Autocorrelation Pet Problem

- Recall Example 2.6.2 in text
  - autocorrelation for $x[n] = a^n u[n]$ is: $r_{xx}[\ell] = \frac{1}{1-a^2} a^{|\ell|}$

    a = real-valued
    $|a| < 1$

  - this implies:

$$a^n u[n] * a^{-n} u[-n] = \frac{1}{1-a^2} a^{|n|}$$

  - or you can replace n by ℓ above
- Now, consider the sequence (DT signal)

$$x[n] = b\,\delta[n] + c\,a^n u[n]$$

b, c are real-valued

$$r_{xx}[\ell] = x[\ell] * x[-\ell]$$

$$= \left(b\,\delta[\ell] + c\,a^{\ell}u[\ell]\right) * \left(b\,\delta[-\ell] + c\,a^{-\ell}u[-\ell]\right)$$

$$= b^2\delta[\ell] + bc\,a^{\ell}u[\ell] + bc\,a^{-\ell}u[-\ell]$$

$$+ \underbrace{\frac{c^2}{1-a^2}\,a^{|\ell|}}$$

$$\frac{c^2}{1-a^2}\,\delta[\ell] + \underbrace{\frac{c^2}{1-a^2}\,a^{\ell}u[\ell-1]}_{\text{for } \ell>0} + \underbrace{\frac{c^2}{1-a^2}\,a^{-\ell}u[-1-\ell]}_{\text{for } \ell<0}$$

- Thus: if $\boxed{\frac{c^2}{1-a^2} = -bc}$ $\Rightarrow r_{xx}[\ell] \propto \delta[\ell]$

$$\Rightarrow \quad b = -\frac{c}{1-a^2} = \frac{c}{a^2-1}$$

- choose: $2bc + b^2 + \dfrac{c^2}{1-a^2} = 1$ $\left.\right\}$ so that: $r_{xx}[\ell] = \delta[\ell]$

- Reader can verify that with

$$b = \frac{1}{a} \quad \text{and} \quad c = \frac{a^2-1}{a}$$

- that $bc = \dfrac{a^2-1}{a^2} = \dfrac{-c^2}{1-a^2}$

- such that the autocorrelation for

$$x[\ell] = \frac{1}{a}\left\{\delta[n] + (a^2-1)\,a^n u[n]\right\} \text{ is}$$

$$r_{xx}[\ell] = \delta[\ell]$$