Autocorrelation Problem

- Recall Example 2.6.2 in text
  - Autocorrelation for $x[n] = a^n u[n]$ is:
    \[ r_{xx}[k] = \frac{1}{1-a^2} a^{2k} \quad a = \text{real-valued} \quad |a| < 1 \]
  - This implies:
    \[ a^n u[n] \ast a^{-n} u[-n] = \frac{1}{1-a^2} a^{1n} \]
  - Or you can replace $n$ by $k$ above

Now, consider the sequence (IIT signal)

\[ x[n] = b \delta[n] + c a^n u[n] \]

$b, c$ are real-valued
\[ r_{xx}[l] = \chi[l] \ast \chi[-l] \]
\[ = (b \delta[l] + c a^l u[l]) \ast (b \delta[l] + c a^{-l} u[-l]) \]
\[ = b^2 \delta[l] + bc a^l u[l] + bc a^{-l} u[-l] \]
\[ + \frac{c^2}{1 - a^2} a^{1 \cdot l} \]
\[ = \frac{c^2}{1 - a^2} \delta[l] + \frac{c^2}{1 - a^2} a^l u[l-1] + \frac{c^2}{1 - a^2} a^{-l} u[-l-1] \]

- For \( l > 0 \)
- For \( l < 0 \)

Thus: if \[ \frac{c^2}{1 - a^2} = -bc \]
\[ \Rightarrow r_{xx}[l] \propto \delta[l] \]

\[ \Rightarrow b = -\frac{c}{1 - a^2} = \frac{c}{a^2 - 1} \]
Choose: 
\[2bc + b^2 + \frac{c^2}{1-a^2} = 1\] 
so that:
\[\rho_{xx}[l] = \delta[l]\]

Reader can verify that with 
\[b = \frac{1}{a} \quad \text{and} \quad c = \frac{a^2-1}{a}\]

that 
\[bc = \frac{a^2-1}{a^2} = -\frac{c^2}{1-a^2}\]

such that the autocorrelation for 
\[\chi[l] = \frac{1}{a} \left\{ \delta[n] + (a^2-1) a^n u[n] \right\}\]
is
\[\rho_{xx}[l] = \delta[l]\]