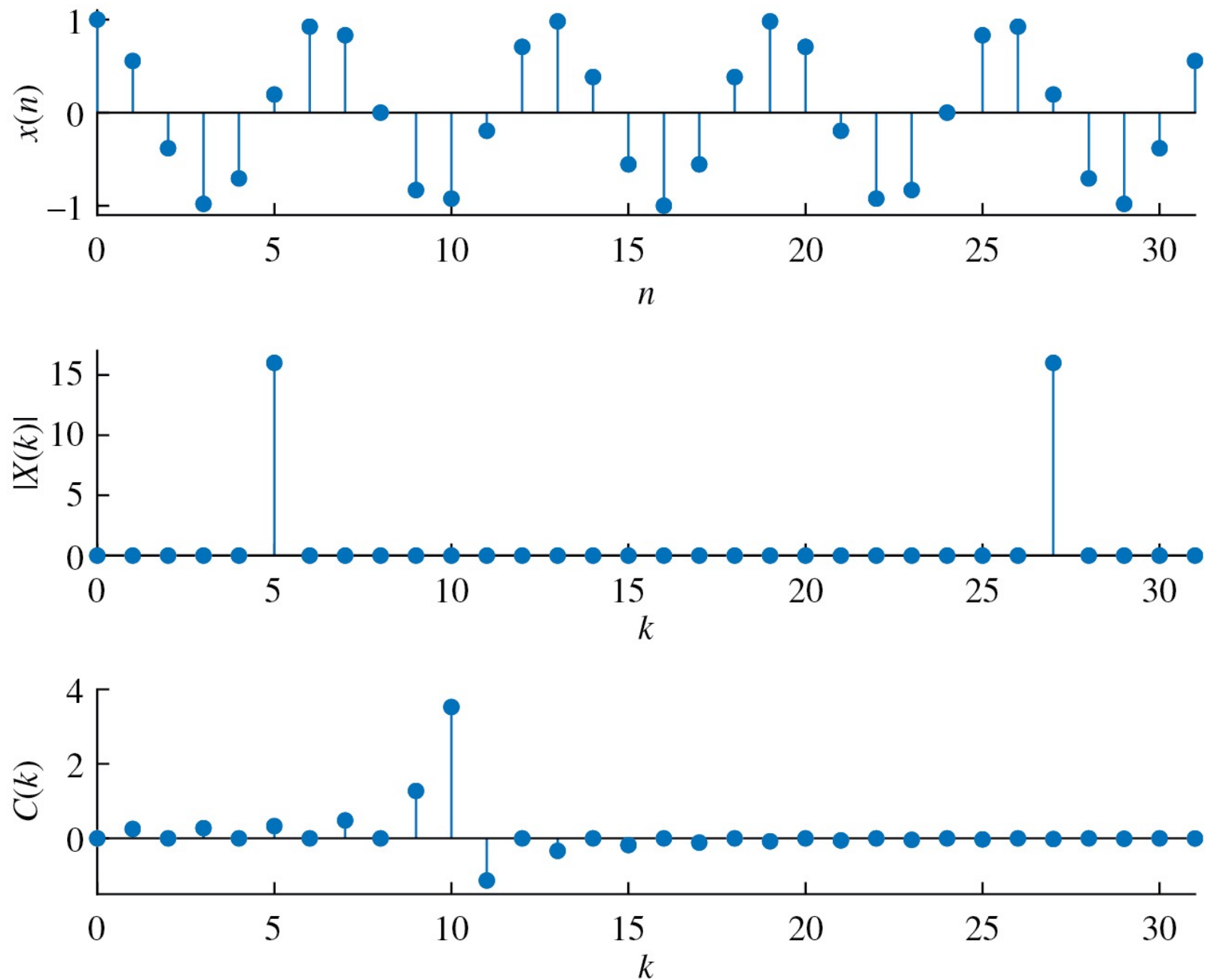
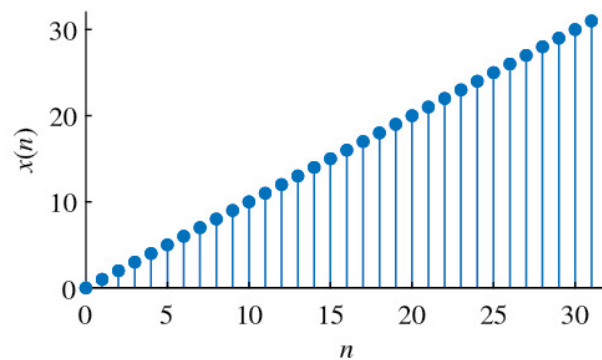
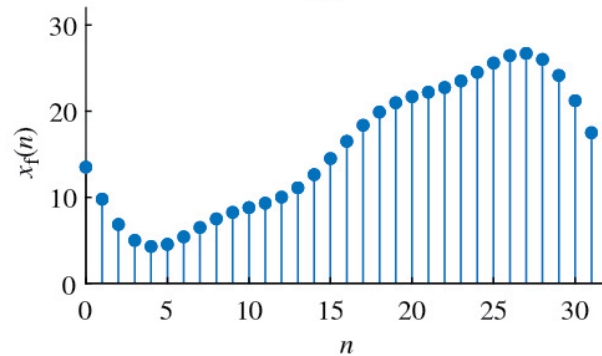


