EE 368

Weeks 3 (Notes)

State of a Queuing System

State: Set of parameters that describe the condition of the system at a point in time.

Why do we need it?

- Average size of Queue
- Average waiting time

How to define it:

M/M/1

Number of items in the system.

Introduction to Queuing Theory

A queue is characterized by:

- Inter-arrival Time PDF
- Service Time PDF
- Number of Servers
- The Queuing Discipline FIFO, Priority,….
- Size of Buffer

Probability Function of the Length of a M/M/1 Queue

P_k = Probability system has k customers (Queue + Server)

λ = arrival rate (average)

μ = service rate (average)

Assume: $\lambda < \mu$

Let: $\rho = \lambda / \mu \qquad < 1$

Probability Function of the Length of a M/M/1 Queue (cont.)

In steady-state:

$P_k = (1-\rho)\,\rho^k$

This distribution is Geometric

Performance Parameters of Queuing System

Average size of the queue:

Average size of the system

Average of random variable $K = N = \rho / (1 - \rho)$

Average size of the Queue = $N-1$

Average waiting time in the Queue *(Tq):*

Average time spent in the system = T

Little's Law:

$N = \lambda T$

$T_q = T - 1/\mu$

Simulation of a Queuing System

Simulation: Modeling of real-world for the purpose of design and evaluation.

Model: An abstract represenation of a system in terms of system state, entities, sets, and events etc.

Event-Driven: Occurrence of events changes the simulated situation.

Events in a M/M/1 Queue.

- A new Arrival
- Completion of a service

Events in **M/M/k** Queuing System???

Concepts of Event-Driven Simulation

Entity: Any object, such as customer, server

System: A collection of entities (example: customers)

State: A collection of variables that contain the information to describe the system at any time.

Event: An instantaneous occurrence that changes the system's state

Set: Collection of related entities, eg. queues of various priorities

Concepts for Event-Driven Simulation

Queues to be maintained:

- **Customer Queues** (can be multiple, such as priority queues)
- **Future Event Queue (EQ)**: Maintains evolution of events. It contains all previously scheduled future events and their associated event times.

Computation of Exponentially Distributed Random Time

Interarrival time PDF

$$f(r) = \lambda e^{-\lambda r}$$

Service time PDF

$$f(r) = \mu e^{-\mu r}$$

Computing exponential PDF from unifrom

$$r = -1/\mu \ (\ln (1 - x))$$

Cumulative Statistics in Simulation

Average waiting time in the queue =

Total waiting time of all the customers / Total number of customers

Probability (wait) =

Number of customers waited / Total number of customers arrived

Probability of idle server:

Average Service Time:

Average time customer spends in the system:

Event-Driven Simulation

- *Simulation* is the imitation of the operation of a real-world process over time
- Why simulation is performed?
  - to study the internal interactions of a complex system
  - To experiment with new designs or policies prior to implementation
  - To observe the effect of a change in input on the system's output

Data structures for simulation of a Queueing System

- An ADT is needed to maintain all the vital information related to the simulated system
- ***Entities*** in a queuing system are objects and servers
- A waiting queue is needed to represent a ***set*** of objects, waiting to be processed by the server
- ***Event*** can be an arrival event or a departure event
- The ***state*** of the queueing system at time *t* is described by the number of objects waiting in their respective queues and the state of the servers (busy/idle) at time t.
- A list of events to occur in the future is maintained and can be termed as ***FEL***. Events in ***FEL*** are ordered in increasing order of time
- ***Queueing Model***
  - Number of servers
  - Number of queues
  - Objects arrival rate
  - Service rate

System Snapshot at simulation time t

Clock	System state	Entities and attribute	Set 1	Set 2	…	FEL	Cumulative statistics and counters
t	(x,y, z,..)					(A,t_1), (B,t_2),(C,t_3), …. Note: $t \leq t_1 \leq t_2 \leq t_3 \leq \ldots$	

Event-scheduling/time-advance algorithm

Step1. Remove an event (imminent event) from FEL. Say (event A, time t_1)

Step2. Advance clock to imminent event time (i.e., advance clock from t to t_1)

