DT System Obtained by Sampling a CT Digital Communications Signal

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$$x(t) = \sum_{k=-\infty}^{\infty} b_k \, p(t - kT_0)$$

b_k: information symbols

$p(t)$: symbol pulse shaping

- Consider received signal modelled as:

$$y(t) = x(t) * g(t)$$

- where $g(t)$ is the impulse response of the multipath propagation channel

- Since convolution satisfies distributive property, we have: ②

$$y(t) = \left\{ \sum_{k=-\infty}^{\infty} b_k \, p(t - kT_0) \right\} * g(t)$$

$$= \sum_{k=-\infty}^{\infty} b_k \, p(t - kT_0) * g(t)$$

$$= \sum_{k=-\infty}^{\infty} b_k \left\{ \delta(t - kT_0) * p(t) * g(t) \right\}$$

$$= \sum_{k=-\infty}^{\infty} b_k \left\{ \delta(t - kT_0) * h(t) \right\}$$

$$= \sum_{k=-\infty}^{\infty} b_k \, h(t - kT_0)$$

where:

$$h(t) = p(t) * g(t)$$

- Sampling at $t = nT_0$, $-\infty < n < \infty$ (3)

$$y[n] = y(nT_0)$$
$$= \sum_{k=-\infty}^{\infty} b_k \, h(nT_0 - kT_0)$$

define: $h[n] = h(nT_0)$
$$= \{p(t) * g(t)\}\Big|_{t=nT_0}$$

Then:

$$y[n] = \sum_{k=-\infty}^{\infty} b_k \, h[n-k]$$
$$= \sum_{k=-\infty}^{\infty} b[k] \, h[n-k]$$
$$= b[n] * h[n]$$

- where the information symbols are viewed as an input sequence to LTI system

$$\underbrace{b[n]}_{\text{information symbols}} \longrightarrow \boxed{h[n]} \longrightarrow y[n]$$

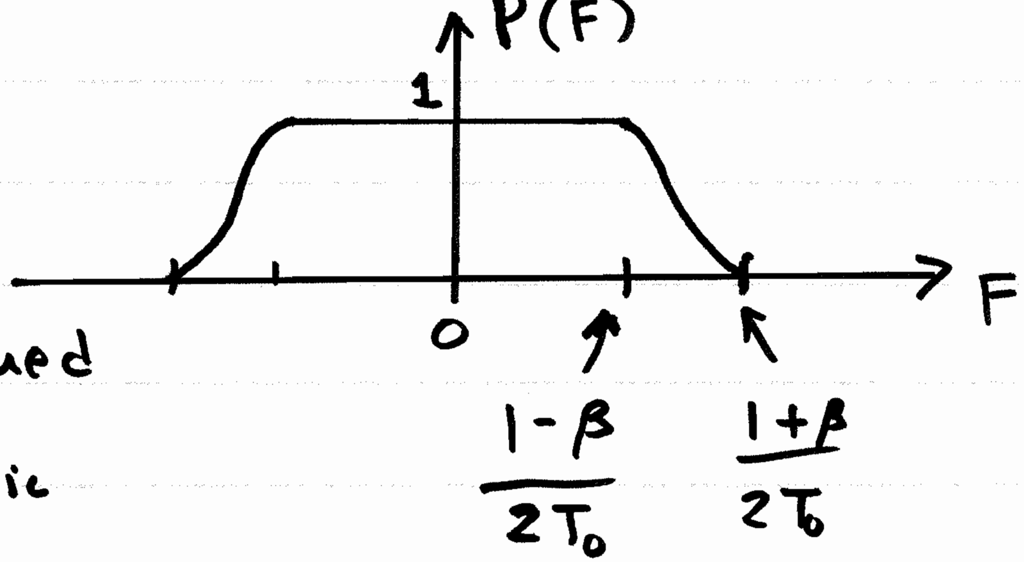
- For example:

$$p(t) = \frac{\sin\left(\pi \frac{t}{T_0}\right)}{\pi \frac{t}{T_0}} \cdot \frac{\cos\left(\beta \pi \frac{t}{T_0}\right)}{1 - 4\beta^2 \left(\frac{t}{T_0}\right)^2}$$

and

$$g(t) = \underbrace{\delta(t) - e^{j\phi}\,\delta(t - T_0)}_{\text{two-ray multipath}}$$