Figure 7.6.3 A discrete time sinusoidal signal and its DFT and DCT representations.



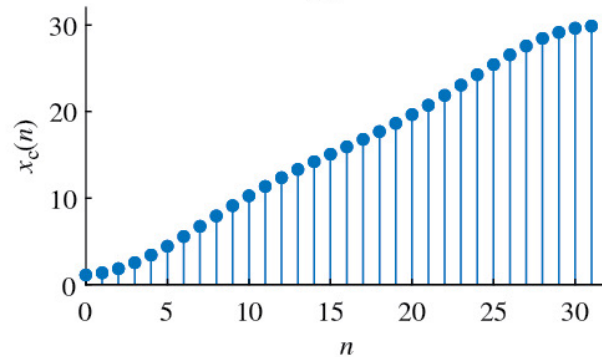
(a)

$k_{\text{zero}} = 27$

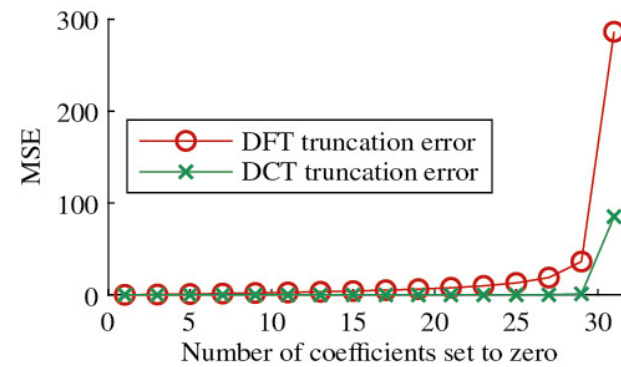


(c)

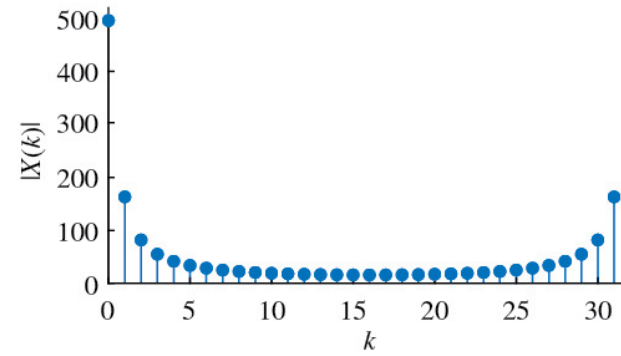
$k_{\text{zero}} = 27$



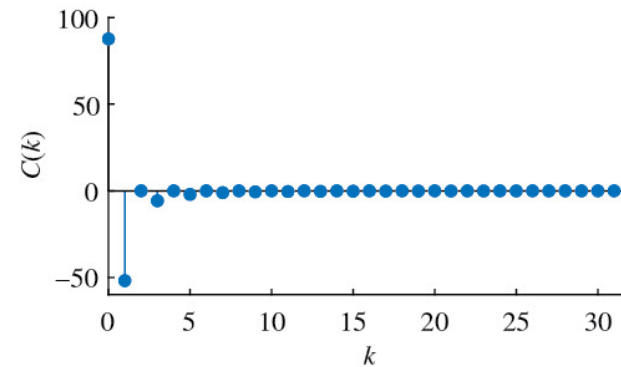
(e)



