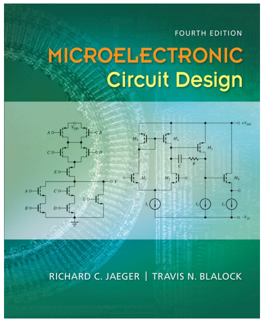
Chapter 17
Amplifier Frequency Response

Microelectronic Circuit Design

Richard C. Jaeger

Travis N. Blalock



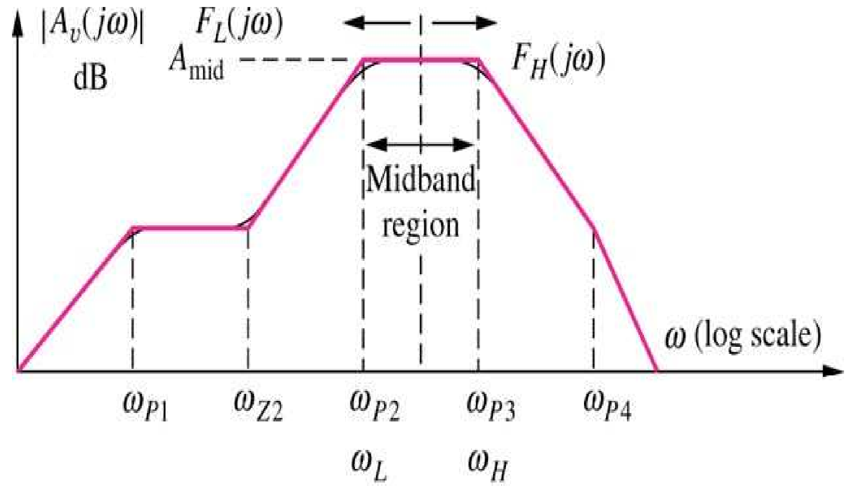
Chapter Goals

- Review transfer function analysis and dominant-pole approximations of amplifier transfer functions.
- Learn partition of ac circuits into low and high-frequency equivalents.
- Learn the short-circuit time constant method to estimate upper and lower cutoff frequencies.
- Develop bipolar and MOS small-signal models with device capacitances.
- Study unity-gain bandwidth product limitations of BJTs and MOSFETs.
- Develop expressions for upper cutoff frequency of inverting, non-inverting and follower configurations.
- Explore high-frequency limitations of single and multiple transistor circuits.

Chapter Goals (contd.)

- Understand Miller effect and design of op amp frequency compensation.
- Develop relationship between op amp unity-gain frequency and slew rate.
- Understand use of tuned circuits to design high-Q band-pass amplifiers.
- Understand concept of mixing and explore basic mixer circuits.
- Study application of Gilbert multiplier as balanced modulator and mixer.

Transfer Function Analysis



$$A_v(s) = \frac{N(s)}{D(s)} = \frac{a_0 + a_1 s + a_2 s^2 + \ldots + a_m s^m}{b_0 + b_1 s + b_2 s^2 + \ldots + b_n s^n}$$

$$= A_{mid} F_L(s) F_H(s)$$

A_{mid} is midband gain between upper and lower cutoff frequencies.

$$F_L(s) = \frac{\left(s+\omega_{Z1}^L\right)\left(s+\omega_{Z2}^L\right)\cdots\left(s+\omega_{Zk}^L\right)}{\left(s+\omega_{P1}^L\right)\left(s+\omega_{P2}^L\right)\cdots\left(s+\omega_{Pk}^L\right)}$$

$$F_H(s) = \frac{\left(1+\frac{s}{\omega_{Z1}^H}\right)\left(1+\frac{s}{\omega_{Z2}^H}\right)\cdots\left(1+\frac{s}{\omega_{Zl}^H}\right)}{\left(1+\frac{s}{\omega_{P1}^H}\right)\left(1+\frac{s}{\omega_{P2}^H}\right)\cdots\left(1+\frac{s}{\omega_{Pl}^H}\right)}$$

$\left|F_H(j\omega)\right| \rightarrow 1$ for $\omega \ll \omega_{Zi}^H, \omega_{Pi}^H$,i =1…l

$\therefore A_L(s) \cong A_{mid} F_L(s)$

$\left|F_L(j\omega)\right| \rightarrow 1$ for $\omega \gg \omega_{Zj}^L, \omega_{Pj}^L$,j =1…k

$\therefore A_H(s) \cong A_{mid} F_H(s)$

Low-Frequency Response

$$F_L(s) \cong \frac{s}{s+\omega_{P2}}$$

$$\omega_L \cong \omega_{P2}$$

Pole ω_{P2} is called the dominant low-frequency pole (> all other poles) and zeros are at frequencies low enough to not affect ω_L.

