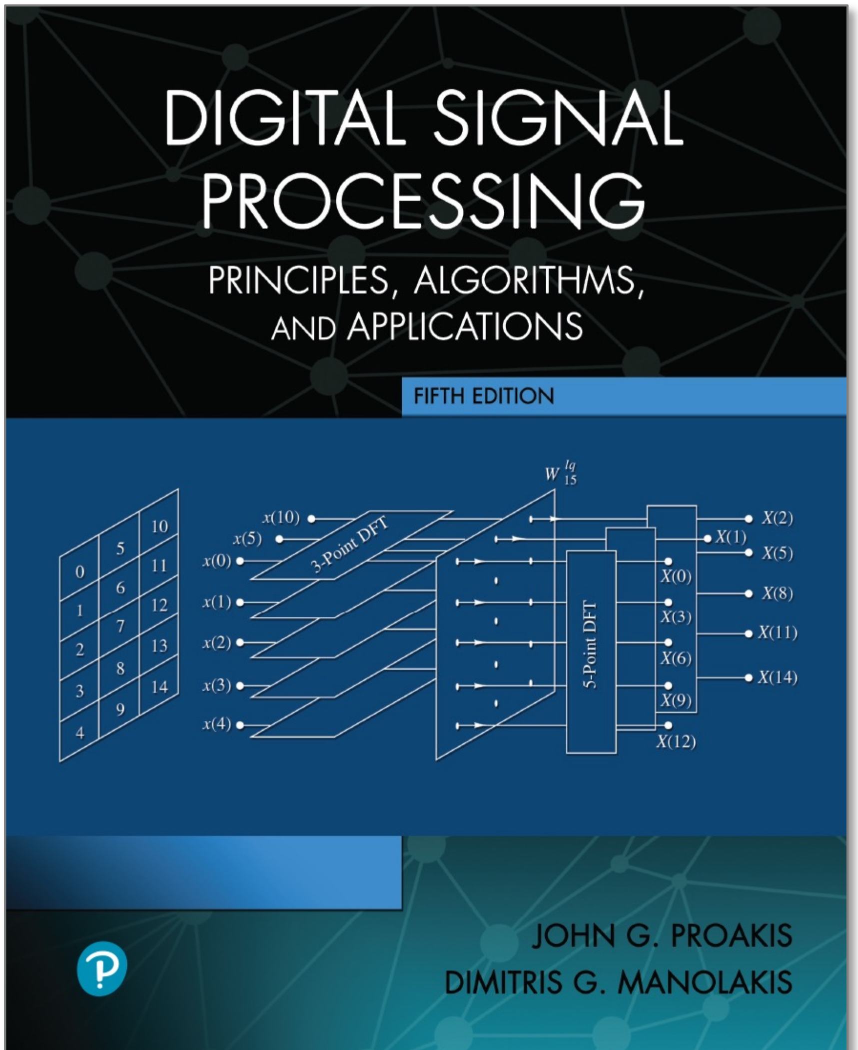
Digital Signal Processing

Fifth Edition



Chapter 7

The Discrete Fourier Transform: Its Properties and Applications

Pearson

Figure 7.1.1 Frequency-domain sampling of the Fourier transform.

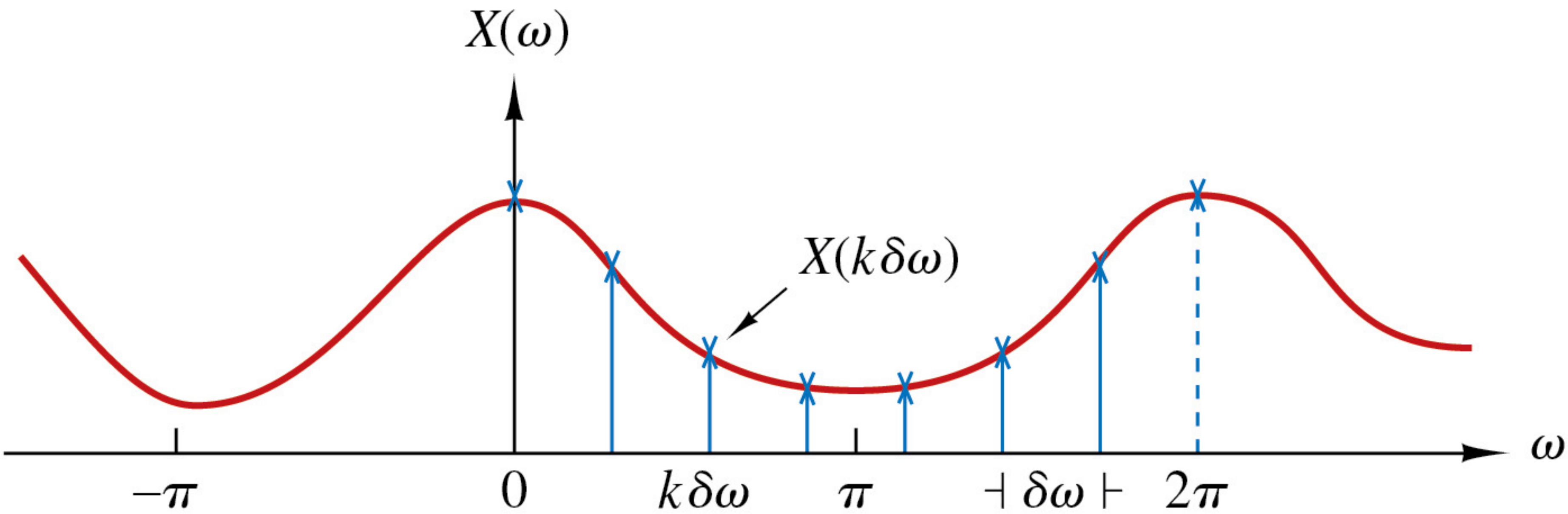


Figure 7.1.2 Aperiodic sequence $x(n)$ of length L and its periodic extension for $N \geq L$ (no aliasing) and $N < L$ (aliasing).

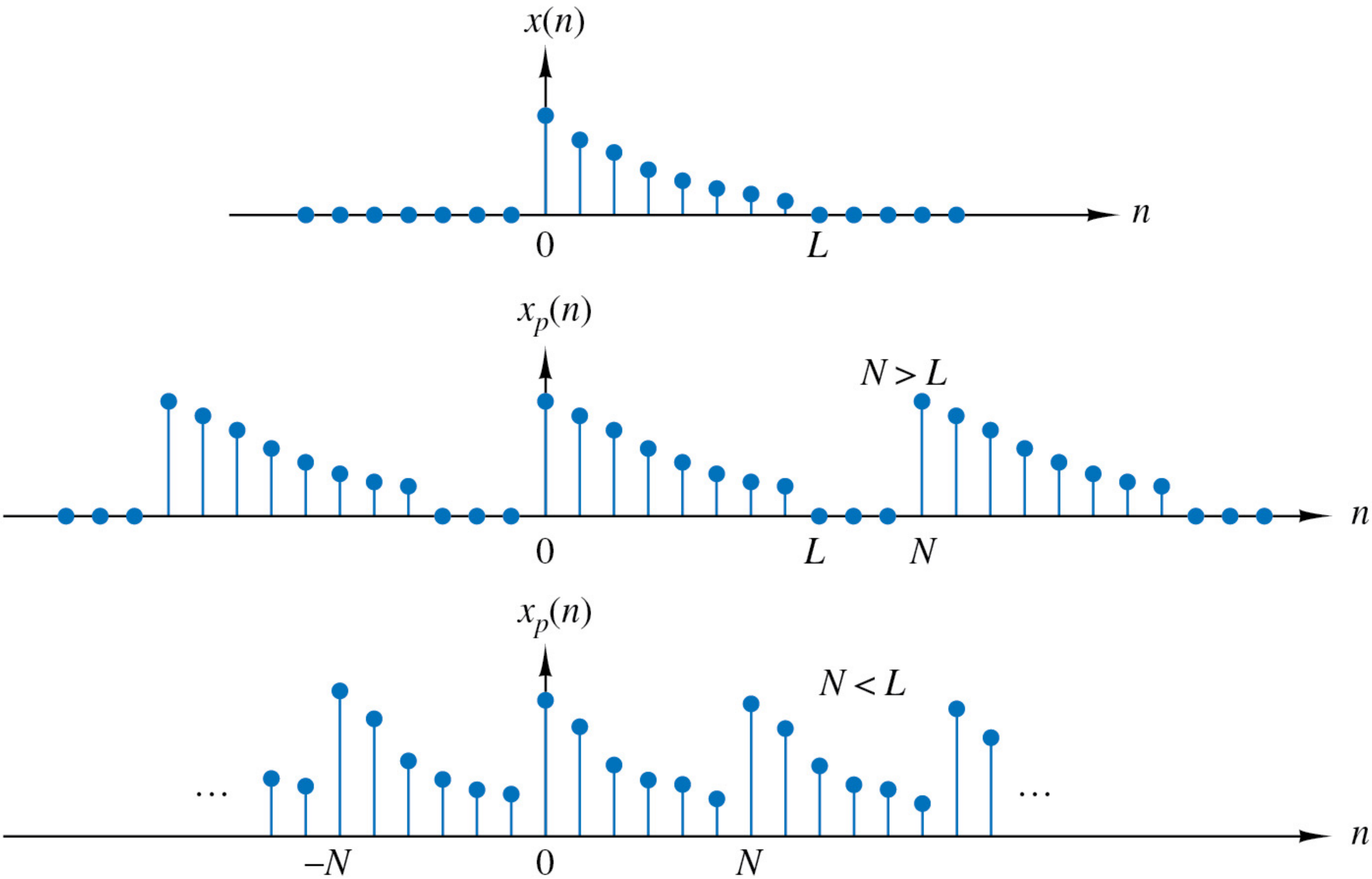


Figure 7.1.3 Plot of the function $[\sin(\omega N/2)]/[N \sin(\omega/2)]$.