(b)

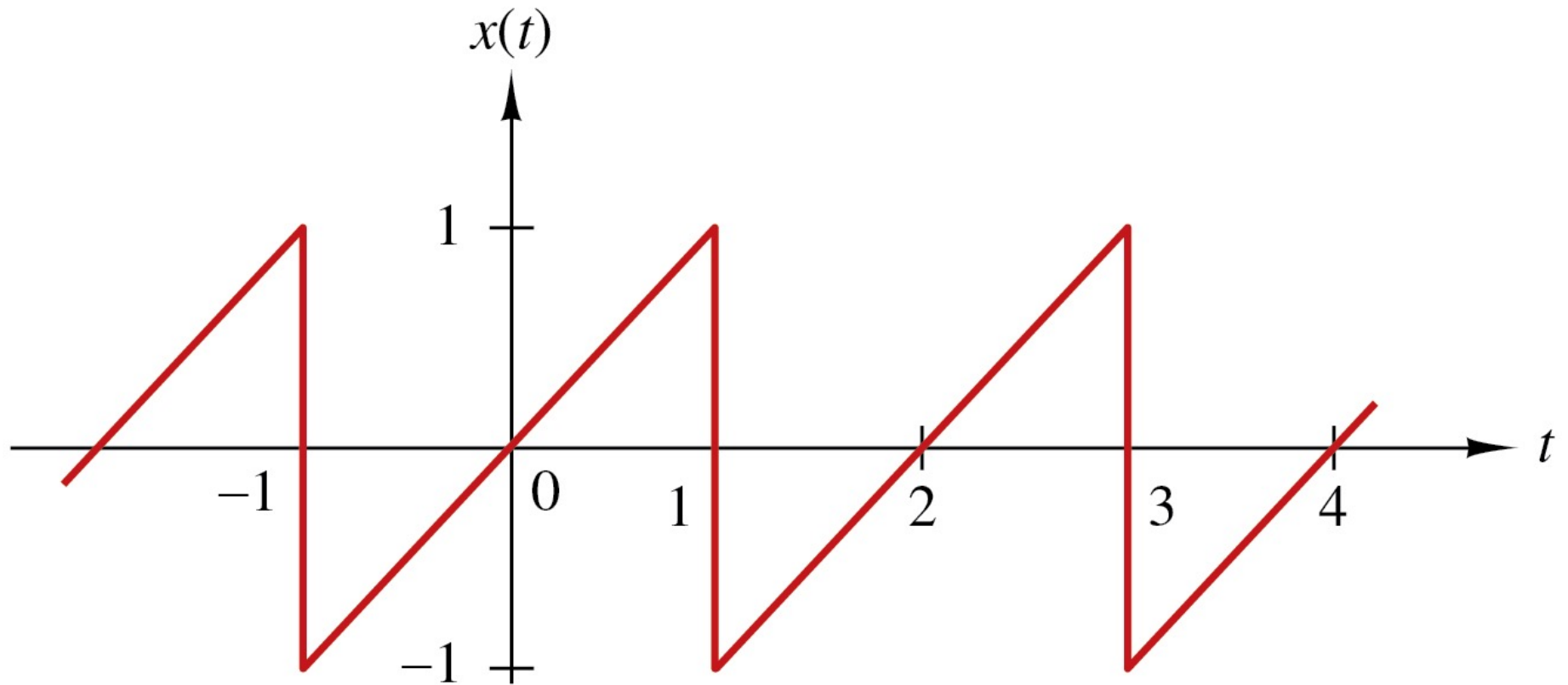


(d)



(f)

Figure CP7.8



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