If there is no dominant pole at low frequencies, poles and zeros interact to determine ω_L.

$$A_L(s) = A_{mid}F_L(s) = A_{mid}\frac{(s+\omega_{Z1})(s+\omega_{Z2})}{(s+\omega_{P1})(s+\omega_{P2})}$$

For $s=j\omega$, at ω_L, $\left|A(j\omega_L)\right| = \frac{A_{mid}}{\sqrt{2}}$

$$\frac{1}{\sqrt{2}} = \sqrt{\frac{(\omega_L{}^2+\omega_{Z1}{}^2)(\omega_L{}^2+\omega_{Z2}{}^2)}{(\omega_L{}^2+\omega_{P1}{}^2)(\omega_L{}^2+\omega_{P2}{}^2)}}$$

$$\Rightarrow \frac{1}{2} = \frac{1+\frac{(\omega_{Z1}{}^2+\omega_{Z2}{}^2)}{\omega_L{}^2}+\frac{(\omega_{Z1}{}^2\omega_{Z2}{}^2)}{\omega_L{}^4}}{1+\frac{(\omega_{P1}{}^2+\omega_{P2}{}^2)}{\omega_L{}^2}+\frac{(\omega_{P1}{}^2\omega_{P2}{}^2)}{\omega_L{}^4}}$$

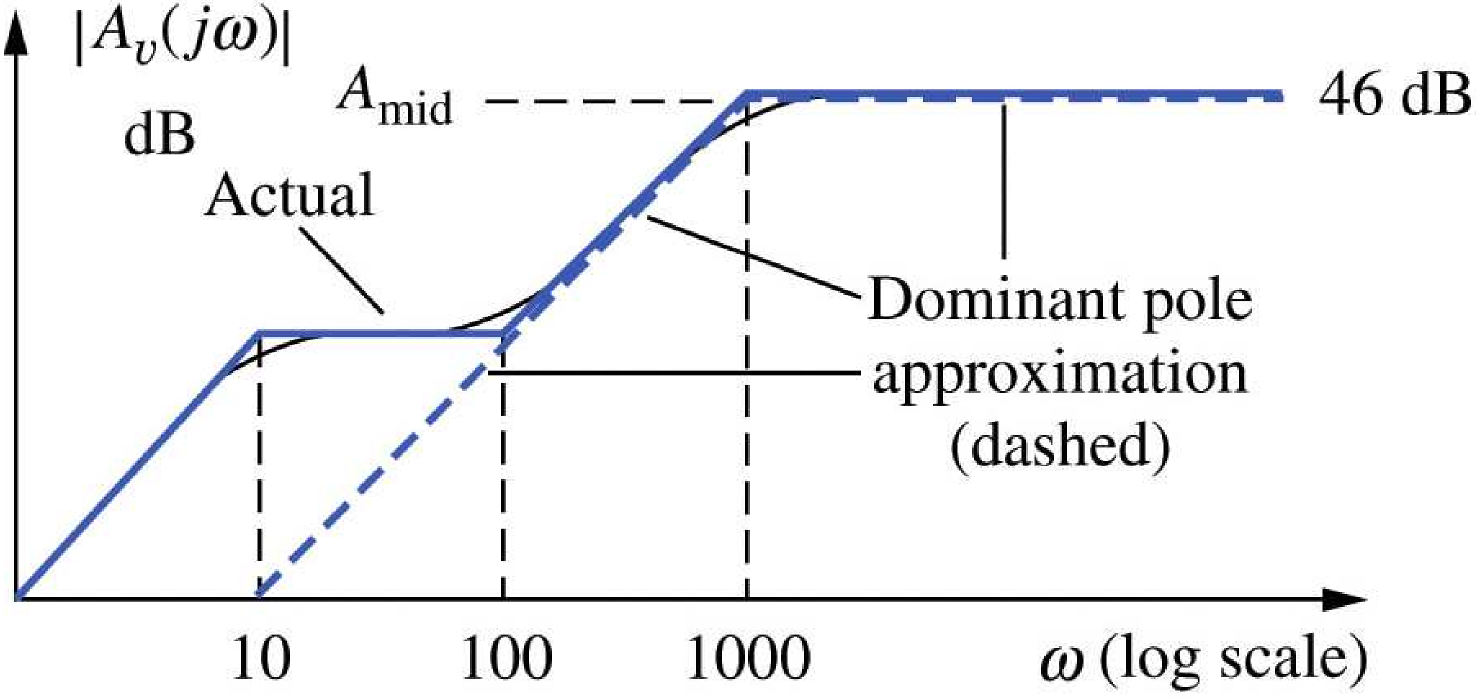
Pole ω_L > all other pole and zero frequencies