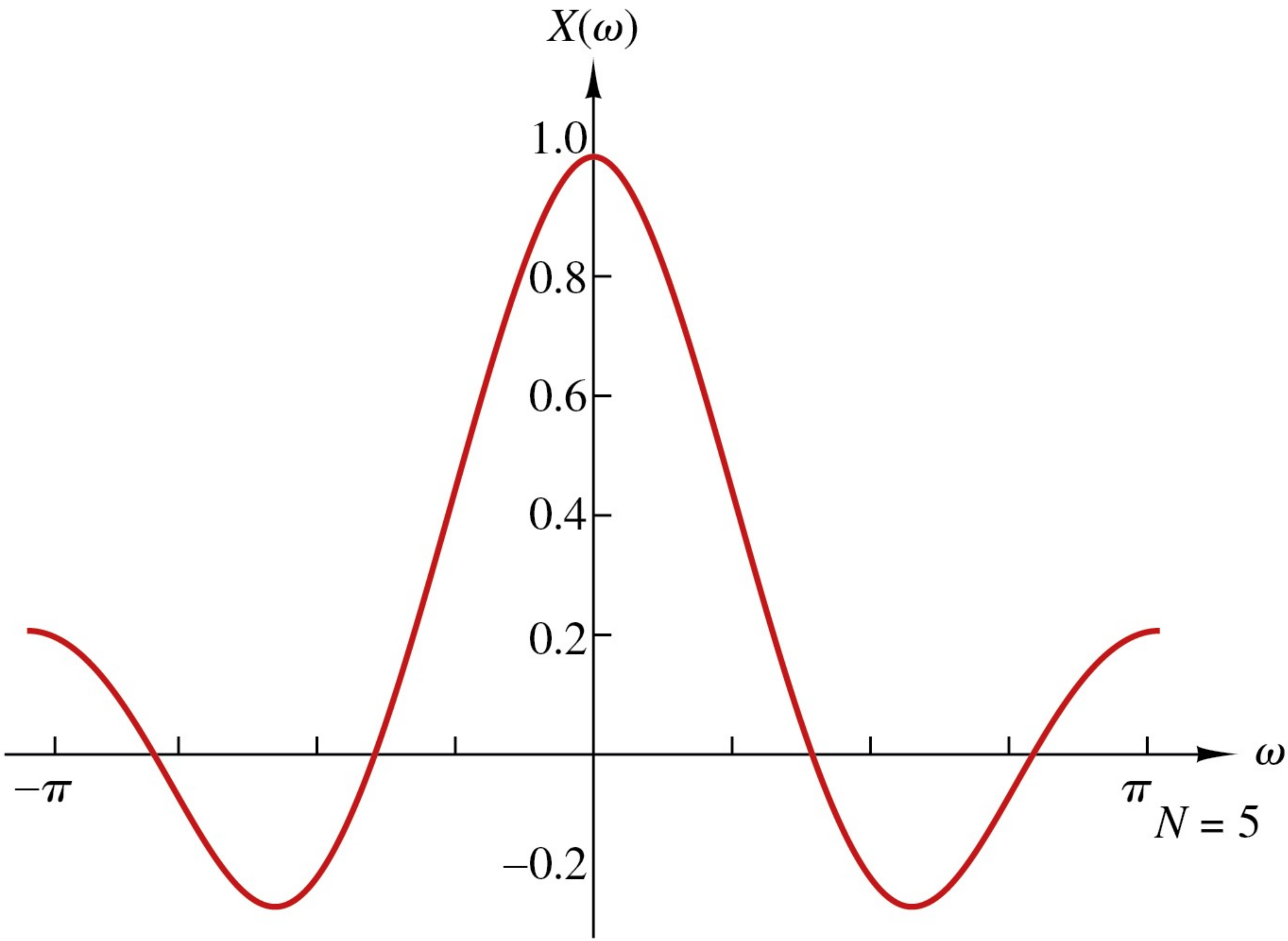


Figure 7.1.4 (a) Plot of sequence $x(n) = (0.8)^n u(n)$; (b) its Fourier transform (magnitude only); (c) effect of aliasing with $N = 5$; (d) reduced effect of aliasing with $N = 50$.

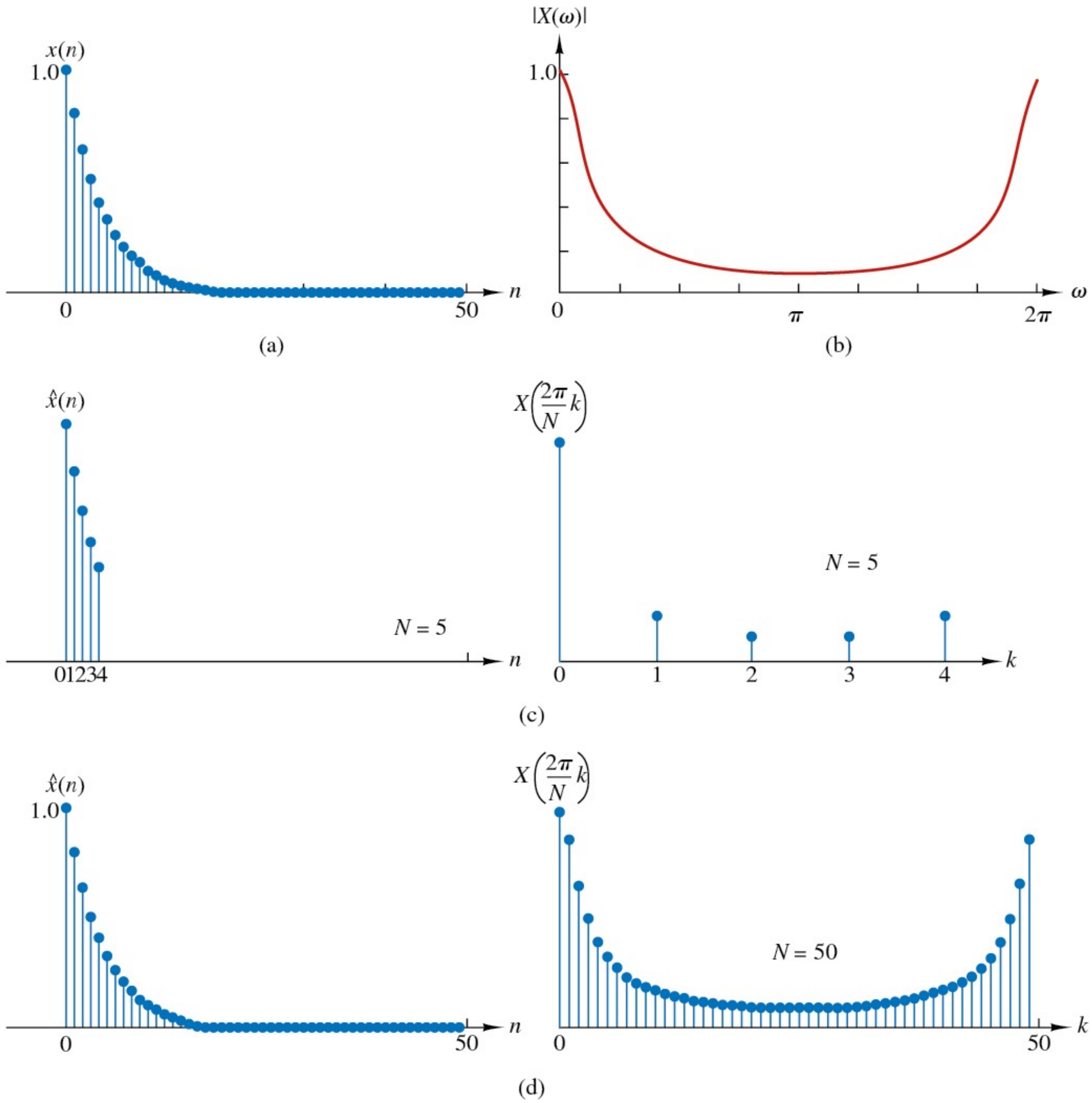


Figure 7.1.5 Magnitude and phase characteristics of the Fourier transform for the signal in Example 7.1.2.

