

Prob. 1 (b):

$$r_{zz}[l] = \underbrace{(1 + (-1)^l)}_{=0 \text{ for } l \text{ odd}} \underbrace{\frac{\sin(\frac{\pi}{2}l)}{\pi l}}_{=0 \text{ for } l \text{ even}}$$

$$\begin{array}{cc} = 0 & \text{for} \\ l & \text{odd} \end{array} \quad \begin{array}{cc} = 0 & \text{for} \\ l & \text{even} \end{array}$$

$$\text{at } l=0: \left. \frac{\sin(\frac{\pi}{2}l)}{\pi l} \right|_{l=0} = \frac{1}{2}$$

$$r_{zz}[l] = \delta[l]$$

• Continuing Ideal Case for $L=3$

$$h_{LP}[n] = \frac{\sin\left(\frac{\pi}{3}n\right)}{\frac{\pi}{3}n} = 3 \frac{\sin\left(\frac{\pi}{3}n\right)}{\pi n}$$

• Three polyphase components:

$$h_\ell[n] = h_{LP}[3n+\ell] = \frac{\sin\left(\frac{\pi}{3}(3n+\ell)\right)}{\frac{\pi}{3}(3n+\ell)}$$

$\ell = 0, 1, 2$

$$h_\ell[n] = \frac{\sin\left(\pi\left(n + \frac{\ell}{3}\right)\right)}{\pi\left(n + \frac{\ell}{3}\right)} \quad -\infty < n < \infty$$

$$\xleftrightarrow{\text{DTFT}} \sum_{k=0}^2 \frac{1}{3} \underbrace{e^{-j2\pi\frac{\ell}{3}k} e^{j\omega\frac{\ell}{3}}}_{e^{j(\omega - k2\pi)\frac{\ell}{3}}} H_{LP}\left(\frac{\omega - k2\pi}{3}\right)$$

For $l=0$:

$$h_0[n] = \frac{\sin(\pi n)}{\pi n} = \delta[n]$$

For $l=1$:

$$h_1[n] = \frac{\sin\left(\pi\left(n + \frac{1}{3}\right)\right)}{\pi\left(n + \frac{1}{3}\right)} \xleftrightarrow{\text{DTFT}}$$

$-\pi < \omega < \pi$:

$$e^{j\frac{\omega}{3}}$$

$\pi < \omega < 3\pi$:

$$e^{j(\omega - 2\pi)\frac{1}{3}} = e^{j\frac{1}{3}(\omega - 2\pi)}$$

$3\pi < \omega < 5\pi$:

$$e^{j(\omega - 4\pi)\frac{1}{3}} = e^{j\frac{1}{3}(\omega - 4\pi)}$$