$$\omega_L \cong \sqrt{\omega_{P1}{}^2+\omega_{P2}{}^2-2\omega_{Z1}{}^2-2\omega_{Z2}{}^2}$$

In general, for *n* poles and *n* zeros,

$$\omega_L \cong \sqrt{\sum_n \omega_{Pn}{}^2 - 2\sum_n \omega_{Zn}{}^2}$$

Low-Frequency Response



Transfer Function Analysis and Dominant Pole Approximation Example

- **Problem:** Find midband gain, $F_L(s)$ and f_L for

$$A_L(s) = 2000 \frac{s\left(\frac{s}{100}+1\right)}{(0.1s+1)(s+1000)}$$

- **Analysis:** Rearranging the given transfer function to get it in standard form,

$$A_L(s) = 200 \frac{s(s+100)}{(s+10)(s+1000)}$$

Now,

$$A_L(s) = A_{mid} F_L(s)$$
$$\therefore A_{mid} = 200$$

and

$$F_L(s) = \frac{s(s+100)}{(s+10)(s+1000)}$$

Zeros are at s=0 and s =-100. Poles are at s= -10, s=-1000

$$f_L = \frac{1}{2\pi}\sqrt{10^2 + 1000^2 - 2(0^2 + 100^2} = 158\text{Hz}$$

All pole and zero frequencies are low and separated by at least a decade. Dominant pole is at ω=1000 and f_L =1000/2π= 159 Hz. For frequencies > a few rad/s: $A_L(s) = 200 \frac{s}{(s+1000)}$

High-Frequency Response

$$F_L(s) \cong \frac{s}{1+(s/\omega_{P3})}$$

$$\omega_H \cong \omega_{P3}$$

Pole ω_{P3} is called the dominant high-frequency pole (< all other poles).

If there is no dominant pole at low frequencies, poles and zeros interact to determine ω_H.

$$A_H(s) = A_{mid} F_H(s)$$

$$= A_{mid} \frac{\left(1+(s/\omega_{Z1})\right)\left(1+(s/\omega_{Z2})\right)}{\left(1+(s/\omega_{P1})\right)\left(1+(s/\omega_{P2})\right)}$$

For $s=j\omega$, at ω_H, $\left|A(j\omega_H)\right| = \frac{A_{mid}}{\sqrt{2}}$

$$\frac{1}{\sqrt{2}} = \sqrt{\frac{\left(1+(\omega_H{}^2/\omega_{Z1}{}^2)\right)\left(1+(\omega_H{}^2/\omega_{Z2}{}^2)\right)}{\left(1+(\omega_H{}^2/\omega_{P1}{}^2)\right)\left(1+(\omega_H{}^2/\omega_{P2}{}^2)\right)}}$$