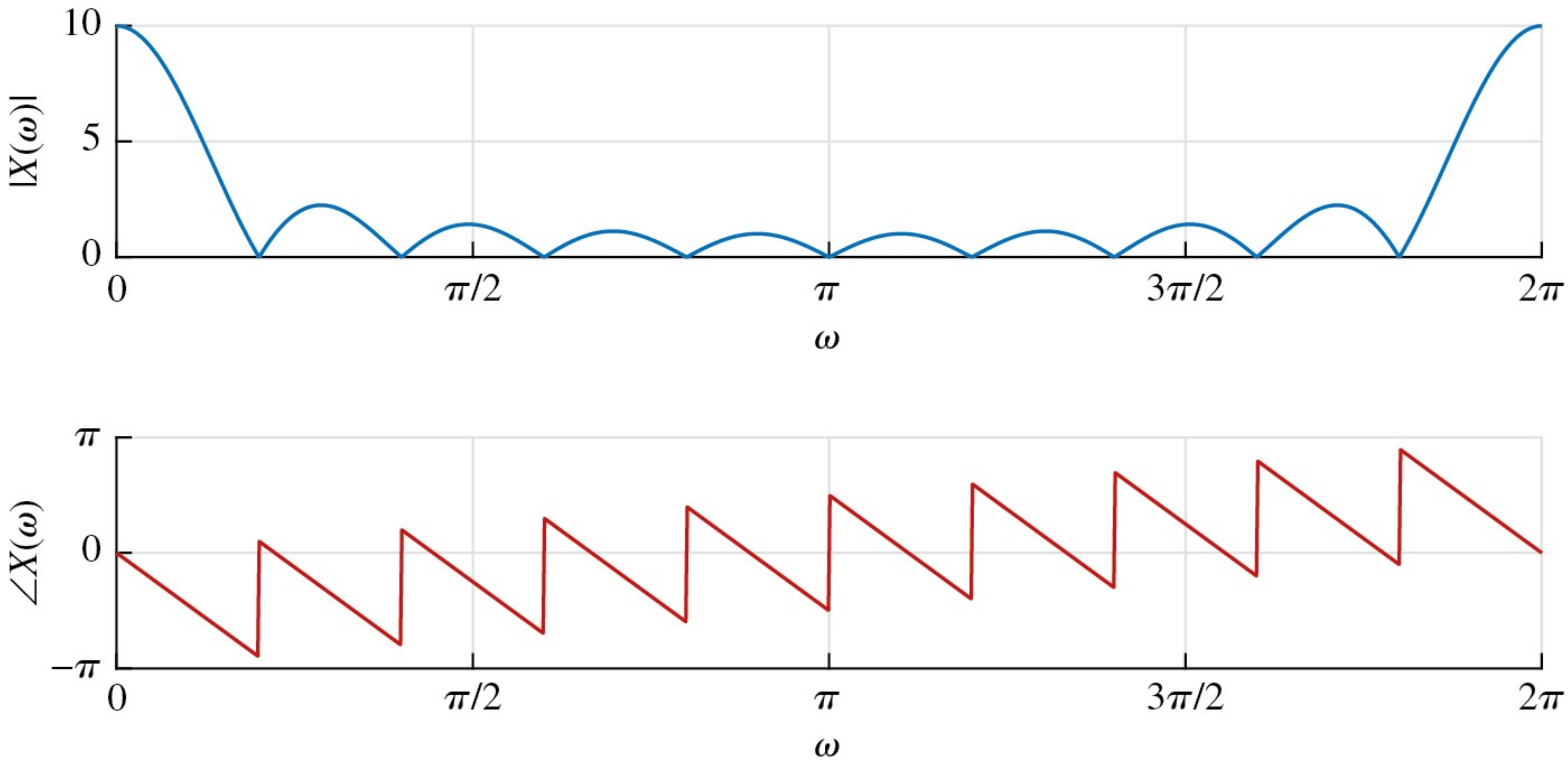
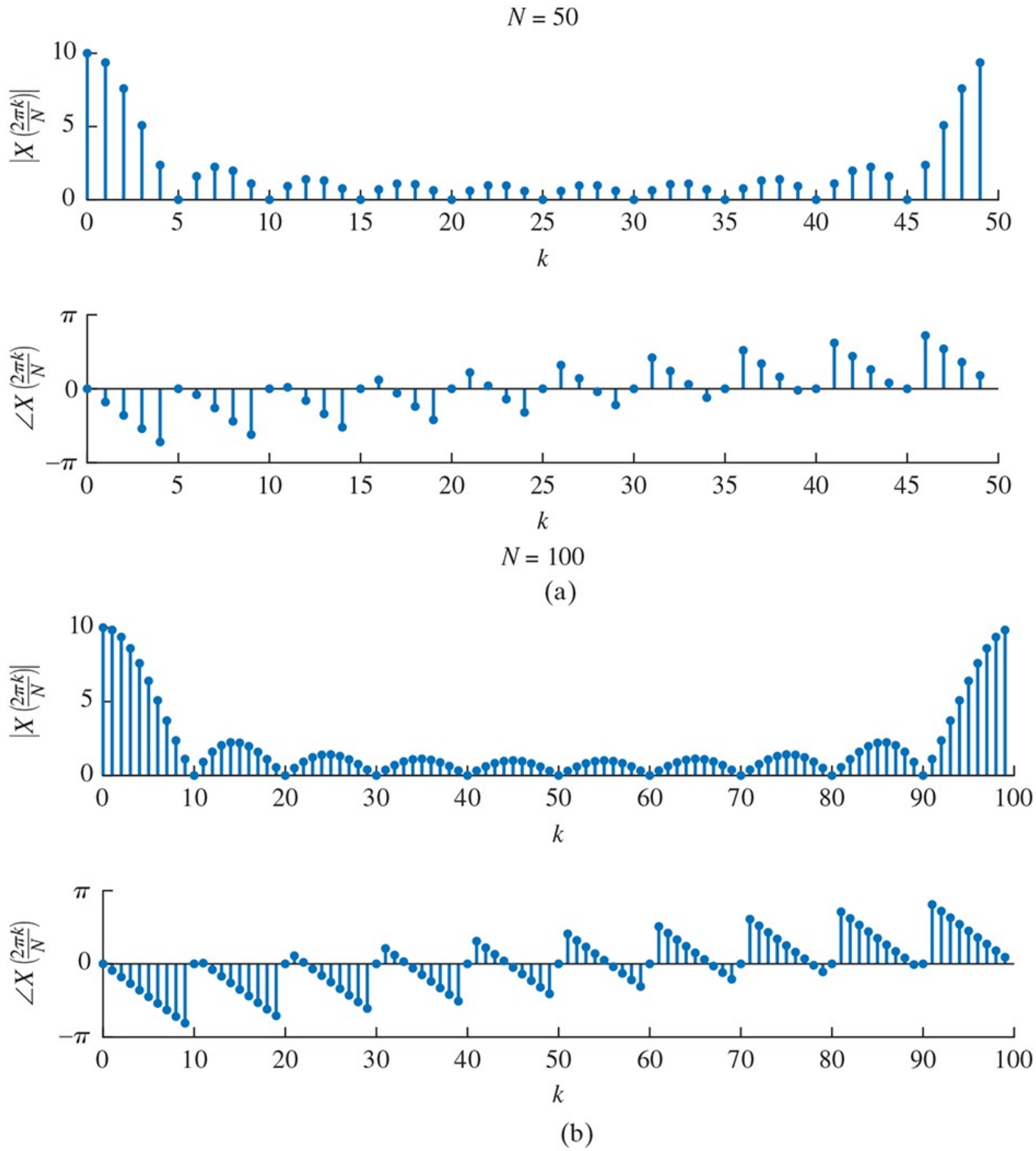
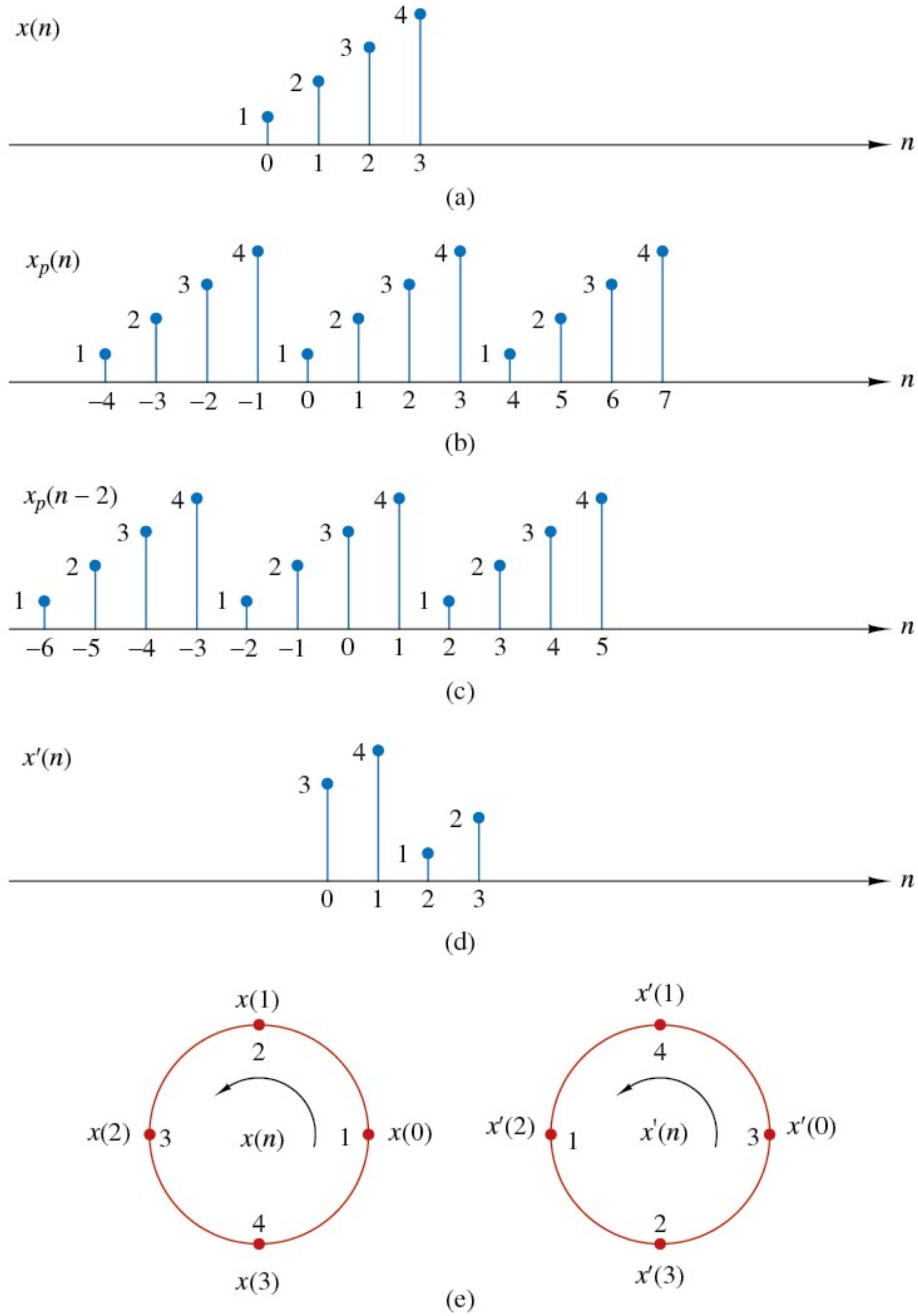


Figure 7.1.6 Magnitude and phase of an N-point DFT in Example 7.1.2; (a) L = 10, N = 50; (b) L = 10, N = 100.



Pearson

Figure 7.2.1 Circular shift of a sequence.



Equation 7.2.31

$$x(n) = x_R^e(n) + x_R^o(n) + jx_I^e(n) + jx_I^o(n)$$

$$X(k) = X_R^e(k) + X_R^o(k) + jX_I^e(k) + jX_I^o(k)$$

Table 7.1 Symmetry Properties of the DFT

N-Point Sequence $x(n)$, $0 \le n \le N-1$	N-Point DFT
$x(n)$	$X(k)$
$x^*(n)$	$X^*(N-k)$
$x^*(N-n)$	$X^*(k)$
$x_R(n)$	$X_{ce}(k) = \frac{1}{2}[X(k) + X^*(N-k)]$
$jX_I(n)$	$X_{co}(k) = \frac{1}{2}[X(k) - X^*(N-k)]$
$x_{ce}(n) = \frac{1}{2}[x(n) + x^*(N-n)]$	$X_R(k)$
$x_{co}(n) = \frac{1}{2}[x(n) - x^*(N-n)]$	$jX_I(k)$
Real Signals	
Any real signal	$X(k) = X^*(N-k)$
$x(n)$	$X_R(k) = X_R(N-k)$
	$X_I(k) = -X_I(N-k)$
	$\lvert X(k) \rvert = \lvert X(N-k) \rvert$
	$\angle X(k) = -\angle X(N-k)$
$x_{ce}(n) = \frac{1}{2}[x(n) + x(N-n)]$	$X_R(k)$
$x_{co}(n) = \frac{1}{2}[x(n) - x(N-n)]$	$jX_I(k)$

Figure 7.2.2 Circular convolution of two sequences.

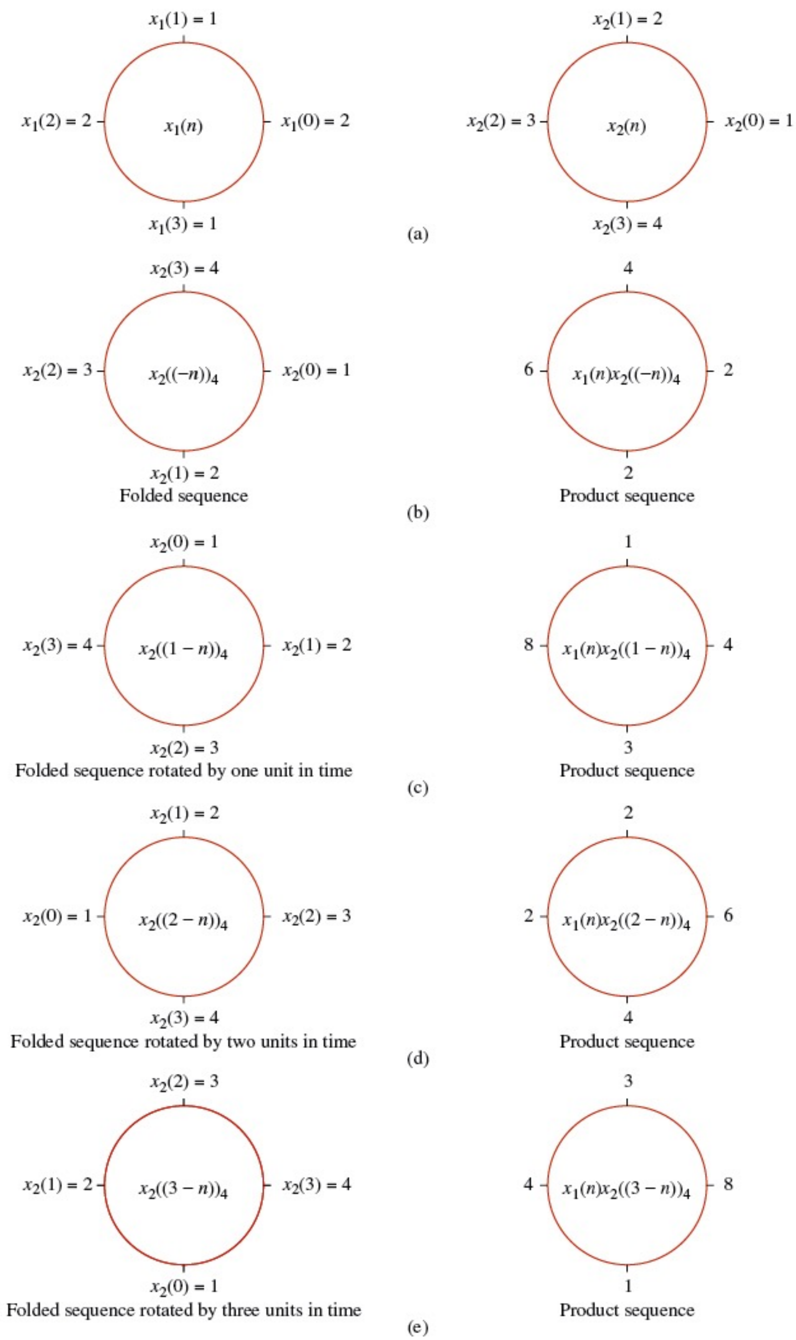


Figure 7.2.3 Time reversal of a sequence.

