Alternative Sol'n to Prob. 3 (c) for Exam 3 F'02

So far:

$$w[n] \longrightarrow \boxed{\dfrac{b_0 + b_1 z^{-1}}{1 - \frac{1}{2} z^{-1}}} \longrightarrow x[n]$$

What are b_0 and b_1?

$r_{ww}[m] = \delta[m]$

$\sigma_w^2 = 1$

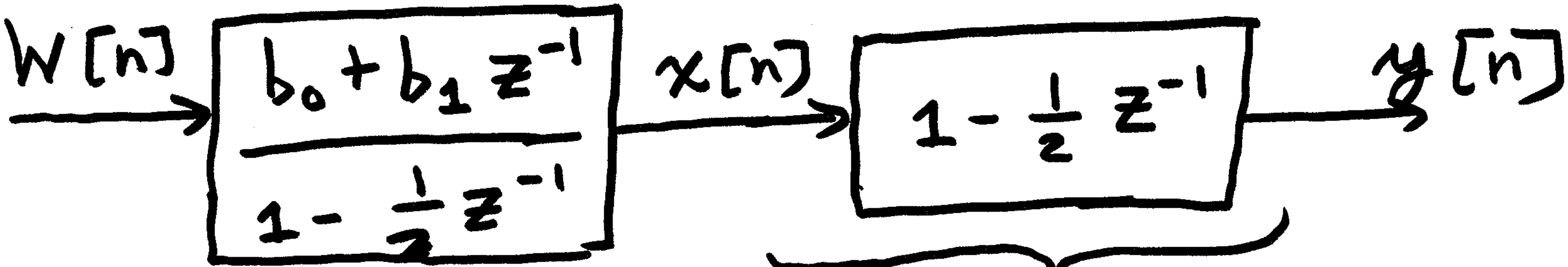
$$\Downarrow$$

$$r_{xx}[m] = 6\left(\frac{1}{2}\right)^{|m|} - 2\delta[m]$$

Use principle behind 2-Step ARMA spectral estimation technique.

Also: recall

$$x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n] \quad \Rightarrow r_{yy}[m] = r_{xx}[m] * r_{hh}[m]$$



$$h[n] = \{\underset{\uparrow}{1}, -\tfrac{1}{2}\} = \delta[n] - \tfrac{1}{2}\delta[n-1]$$

$$r_{hh}[m] = h[m] * h[-m]$$

$$= \{\underset{\uparrow}{1}, -\tfrac{1}{2}\} * \{-\tfrac{1}{2}, \underset{\uparrow}{1}\} =$$

$$= \left\{-\frac{1}{2}, \underset{\uparrow}{\frac{5}{4}}, -\frac{1}{2}\right\}$$

$$= -\frac{1}{2}\delta[m+1] + \frac{5}{4}\delta[m] - \frac{1}{2}\delta[m-1]$$

Thus:

$$r_{yy}[m] = -\frac{1}{2} r_{xx}[m+1] + \frac{5}{4} r_{xx}[m] - \frac{1}{2} r_{xx}[m-1]$$

$$= -3\left(\frac{1}{2}\right)^{|m+1|} + \delta[m+1] + \frac{15}{2}\left(\frac{1}{2}\right)^{|m|} - \frac{5}{2}\delta[m]$$

$$-3\left(\frac{1}{2}\right)^{|m-1|} + \delta[m-1]$$

$m = 0$: $-\frac{3}{2} + \frac{15}{2} - \frac{5}{2} - \frac{3}{2} = 2$

$m = 1$: $-\frac{3}{4} + \frac{15}{4} - \frac{12}{4} + \frac{4}{4} = 1$

$m \geq 2$:

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$$-3\left(\frac{1}{2}\right)^{m+1} + \frac{15}{2}\left(\frac{1}{2}\right)^{m} - 3\left(\frac{1}{2}\right)^{m-1}$$

$$= \left(\frac{1}{2}\right)^{m}\left\{-\frac{3}{2} + \frac{15}{2} - \frac{12}{2}\right\} = 0 \,!$$

$$r_{yy}[m] = \{1, \underset{\underset{m=0}{\uparrow}}{2}, 1\}$$

At the same time:

$$w[n] \longrightarrow \underbrace{\boxed{b_0 + b_1 z^{-1}}} \longrightarrow y[n]$$

$$r_{ww}[m] = \delta[m] \qquad h'[n] = \{\underset{\uparrow}{b_0}, b_1\} = b_0\delta[n] + b_1\delta[n-1]$$

$$r_{h'h'}[m] = \{\underset{\uparrow}{b_0}, b_1\} * \{b_1, \underset{\uparrow}{b_0}\}$$

$$= \{b_0 b_1, \underset{\uparrow}{b_0^2 + b_1^2}, b_0 b_1\}$$

$$r_{yy}[m] = r_{ww}[m] * r_{h'h'}[m]$$

$$= r_{h'h'}[m] \qquad (= r_{bb}[m])$$

Thus: $b_0^2 + b_1^2 = 2$

$$b_0 b_1 = 1$$

$$b_0^2 + \left(\frac{1}{b_0}\right)^2 = 2 \Rightarrow b_0^4 + 1 = 2 b_0^2$$

$$b_0^4 - 2 b_0^2 + 1 = 0 \qquad x = b_0^2$$

$$x^2 - 2x + 1 = 0 \Rightarrow (x-1)^2 = 0$$

$$\Rightarrow x = 1 \qquad b_0 = 1 \quad (\text{or } -1)$$

$$b_1 = \frac{1}{b_0} \Rightarrow b_1 = 1 \quad (\text{or } -1)$$

End of Alternative Sol'n.