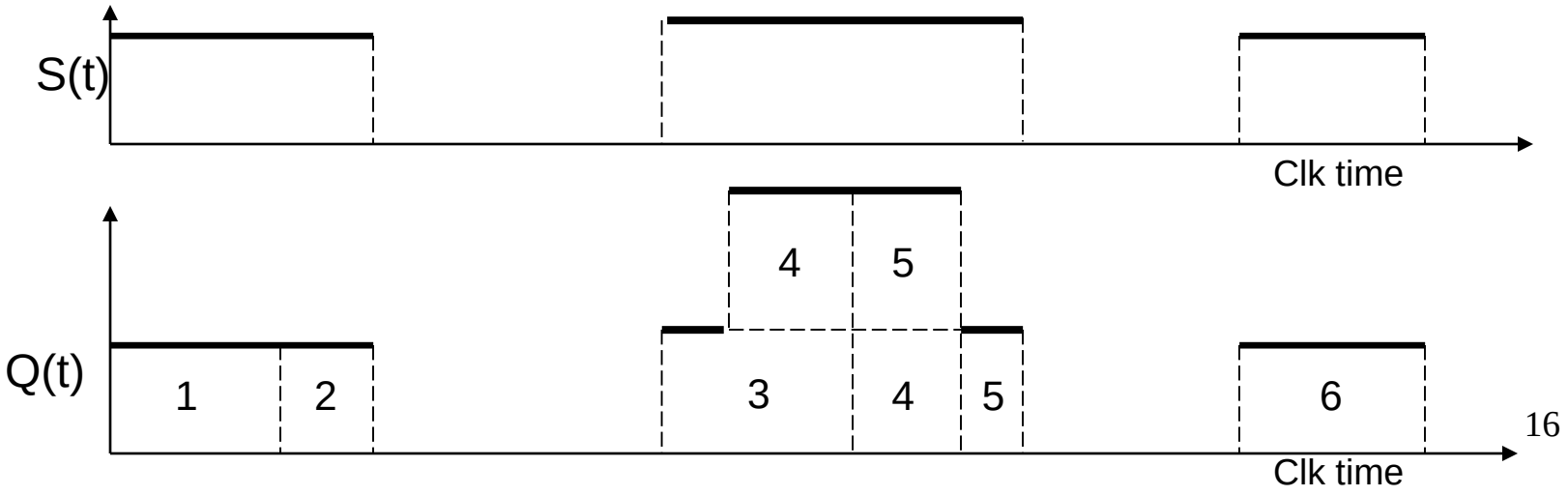
Step3. Execute imminent event: update system state, change entity attributes, and set membership as needed. For example, if the imminent event is an arrival of a new customer and the server is busy, insert the customer in the waiting queue

Step4. Generate future events (if necessary) and place on FEL.

Step5. Update cumulative statistics and counters.

Example of an M/M/1 Queue

- A small grocery store that has only one checkout counter
- *Simulate this queue from t= 0 to t = 25*
- System State variables: Q(t) and S(t)
  - Q(t) = # of customers in the system at time t
  - S(t) = 1 server busy, 0 otherwise



Example of an M/M/1 Queue (Contd.)

- Events:

  Arrival (A)

  Deprture (D)

  Stopping Event (E) Scheduled to occur at time t = 25

- Event Notices:

  (A,t) arrival of a customer at future time t

  (D,t) departure of a customer at time t

  (E,25) stopping event at t=25

Example of an M/M/1 Queue (Contd.)