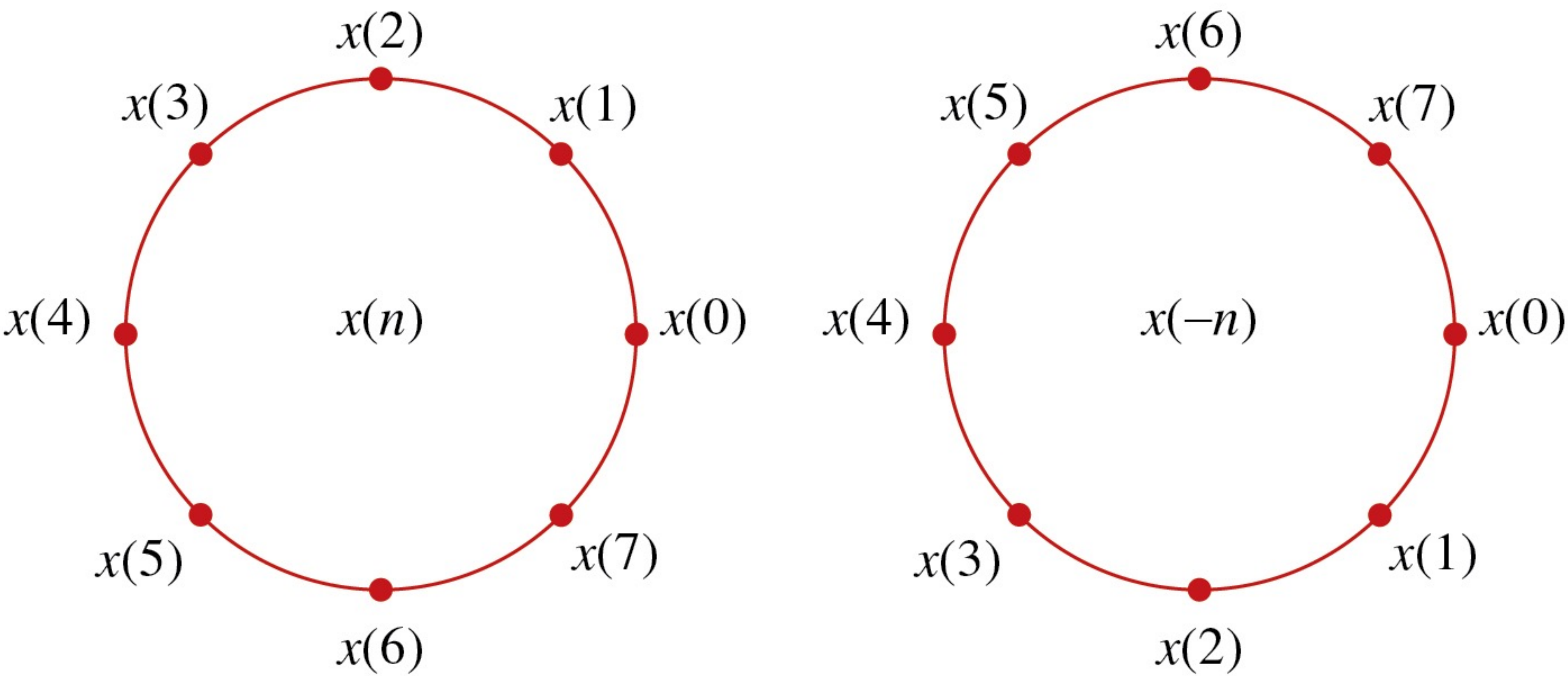


Table 7.2 Properties of the DFT

Property	Time Domain	Frequency Domain
Notation	$x(n), y(n)$	$X(k), Y(k)$
Periodicity	$x(n) = x(n+N)$	$X(k) = X(k+N)$
Linearity	$a_1x_1(n) + a_2x_2(n)$	$a_1X_1(k) + a_2X_2(k)$
Time reversal	$x(N-n)$	$X(N-k)$
Circular time shift	$x((n-l))_N$	$X(k)e^{-j2\pi kl/N}$
Circular frequency shift	$x(n)e^{j2\pi ln/N}$	$X((k-l))_N$
Complex conjugate	$x^*(n)$	$X^*(N-k)$
Circular convolution	$x_1(n) \textcircled{N} x_2(n)$	$X_1(k)X_2(k)$
Circular correlation	$x(n) \textcircled{N} y^*(-n)$	$X(k)Y^*(k)$
Multiplication of two sequences	$x_1(n)x_2(n)$	$\frac{1}{N}X_1(k) \textcircled{N} X_2(k)$
Parseval's theorem	$\sum_{n=0}^{N-1} x(n)y^*(n)$	$\frac{1}{N}\sum_{k=0}^{N-1} X(k)Y^*(k)$

Figure 7.3.1 Linear FIR filtering by the overlap-save method.

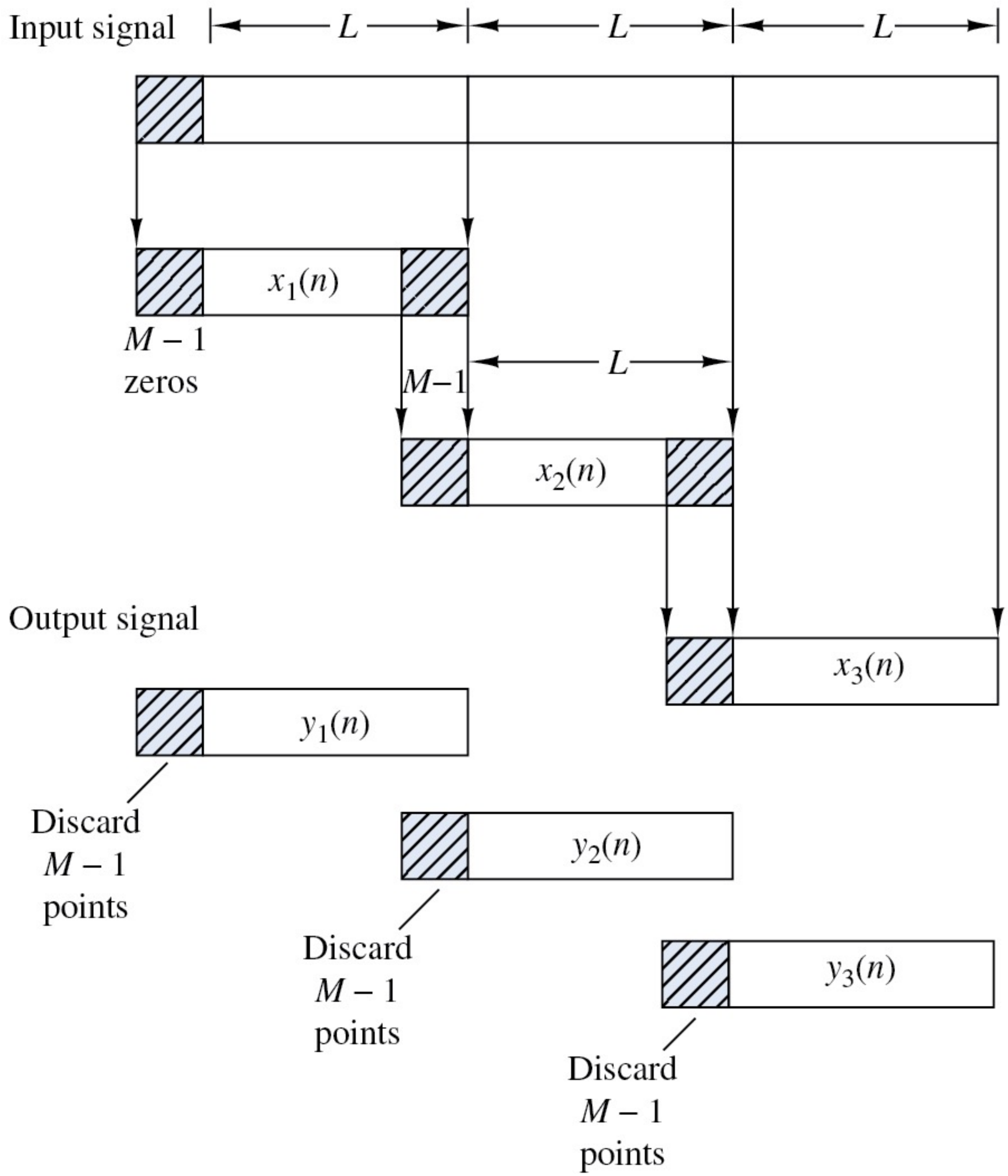
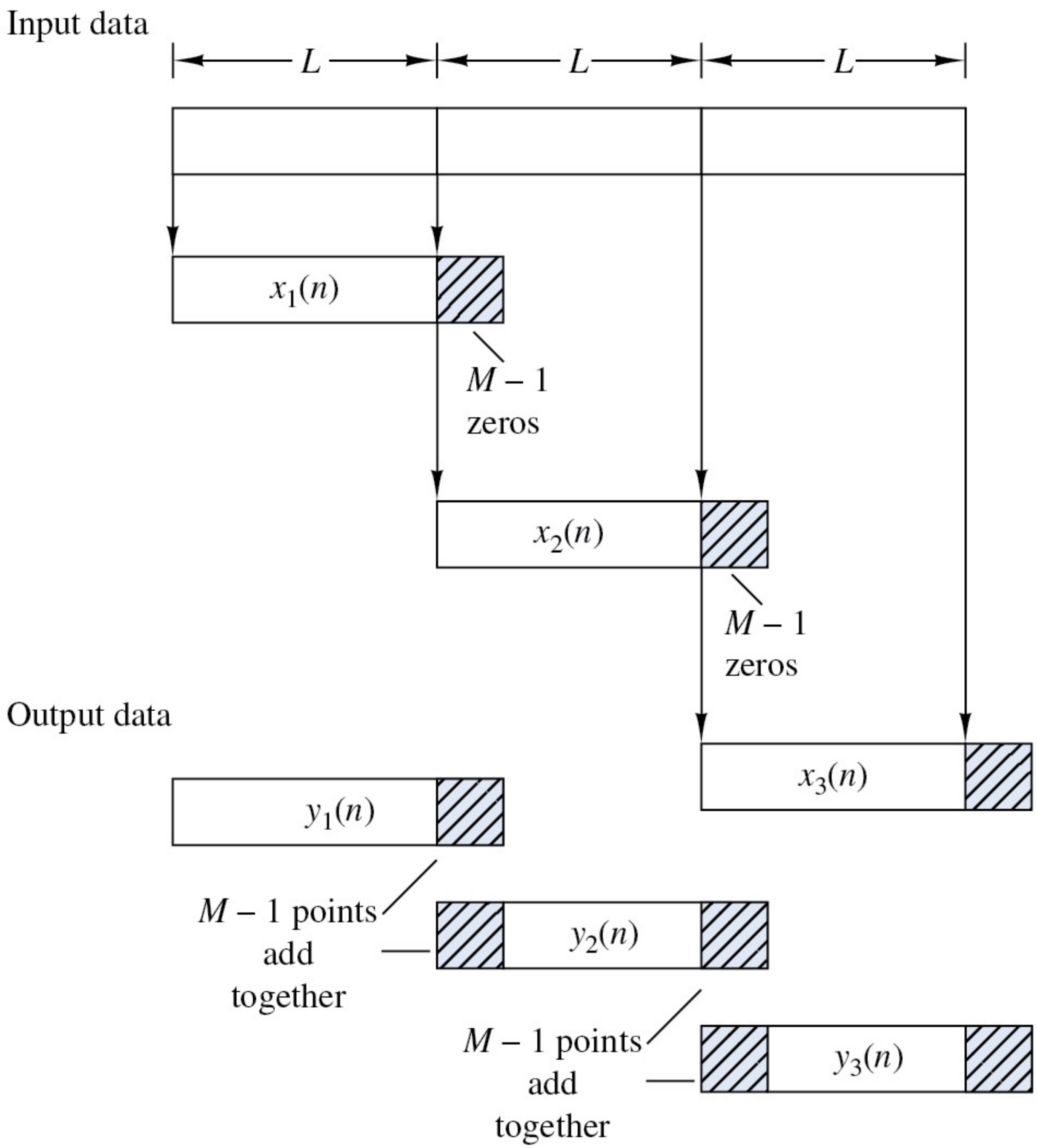


Figure 7.3.2 Linear FIR filtering by the overlap-add method.