- $p(t)$ is a standard pulse shape used in Digital Communications, having a raised-cosine spectrum with $0 < \beta < 1$ (5)



real-valued and symmetric

- Note:

$$p[n] = p(nT_0) = \underbrace{\frac{\sin(\pi n)}{\pi n}}_{\delta[n]} \frac{\cos(\beta \pi n)}{1 - 4\beta^2 n^2}$$

$$\Rightarrow \quad p[n] = \delta[n]$$

- Back to problem:

$$h(t) = p(t) * g(t)$$

$$= p(t) * \{\delta(t) + e^{j\phi}\delta(t - T_0)\}$$

$$= p(t) + e^{j\phi} p(t - T_0)$$

- Thus:

$$h[n] = p[n] + e^{j\phi} p[n-1]$$

$$= \delta[n] + e^{j\phi}\delta[n-1]$$

- Take DTFT to examine frequency response of channel:

$$H(\omega) = 1 + e^{j\phi} e^{-j\omega}$$

• Using half-angle trick:

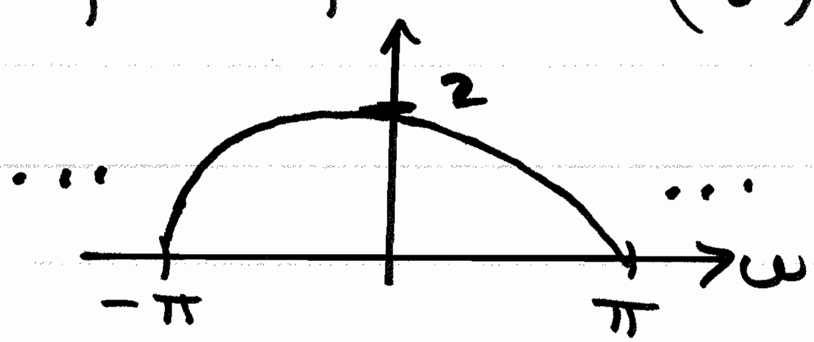
$$H(\omega) = e^{j\frac{(\phi-\omega)}{2}}\left\{ e^{-j\frac{(\phi-\omega)}{2}} + e^{j\frac{(\phi-\omega)}{2}} \right\}$$

$$= 2\,e^{j\frac{(\phi-\omega)}{2}} \cos\left(\frac{\phi-\omega}{2}\right)$$

$$= 2\cos\left(\frac{\omega-\phi}{2}\right) e^{j\frac{(\phi-\omega)}{2}}$$

$$|H(\omega)| = 2\left|\cos\left(\frac{\omega-\phi}{2}\right)\right|$$

i) $\phi = 0 \Rightarrow |H(\omega)| = 2\cos\left(\frac{\omega}{2}\right)$



(ii): $\phi = \frac{\pi}{2} \Rightarrow |H(\omega)| = 2 \left|\cos\left(\frac{\omega - \pi/2}{2}\right)\right|$

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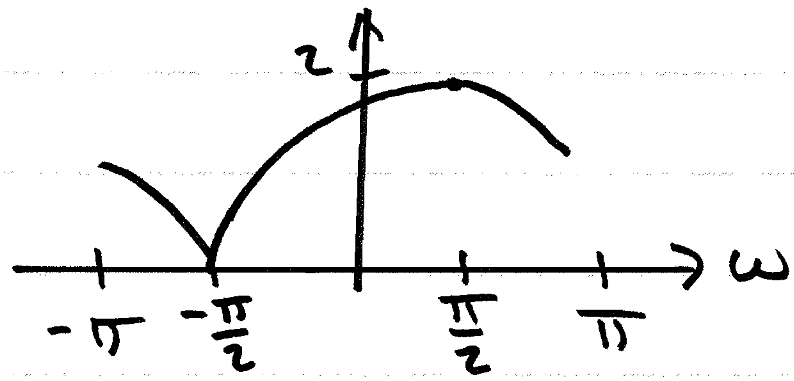

(notch)

multipath induces a null in the frequency response

- Note: problem is trickier if multipath delay is not an integer multiple of T_0
- For example:

$$g(t) = \delta(t) + e^{j\phi}\delta\left(t - \frac{T_0}{2}\right)$$

$$h(t) = p(t) + e^{j\phi} p\left(t - \frac{T_0}{2}\right)$$

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$$h[n] = p(nT_0) + e^{j\phi} p\left(nT_0 - \frac{T_0}{2}\right)$$

$$= \delta[n] + e^{j\phi} \underbrace{\qquad\qquad}_{\text{can't simplify this much}}$$