Prob. 1 (b):

$$r_{zz}[\ell] = \underbrace{\left(1+(-1)^{\ell}\right)}_{\substack{=0 \text{ for} \\ \ell \text{ odd}}} \underbrace{\frac{\sin\left(\frac{\pi}{2}\ell\right)}{\pi \ell}}_{\substack{=0 \text{ for} \\ \ell \text{ even}}}$$

at $\ell = 0$: $\left.\dfrac{\sin\left(\frac{\pi}{2}\ell\right)}{\pi \ell}\right|_{\ell=0} = \dfrac{1}{2}$

$$r_{zz}[\ell] = \delta[\ell]$$

- Continuing Ideal Case for $L=3$
- $$h_{LP}[n] = \frac{\sin\left(\frac{\pi}{3}n\right)}{\frac{\pi}{3}n} = 3\,\frac{\sin\left(\frac{\pi}{3}n\right)}{\pi n}$$
- Three polyphase components:

$$h_\ell[n] = h_{LP}[3n+\ell] = \frac{\sin\left(\frac{\pi}{3}(3n+\ell)\right)}{\frac{\pi}{3}(3n+\ell)}$$

$\ell = 0, 1, 2$

$$h_\ell[n] = \frac{\sin\left(\pi\left(n+\frac{\ell}{3}\right)\right)}{\pi\left(n+\frac{\ell}{3}\right)} \qquad -\infty < n < \infty$$

$$\xleftrightarrow{\text{DTFT}} \sum_{k=0}^{2} \frac{1}{3} \underbrace{e^{-j2\pi\frac{\ell}{3}k}\, e^{j\omega\frac{\ell}{3}}}_{e^{j(\omega-k2\pi)\frac{\ell}{3}}} H_{LP}\left(\frac{\omega-k2\pi}{3}\right)$$

For $\ell = 0$:

$$h_0[n] = \frac{\sin(\pi n)}{\pi n} = \delta[n]$$

For $\ell = 1$:

$$h_1[n] = \frac{\sin\left(\pi\left(n+\frac{1}{3}\right)\right)}{\pi\left(n+\frac{1}{3}\right)} \xleftrightarrow{\text{DTFT}}$$

$-\pi < \omega < \pi$: $e^{j\frac{\omega}{3}}$

$\pi < \omega < 3\pi$: $e^{j(\omega-2\pi)\frac{1}{3}} = e^{j\frac{1}{3}(\omega-k2\pi)}$

$3\pi < \omega < 5\pi$: $e^{j(\omega-4\pi)\frac{1}{3}} = e^{j\frac{1}{3}(\omega-k4\pi)}$