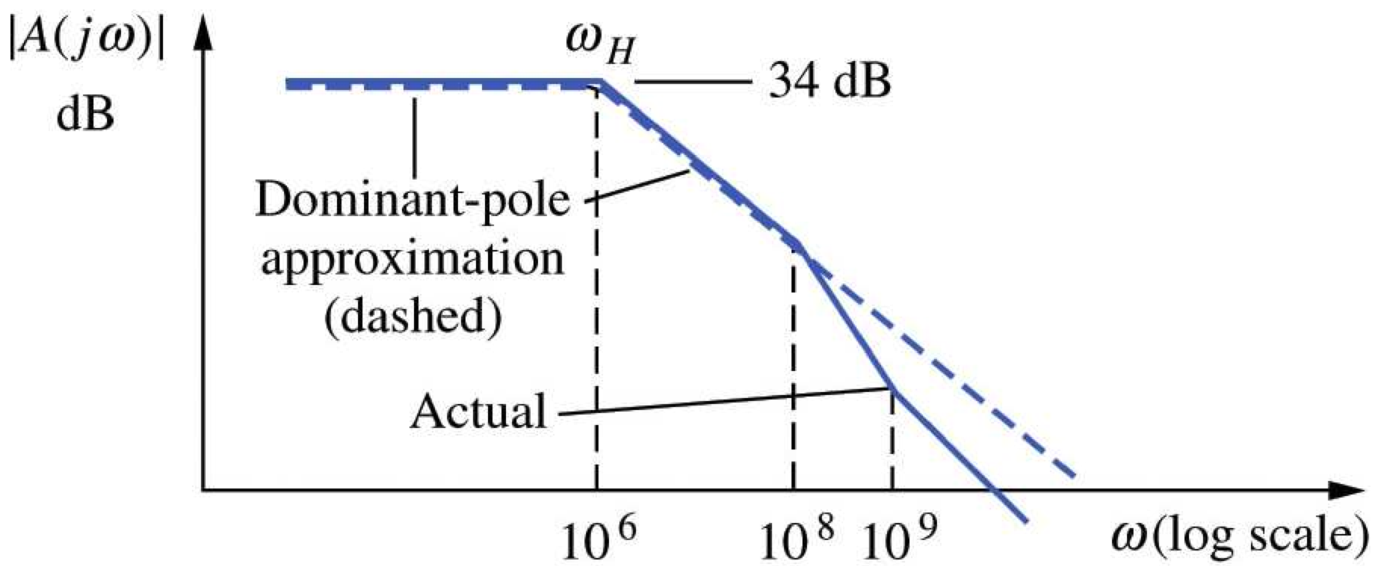
$$\Rightarrow \frac{1}{2} = \frac{1+\dfrac{\omega_H{}^2}{\omega_{Z1}{}^2}+\dfrac{\omega_H{}^2}{\omega_{Z2}{}^2}+\dfrac{\omega_H{}^4}{\omega_{Z1}{}^2\omega_{Z2}{}^2}}{1+\dfrac{\omega_H{}^2}{\omega_{P1}{}^2}+\dfrac{\omega_H{}^2}{\omega_{P2}{}^2}+\dfrac{\omega_H{}^4}{\omega_{P1}{}^2\omega_{P2}{}^2}}$$

Pole ω_H < all other pole and zero frequencies

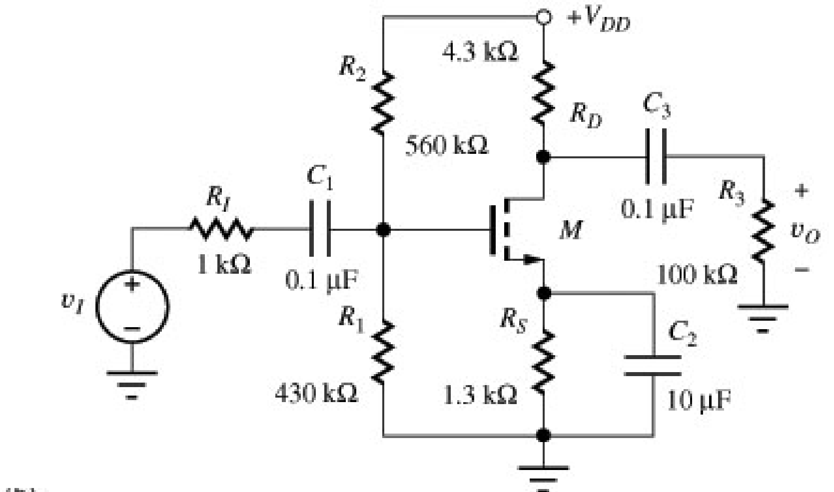
$$\omega_H \cong \frac{1}{\sqrt{\dfrac{1}{\omega_{P1}{}^2}+\dfrac{1}{\omega_{P2}{}^2}-\dfrac{2}{\omega_{Z1}{}^2}-\dfrac{2}{\omega_{Z2}{}^2}}}$$