Pearson

Figure 7.4.1 Magnitude spectrum for L = 25 and N = 2048, illustrating the occurrence of leakage.

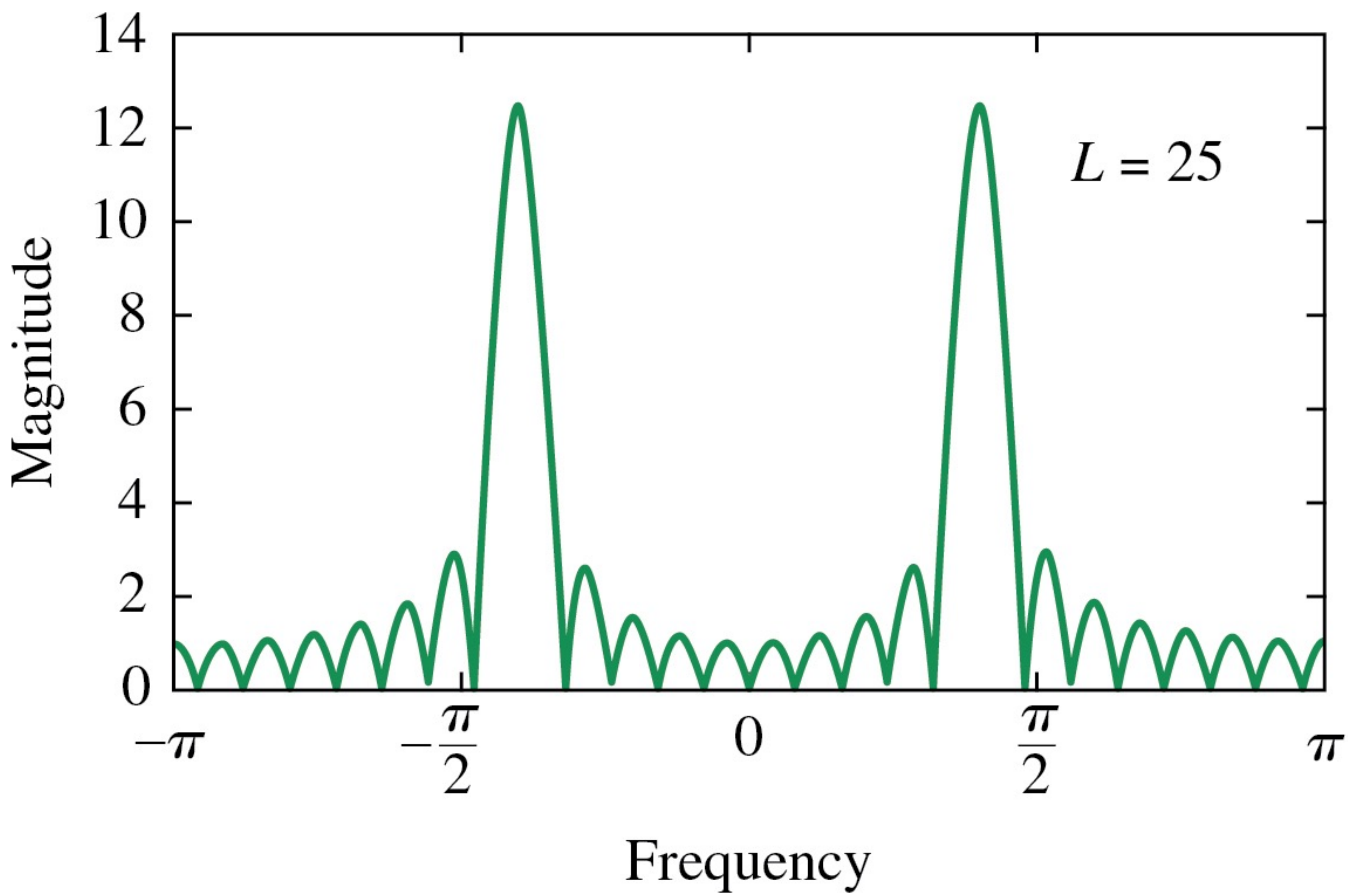


Figure 7.4.2 Magnitude spectrum for the signal given by (7.4.8), as observed through a rectangular window.

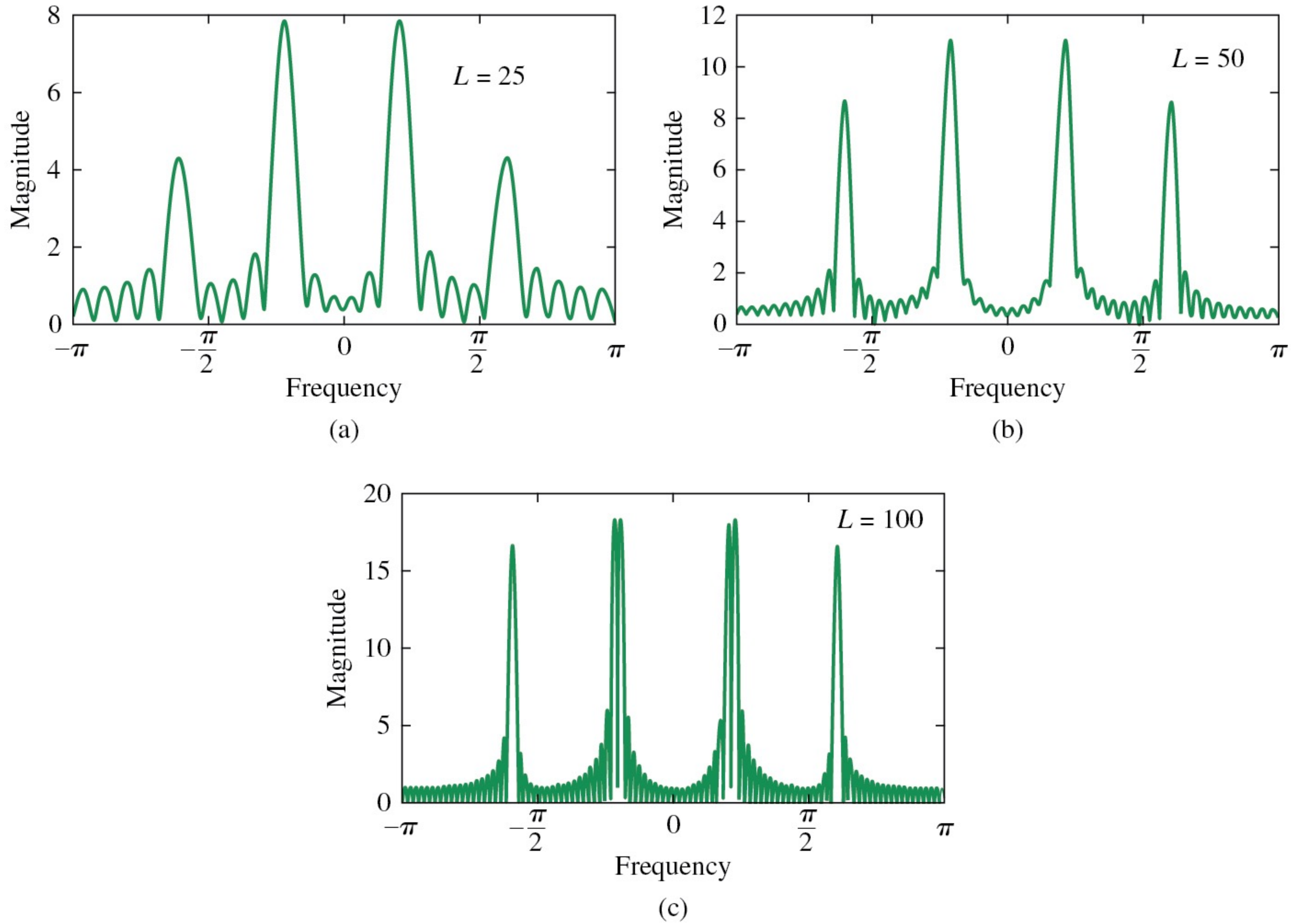


Figure 7.4.3 Magnitude spectrum of the Hanning window.

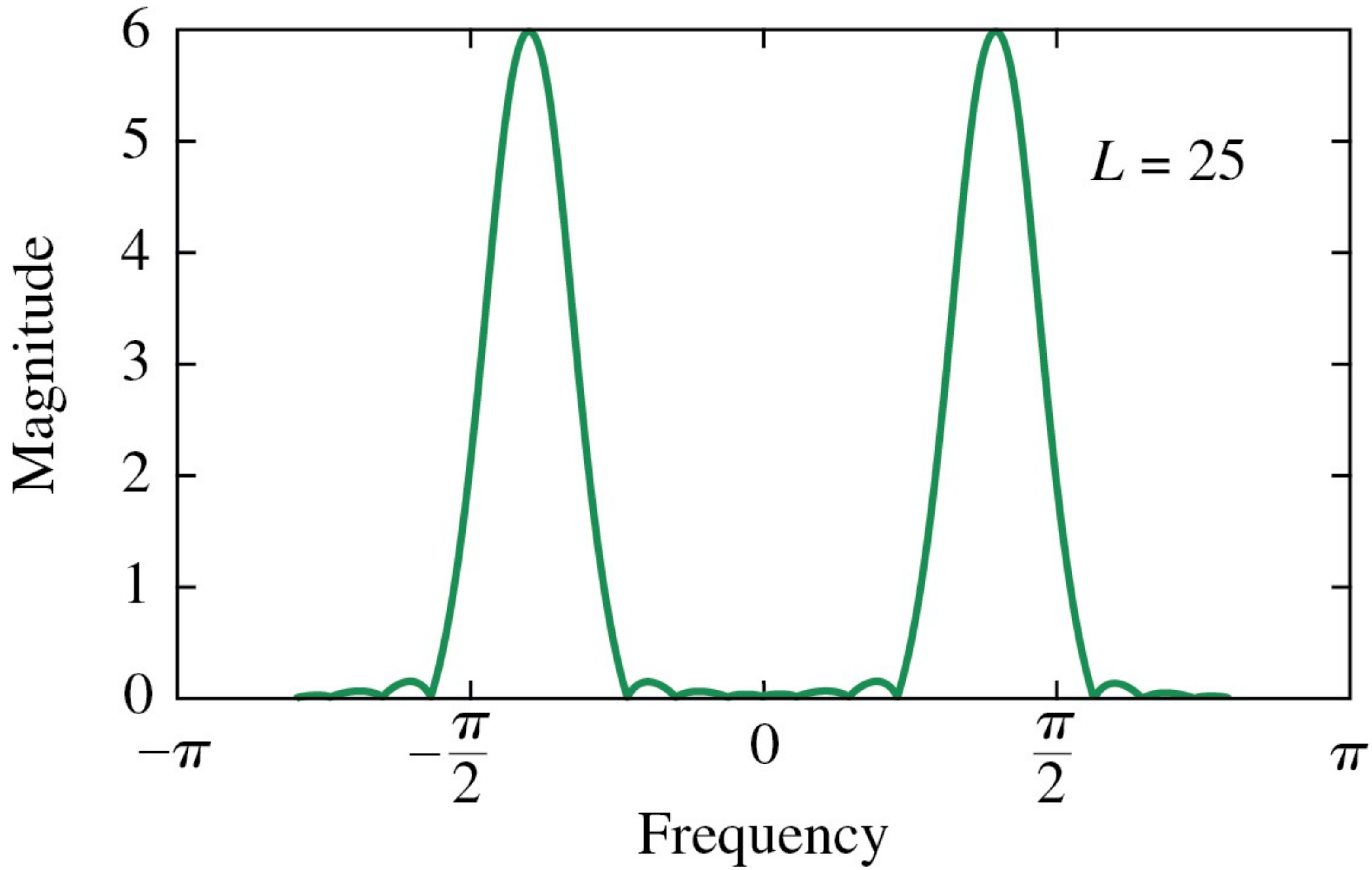


Figure 7.4.4 Magnitude spectrum of the signal in (7.4.8) as observed through a Hanning window.

