

Approximating area under curve via sum of area under rectangles:

$$\int_{-\infty}^{\infty} f(x) dx = \sum_{k=-\infty}^{\infty} \underbrace{f(k\Delta x)}_{\text{height}} \underbrace{\Delta x}_{\text{width}}$$

Applied to convolution integral:

$$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau$$

$$= \sum_{k=-\infty}^{\infty} x(k\Delta\tau) h(t-k\Delta\tau) \Delta\tau$$

Then discretize  $t$  (or sample  $y(t)$ ):

$$y[n] = y(nT_s) = y(n\Delta\tau)$$

$$y[n] = \sum_k x(k\Delta\tau) h((n-k)\Delta\tau) \Delta\tau$$
$$= \sum_k x[k] h[n-k] \Delta\tau$$

where:

$$x[k] = x(k\Delta\tau)$$

$$h[k] = h(k\Delta\tau)$$