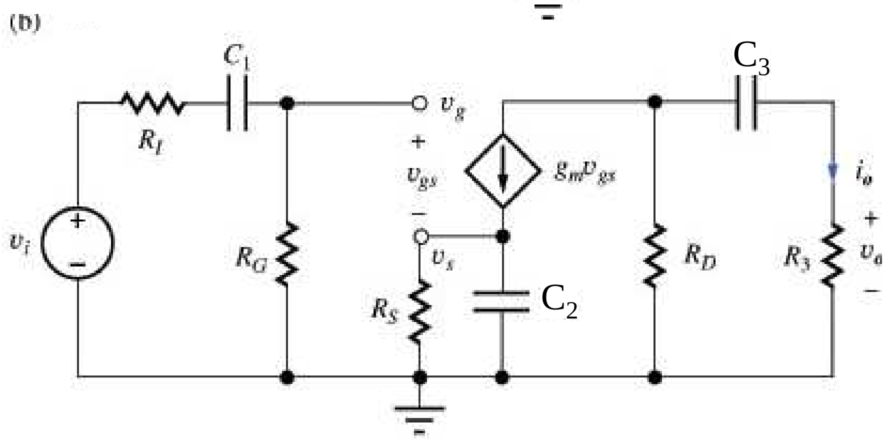
In general, $\omega_H \cong \dfrac{1}{\sqrt{\sum_n \dfrac{1}{\omega_{Pn}{}^2} - 2\sum_n \dfrac{1}{\omega_{Zn}{}^2}}}$

High-Frequency Response



Direct Determination of Low-Frequency Poles and Zeros: C-S Amplifier





$$V_o(s) = I_o(s)R_3 = -g_m V_{gs}(s)\frac{R_D}{R_3 + (1/sC_3) + R_3}R_3$$

$$= -g_m(R_3 \| R_D)\frac{s}{s + \dfrac{1}{C_3(R_D + R_3)}}V_{gs}(s)$$

$$V_g(s) = \frac{s + C_1 R_G}{s + C_1(R_I + R_G) + 1}V_i(s)$$

$$V_{gs}(s) = V_g - V_s = \frac{s + (1/C_2 R_S)}{s + \dfrac{1}{C_2\left[(1/g_m)\|R_S\right]}}V_g(s)$$

$$A_V(s) = \frac{V_o(s)}{V_i(s)} = A_{mid}F_L(s)$$

$$A_{mid} = -g_m(R_3 \| R_D)\frac{R_G}{R_G + R_I}$$

Direct Determination of Low-Frequency Poles and Zeros: C-S Amplifier (contd.)

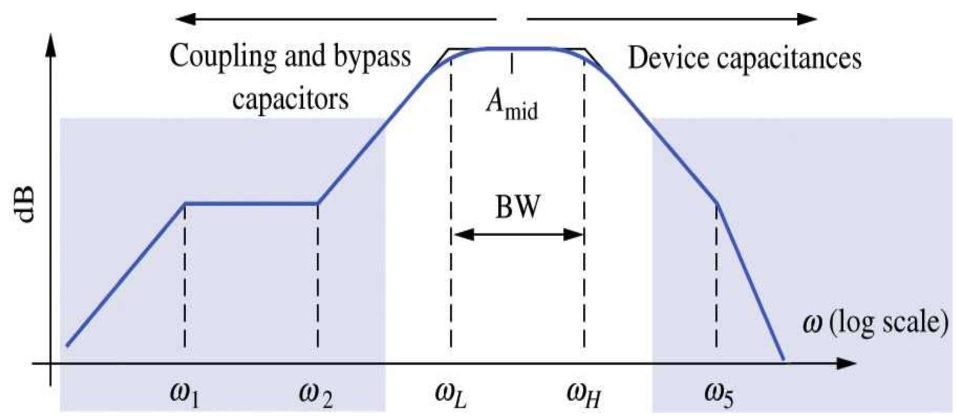
$$F_L(s) = \frac{s^2\left(s + (1/C_2 R_S)\right)}{\left(s + \dfrac{1}{C_1(R_I + R_G)}\right)\left(s + \dfrac{1}{C_2\left[(1/g_m) \| R_S\right]}\right)\left(s + \dfrac{1}{C_3(R_D + R_3)}\right)}$$