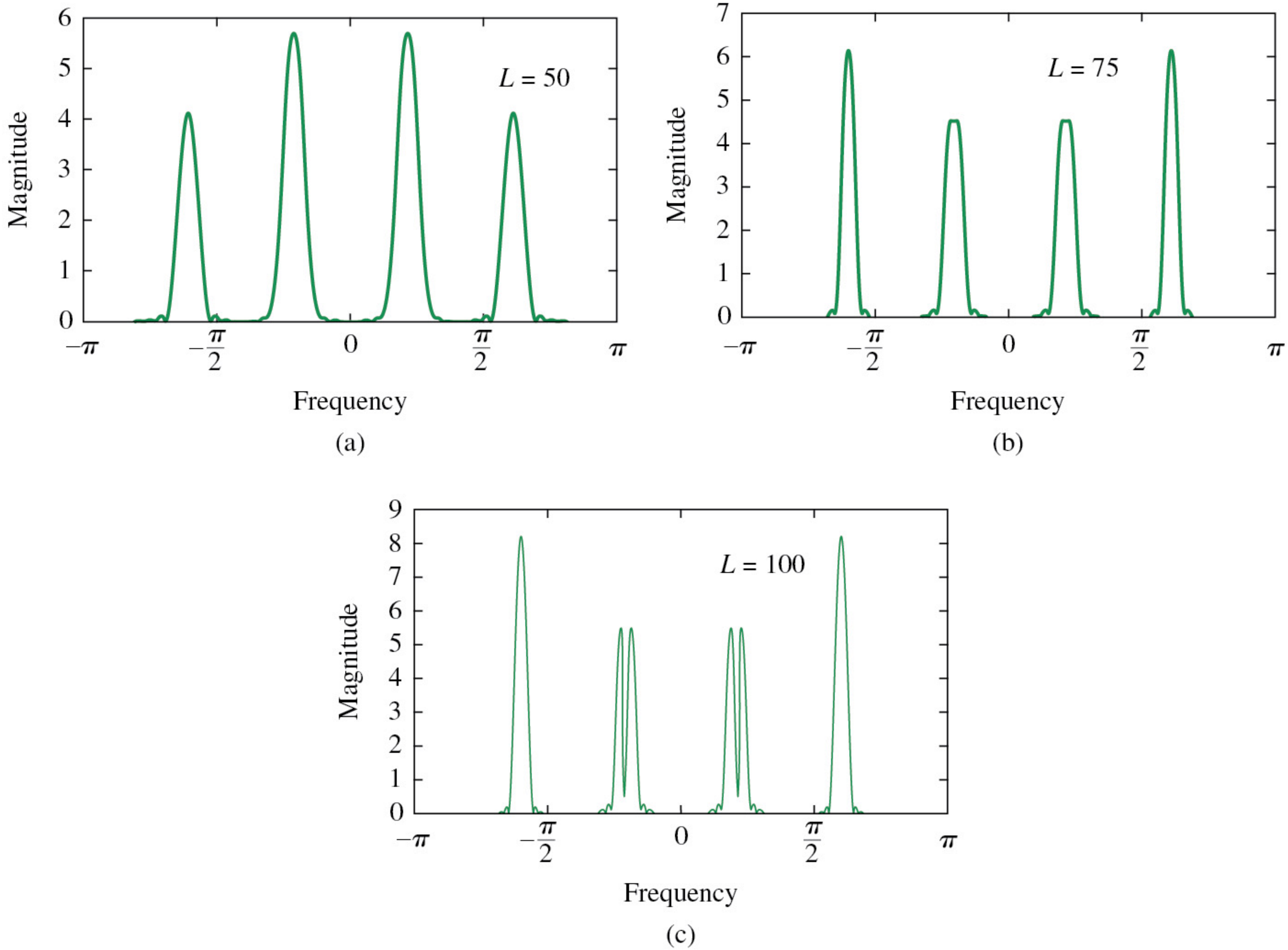
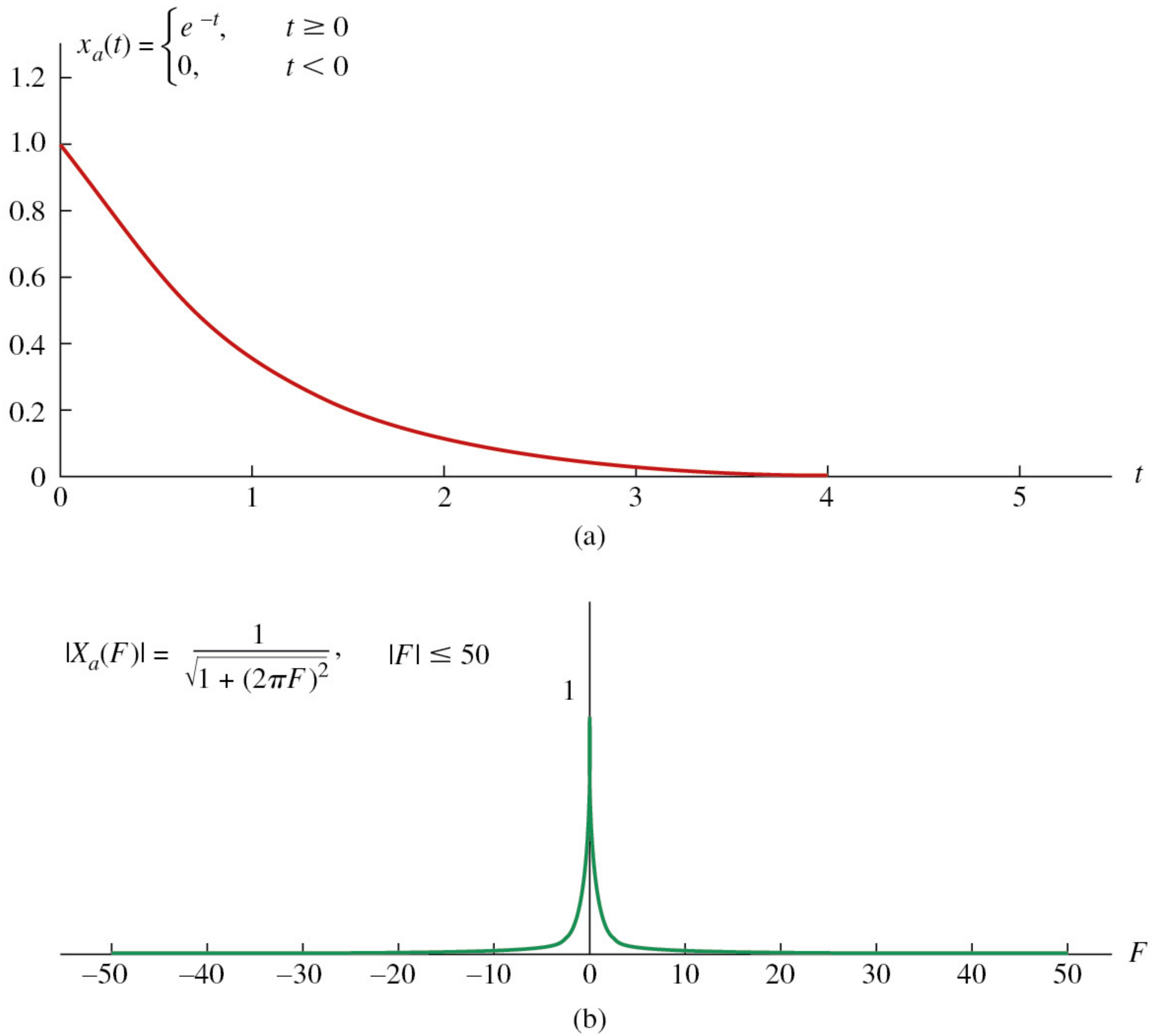
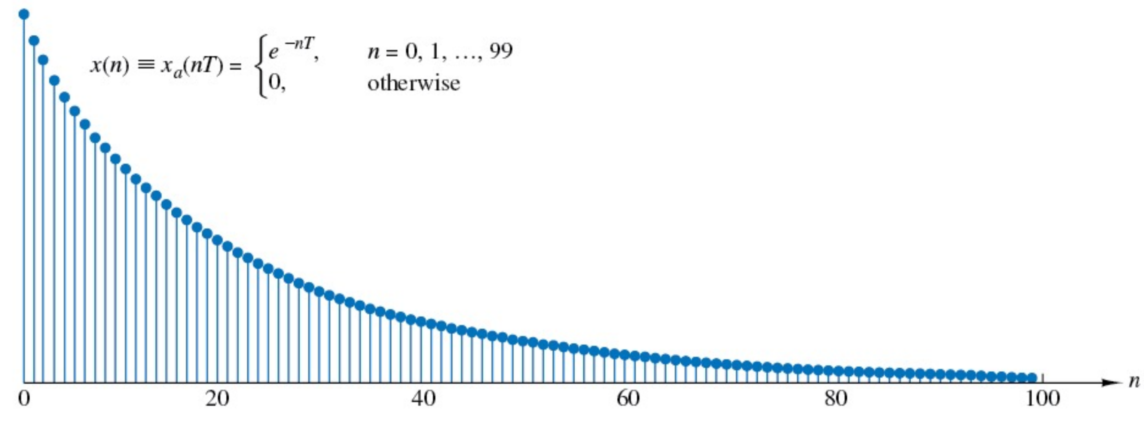


Figure 7.4.5 Effect of windowing (truncating) the sampled version of the analog signal in Example 7.4.1.

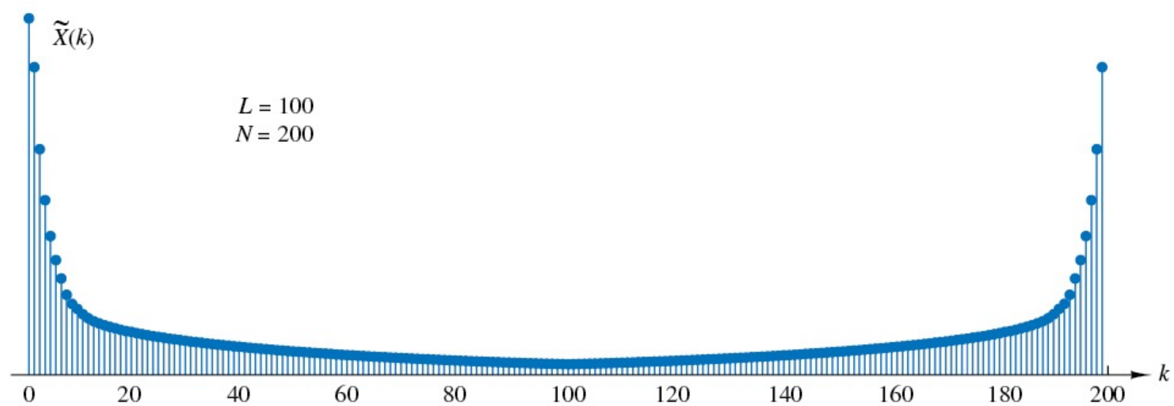


Pearson

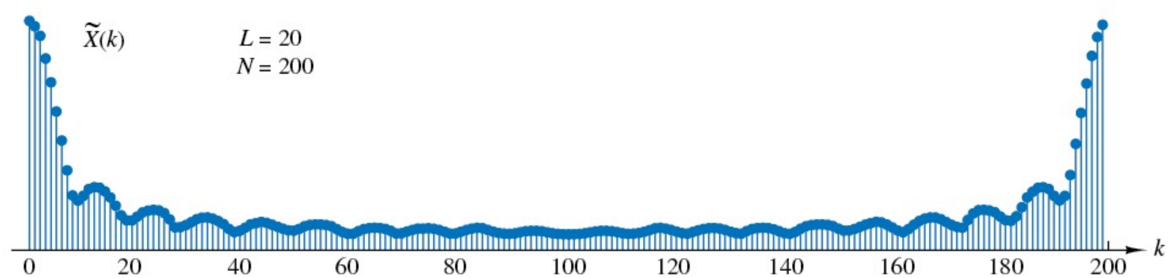
Figure 7.4.5 *(Continued)*



(c)



(d)



(e)

Figure 7.4.6 Illustration of bandlimited interpolation using the DFT.

