Note wrt Hilbert Transform and Single Sideband Modulation (SSM)

- In analog communications textbook, SSM expressed as:

$$\tilde{x}_a(t) = x_a(t)\cos(2\pi f_c t) - \hat{x}_a(t)\sin(2\pi f_c t)$$

where: $\hat{x}_a(t) = x_a(t) * \frac{1}{\pi t}$ OR:

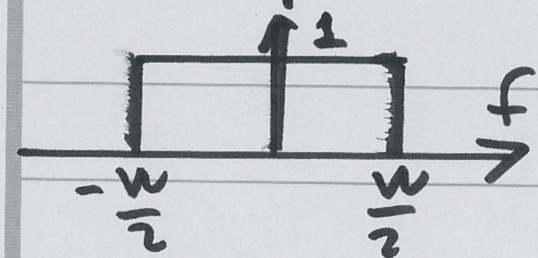
$$h_{HB}(t) = \frac{1}{\pi t} \overset{F}{\longleftrightarrow} H_{HB}(f) = \begin{cases} -j, & f>0 \Rightarrow 0<f<\infty \\ +j, & f<0 \Rightarrow -\infty<f<0 \end{cases}$$

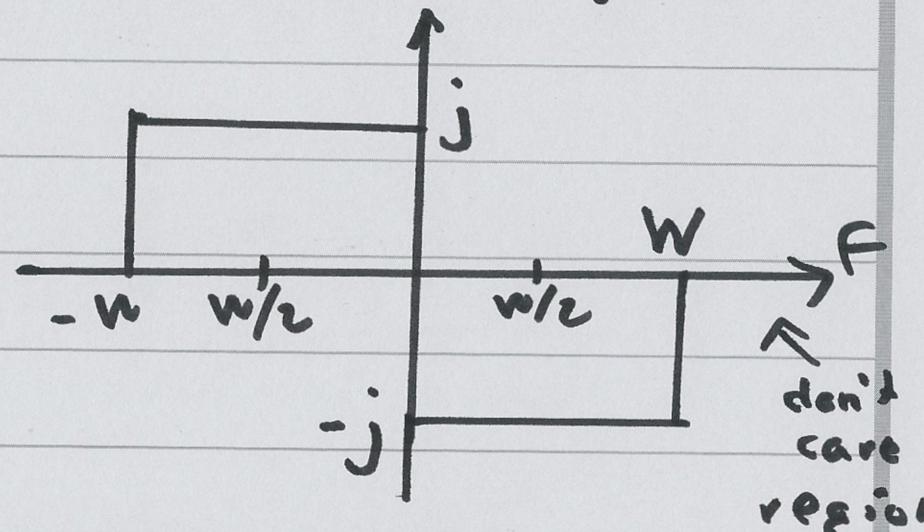
- But real-world signals are always bandlimited: such that $X_a(f) = 0$ for $|f| > W$

Then:

$\hat{x}_a(t) = x_a(t) * h(t)$ where: $2\,\frac{\sin^2(2\pi \frac{W}{4} t)}{\pi t}$

$$\underbrace{\frac{\sin\left(2\pi \frac{W}{2} t\right)}{\pi t}}\; 2\sin\left(2\pi \frac{W}{2} t\right) \overset{\mathcal{F}}{\longleftrightarrow}$$

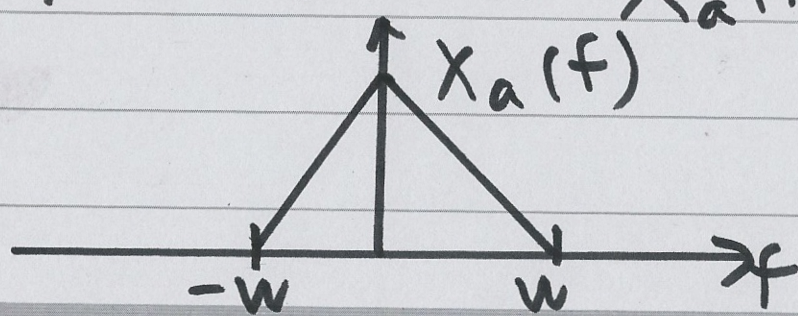




That is:

$$\boxed{x_a(t) * \frac{1}{\pi t} = x_a(t) * \frac{2\sin^2\left(2\pi \frac{W}{2} t\right)}{\pi t}}$$

Might help to visualize $X_a(f)$ as



• define: bandlimited Hilbert Transformer: 3

$$h_{BHT}(t) = 2\,\frac{\sin^2\left(2\pi \frac{W}{2} t\right)}{\pi t}$$

and, again, assume $X_a(f) = 0$ for $|f| > W$

• <u>Recall:</u> $\hat{x}_a(t) = x_a(t) * h_{BHT}(t)$

$$\hat{x}_a[n] = x_a[n] * h_{BHT}[n]\, T_s \qquad \text{Eqn: A}$$

• <u>where:</u> $\hat{x}_a[n] = \hat{x}_a(nT_s)$

$$x_a[n] = x_a(nT_s) \qquad F_s = \frac{1}{T_s} > 2W$$

$$h_{BHT}[n] = h_{BHT}(nT_s)$$

• <u>Assume:</u> Nyquist Rate Sampling:

$$F_s = 2W \Rightarrow W = \frac{F_s}{2}$$ } subsitute above

$$h_{BHT}(t) = \frac{2\sin^2\left(2\pi \frac{F_s}{4} t\right)}{\pi t}$$

Thus:

$$h_{BHT}[n] = h_{BHT}(nT_s) = h_{BHT}\left(\frac{n}{F_s}\right)$$

$$= F_s \, 2\,\frac{\sin^2\left(\frac{\pi}{2}n\right)}{\pi n}$$

in A

- Absorb multiplicative scalar T_s into definition of $h_{BHT}[n]$ ⇒ cancels F_s ⇒ Finally, we have:

$$h_{HBT}[n] = 2\,\frac{\sin\left(\frac{\pi}{2}n\right)}{\pi n}\sin\left(\frac{\pi}{2}n\right) \xleftrightarrow{\text{DTFT}}$$