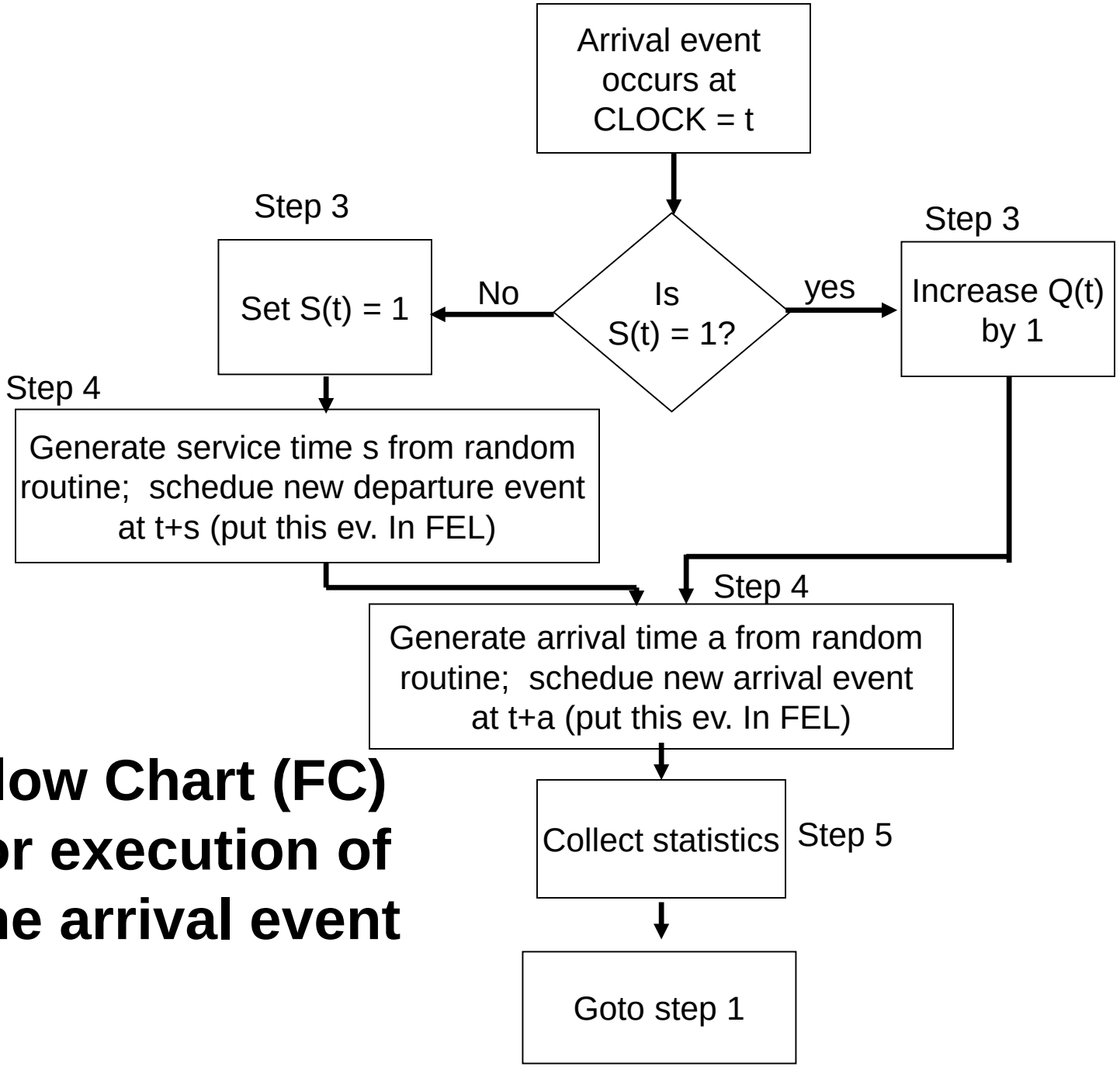
- Activities

  Interarrival time, uniformly distributed between 1 to 8 minutes

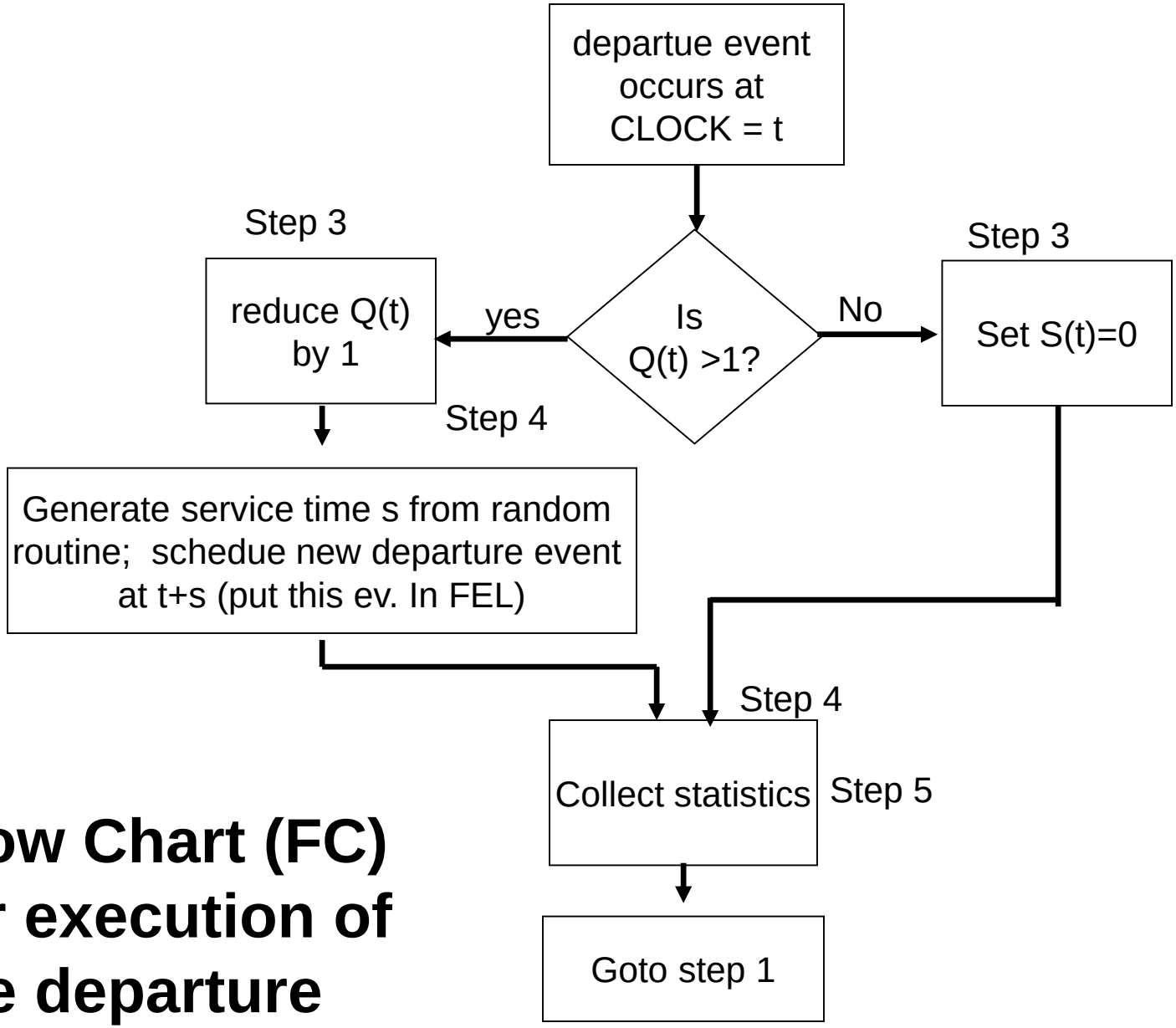
  Service time, exponentially distributed with mean time = 3 minutes

Interarrival times	8	6	1	8	3	8	…..
Service times	4	1	4	3	2	4	…..

- Cumulative statistics
  - B, server utilization
  - MQ, maximum queue length
  - W, cumulative waiting time of all customers



Flow Chart (FC) for execution of the arrival event



Flow Chart (FC) for execution of the departure event

Simulation table for checkout counter (Single channel queue)

t	Event removed	System State Q(t)	S(t)	Future Event List (FEL)	Comment	Cum. Statistics B	MQ	W
< 0		0	0	**(A_1,0)** , (E,25)		0	0	0
0	A_1	0	1	**(D_1,4)** , (A_2,8), (E,25)	A_1 occurs (following LHS of FCA) (s=4) schedule D_1 (a=8) schedule A_2	0	0	0
4	D_1	0	0	**(A_2,8)** , (E,25)	D_1 occurs.(following RHS of FCD)	4	0	0
8	A_2	0	1	**(D_2,9) ,** (A_3,14), (E,25)	A_2 occurs, (following LHS of FCA) (s=1) schedule D_2 (a=6) schedule A_3	4	0	0
9	D_2	0	0	**(A_3,14)**, (E,25)	D_2 occurs,(following RHS of FCD)	5	0	0
14	A_3	0	1	**(A_3,15)**, (D_3,18), (E,25)	A_3 occurs, (following LHS of FCA) (s* = 4) schedule D_3 (a*=1) schedule A_4	5	0	0
15	A_4	1	1	**(D_3,18),** (A_5,23), (E,25)	A_4 occurs,(following RHS of FCA) Customer delayed (a*=8) schedule A_5	6	1	0
18	D_3	0	1	**(D_4,21),** (A_5,23), (E,25)	D_3 occurs,(following LHS of FCD) (s* = 3), schedule D_4	9	1	3