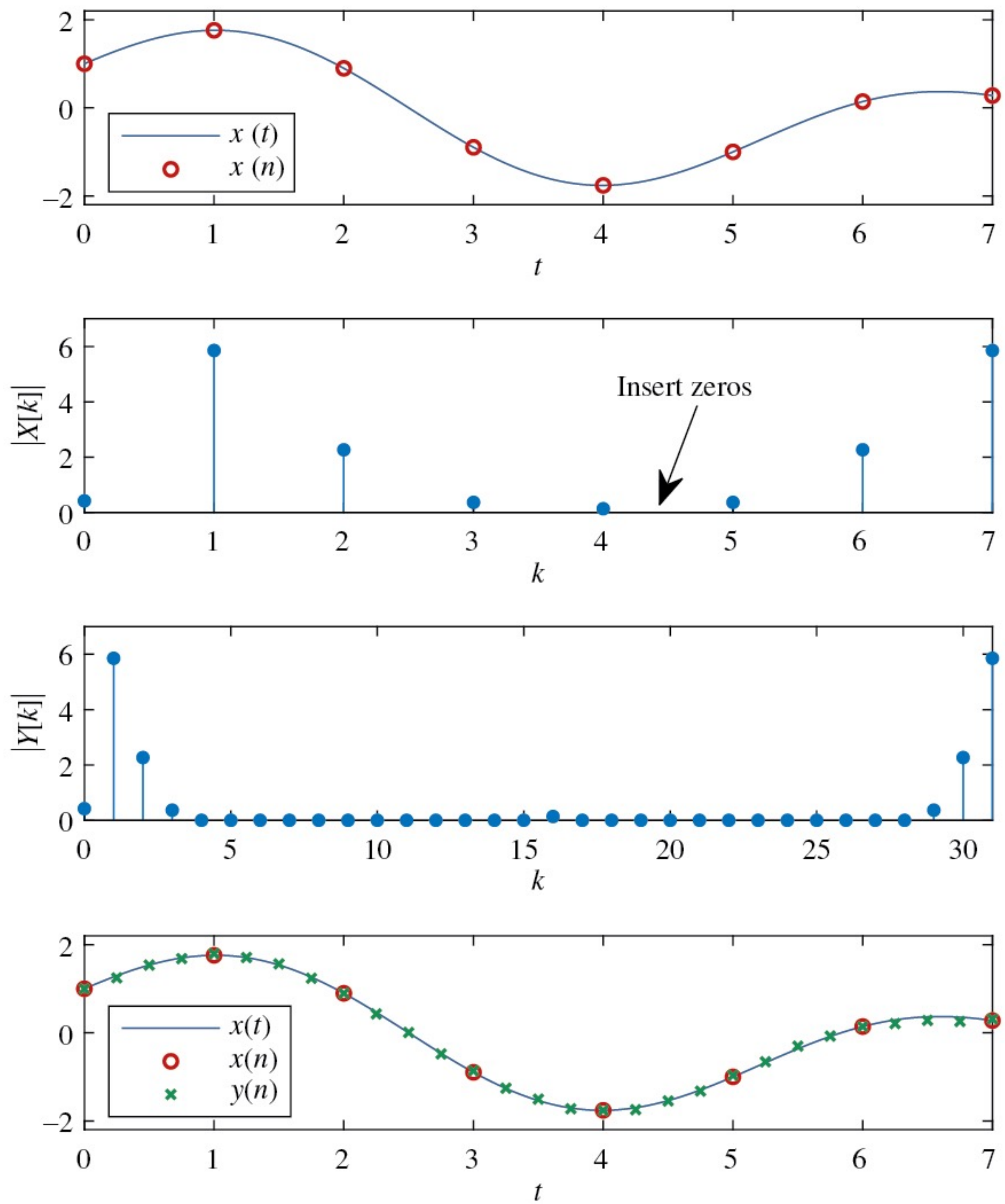


Figure 7.5.1 The STFT for the sinusoidal signal defined by equation (7.5.11).

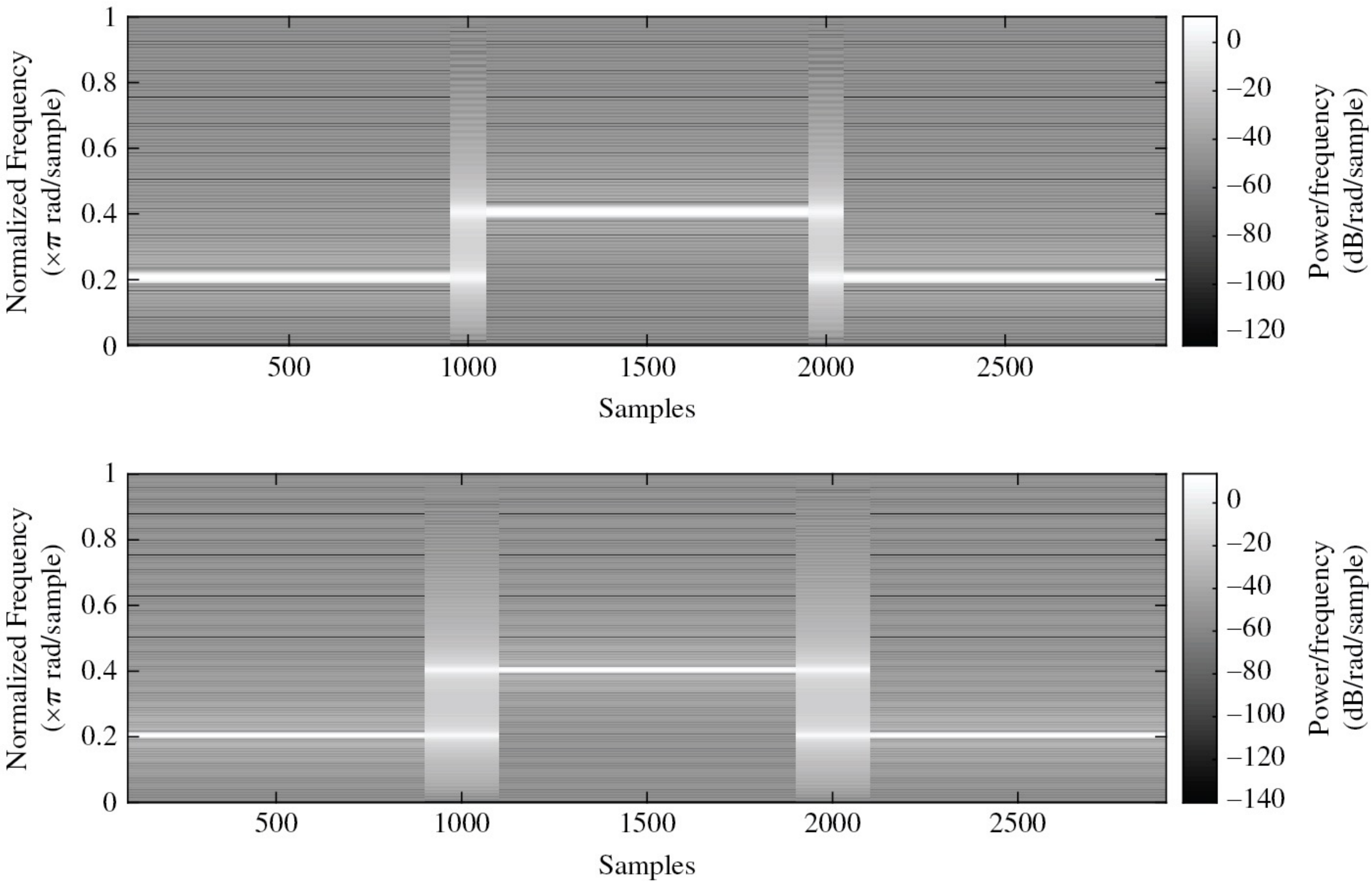


Figure 7.6.1 Original sequence $x(n)$, $0 \le n \le N-1$ and its $2N$-point even extension $s(n)$, $0 \le n \le 2N-1$.

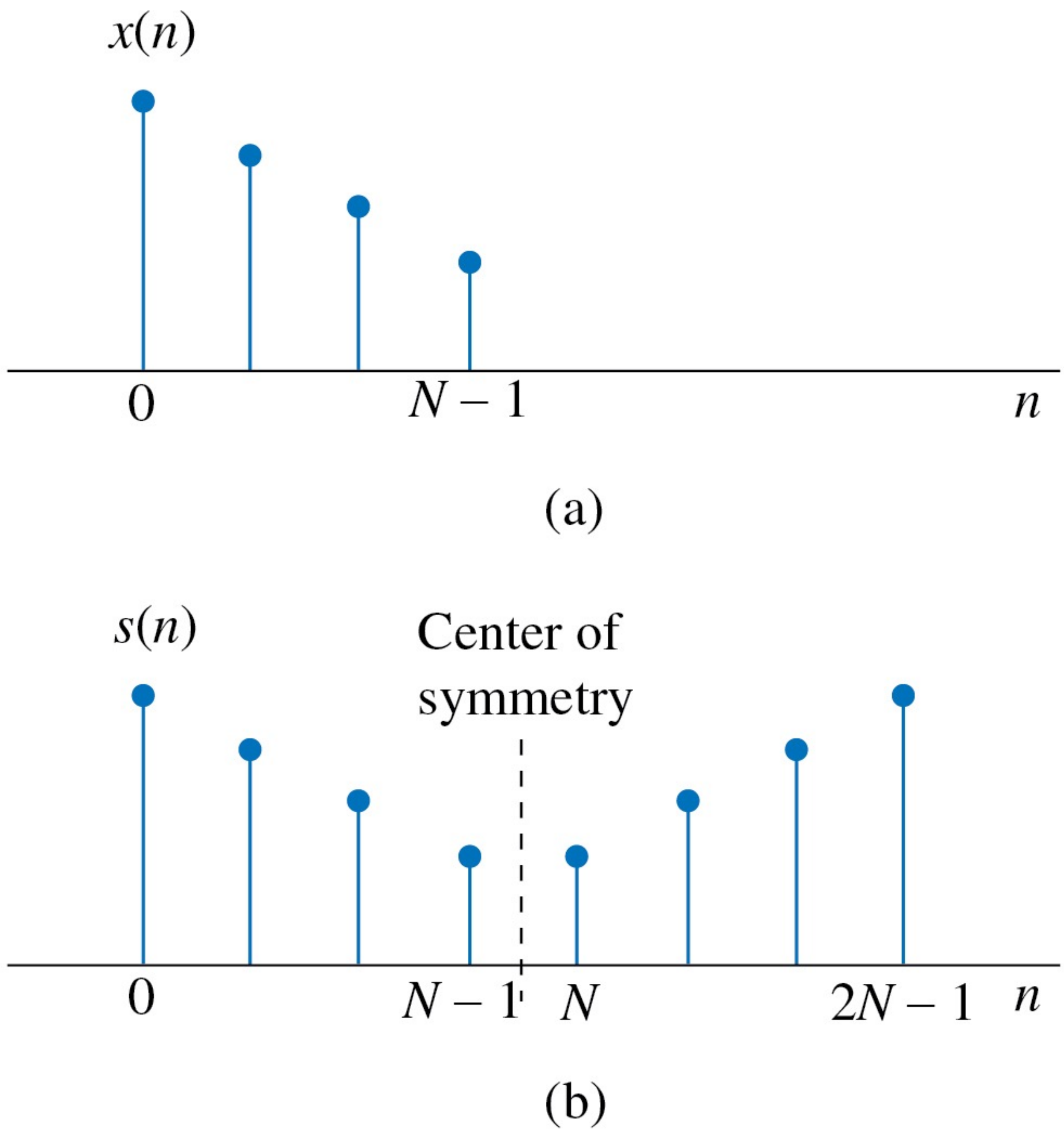


Figure 7.6.2 A discrete-time sinusoidal signal and its DFT and DCT representations.

