Basic DFT Result:

$$\sum_{\ell=-\infty}^{\infty} x[n-\ell N]\, w_R(n) \xleftrightarrow[N]{DFT} X(\omega)\Big|_{\omega = k\frac{2\pi}{N}} \triangleq X_N(k)$$

$$w_R[n] = u[n] - u[n-N] \qquad k = 0, 1, \ldots, N-1$$

where: $x[n] \xleftrightarrow{DTFT} X(\omega)$

If $N > L$ = "length" of signal, then:

$$X(\omega) = \sum_{k=0}^{N-1} \underbrace{X_N(k)}_{X\left(\frac{2\pi k}{N}\right)} \frac{\sin\left(\frac{N}{2}\left(\omega - k\frac{2\pi}{N}\right)\right)}{N\sin\left(\frac{1}{2}\left(\omega - k\frac{2\pi}{N}\right)\right)} e^{-j\frac{(N-1)}{2}\left(\omega - k\frac{2\pi}{N}\right)}$$

and no time-domain aliasing

$$e^{j\left(\frac{2\pi}{N}k_0\right)n}\left\{u[n]-u[n-N]\right\} \xleftrightarrow[N]{DFT} N\,\delta[k-k_0]$$

$$k_0 \in \{0, 1, \ldots, N-1\}$$

$$\cos\left(\frac{2\pi k_0}{N}n\right)\left\{u[n]-u[n-N]\right\} \xleftrightarrow[N]{DFT} \frac{N}{2}\delta[k-k_0] + \frac{N}{2}\delta[k-(N-k_0)]$$

$$\boxed{\frac{1}{2}e^{-j\frac{2\pi k_0}{N}n} \overset{\text{iy}}{=} \frac{1}{2}e^{j\frac{2\pi(N-k_0)}{N}n}}$$

$$\cos\left(2\pi\frac{k_0}{N}n\right) = \frac{1}{2}e^{j\frac{2\pi k_0}{N}n} + \frac{1}{2}e^{-j\frac{2\pi k_0}{N}n}$$