Post-lab 4a Tips

1 Electromagnetic Torque Equation

Question 4 in post-lab 4a asks you to plot predicted electromangetic torque versus position for each phase with a current $I = 10/r_s$. The derivation of torque equation (9) in the manual is for the example stepper motor considered in the text. For the machine studied in lab, the expression for inductance versus position is different from the one in the text and hence the torque equation will also be different. The steps involved in deriving torque due to a-phase in Equation (9) are given below which can be used in your postlab.

1.1 Torque due to a-phase

Electromagnetic torque derivation is a two step process.

1. First, So-called coenergy in a-phase $W_{c,as}$ is calculated. Current through the a-phase i_{as} and the rotor position θ_{rm} are chosen as independent variables. Flux liniking a-phase λ_{as} is the dependent variable. Then the coenergy is computed using the formula

$$W_{c,as}(i_{as}, \theta_{rm}) = \int_0^{i_{as}} \lambda_{as}(\xi, \theta_{rm})\, d\xi \tag{1.1}$$

where ξ is a dummy variable used instead of $i_a s$. Most of the electromechanical devices studied in ECE 321 and ECE 323 are assumed to be magnetically linear. Then the equation for flux linkage λ_{as} is the first row of equation (4) in the manual.

$$\lambda_{as}(i_{as}, \theta_{rm}) = L_{asas} i_{as} \tag{1.2}$$

where $L_{asas} = L_A + L_B \cos(RT\,\theta_{rm})$. Inductance L_{asas} is a function of rotor position θ_{rm}. Hence, the flux linkage is a function of both current i_{as} and rotor position θ_{rm}. (1.2) is plugged in to (1.1) and i_{as} is replaced with dummy variable ξ. After intergrating and evalutating the limits, the following equation is obtained.

$$W_{c,as}(i_{as}, \theta_{rm}) = \frac{1}{2} L_{asas} i_{as}^2 \tag{1.3}$$

If you remember the expression for energy stored in an inductor $(1/2Li^2)$, the equation above should look very familiar.

2. The electromagnetic torque due to a-phase $T_{e,as}$ is computed using the following formula

$$T_{e,as}(i_{as}, \theta_{rm}) = \frac{\partial}{\partial \theta_{rm}} W_{c,as}(i_{as}, \theta_{rm}) \tag{1.4}$$

$$= \frac{\partial}{\partial \theta_{rm}} \frac{i_{as}^2}{2} \left[L_A + L_B \cos(RT\,\theta_{rm})\right] \tag{1.5}$$

$$= -\frac{RT\, i_{as}^2}{2} L_B \sin(RT\,\theta_{rm}) \tag{1.6}$$

The torque due to a-phase (1.6) is the first term in Equation (9) in the manual.

A similar derivation is needed for torque due to a-phase in postlab where the inductance is approximated as $L_{asas} = L_A + L_C \cos(RT\theta_{rm}) + L_S \sin(\theta_{rm})$. The derivations for b-phase and c-phase are similar except for the change in numerial value of the constants L_A, L_C, and L_S.