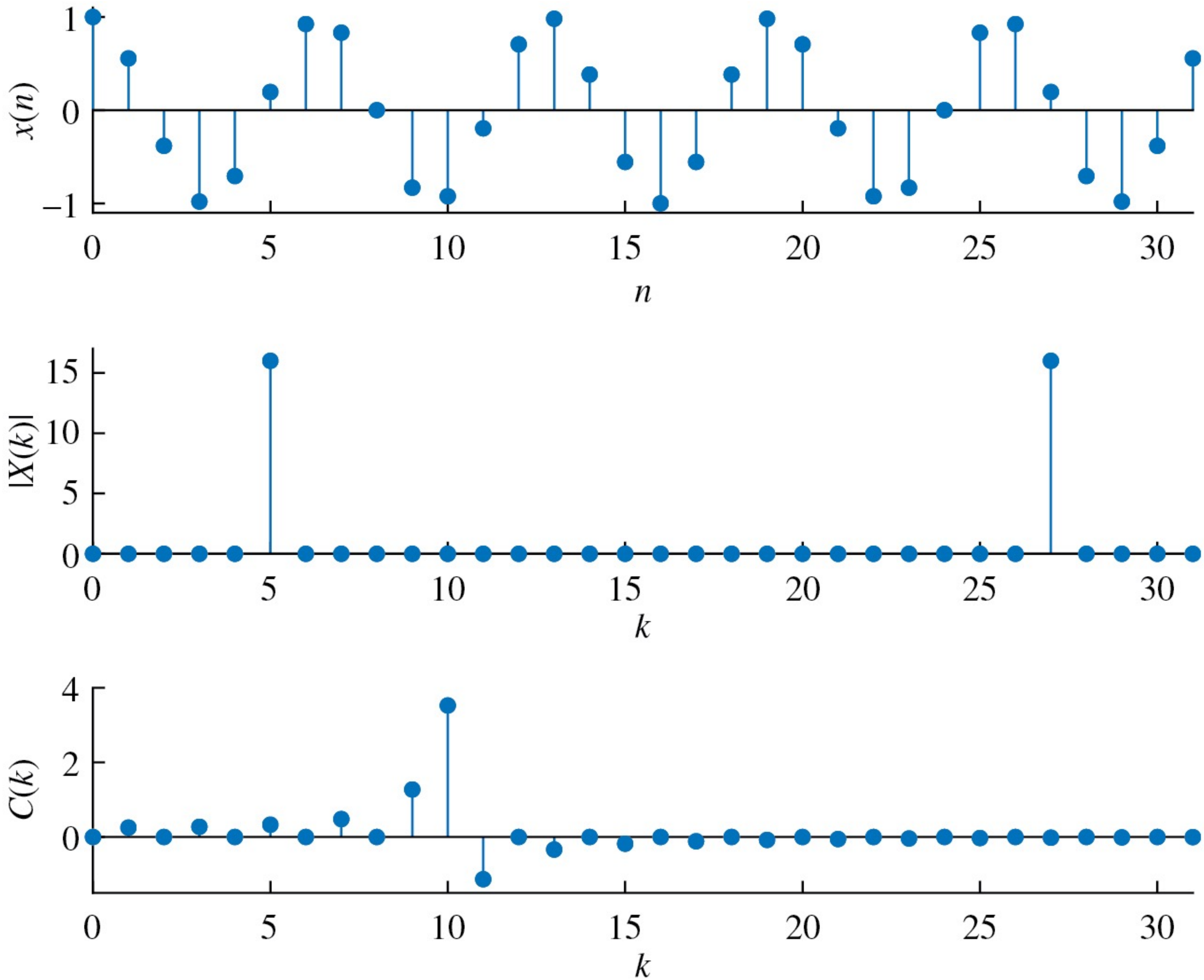


Figure 7.6.3 A discrete time sinusoidal signal and its DFT and DCT representations.

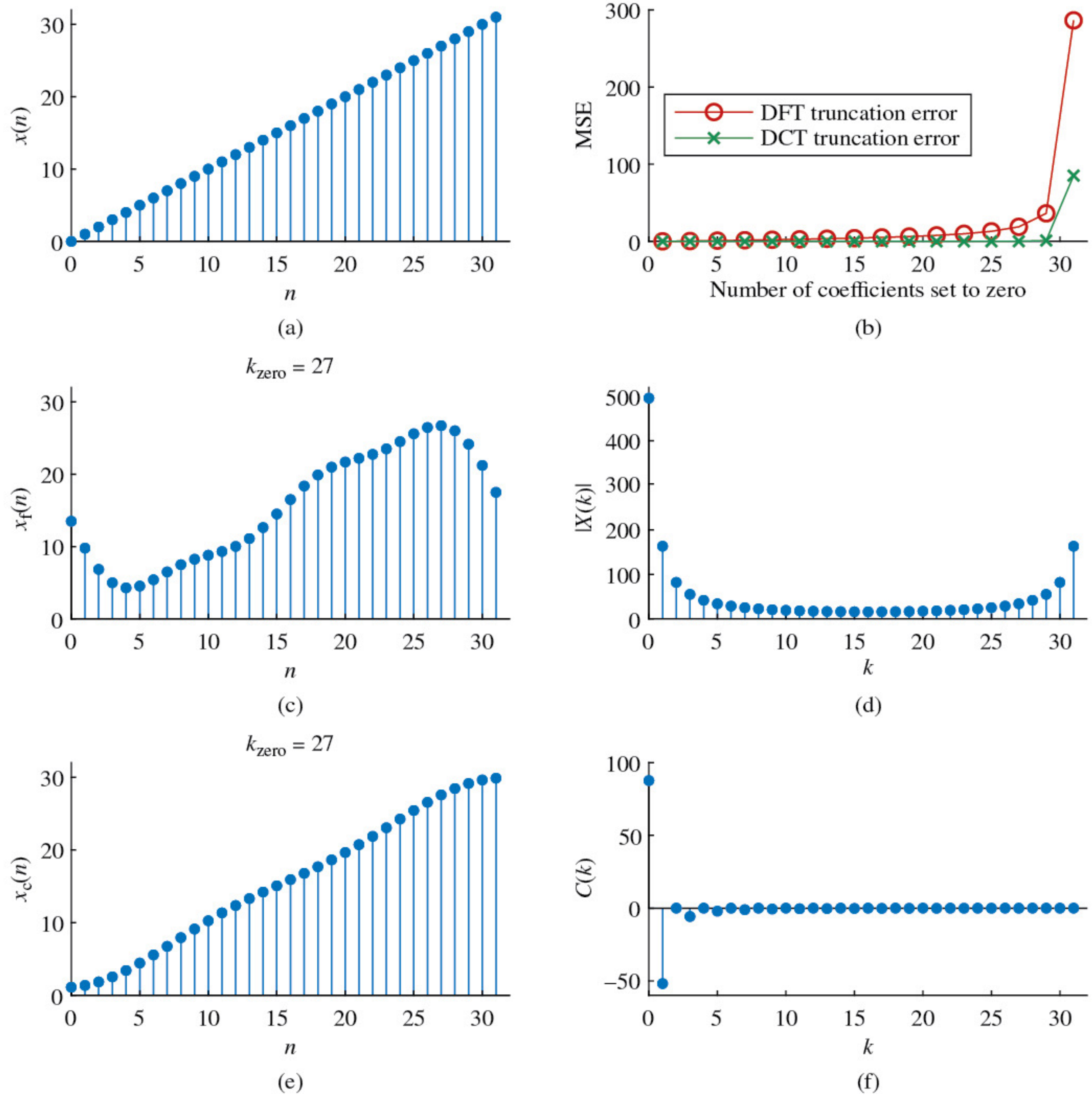
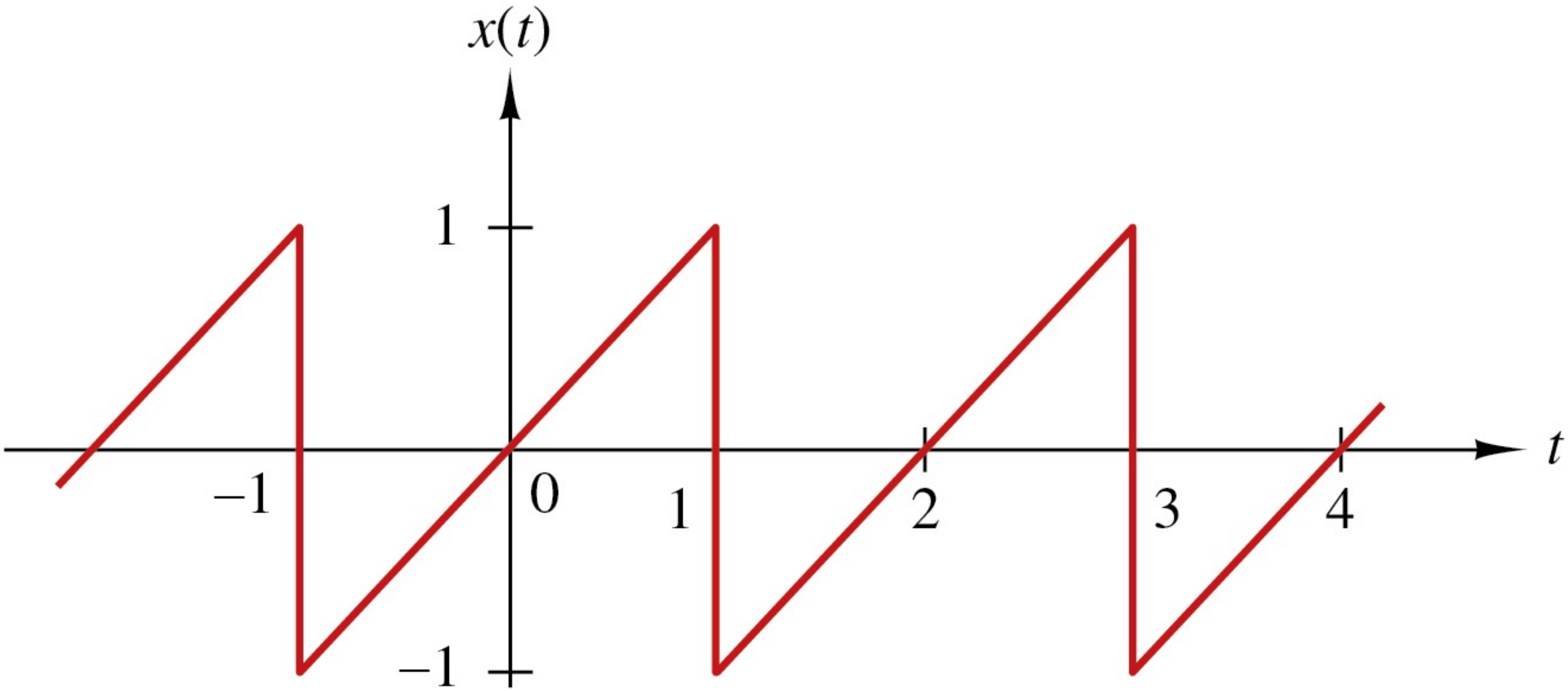


Figure CP7.8



Copyright