The three zero locations are: s = 0, 0, $-1/(R_S C_2)$.

The three pole locations are: $s = -\dfrac{1}{C_1(R_I + R_G)}, -\dfrac{1}{C_2\left[(1/g_m) \| R_S\right]}, -\dfrac{1}{C_2(R_D + R_3)}$

Each independent capacitor in the circuit contributes one pole and one zero. Series capacitors C_1 and C_3 contribute the two zeros at s=0 (dc), blocking propagation of dc signals through the amplifier. The third zero due to parallel combination of C_2 and R_S occurs at frequency where signal current propagation through MOSFET is blocked (output voltage is zero).

Short-Circuit Time Constant Method to Determine ω_L



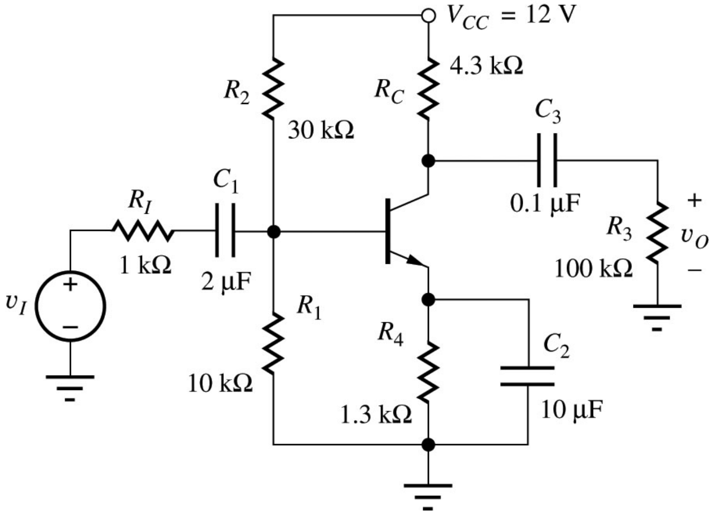
Midband gain and upper and lower cutoff frequencies that define bandwidth of amplifier are of more interest than complete transfer function.

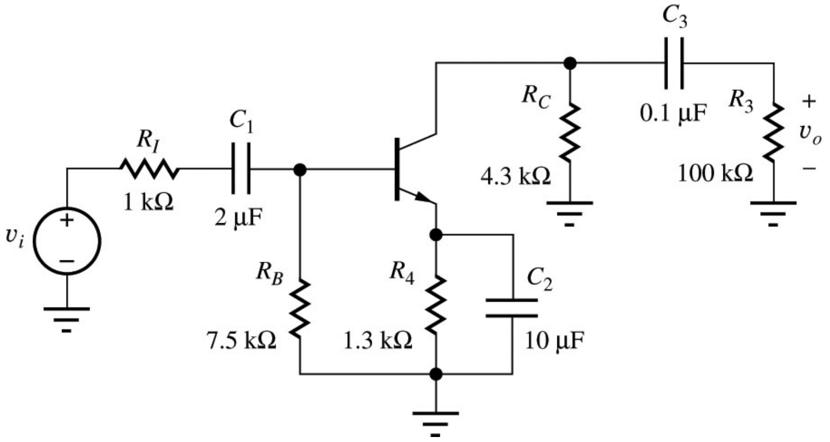
- Lower cutoff frequency for a network with n coupling and bypass capacitors is given by:

$$\omega_L \cong \sum_{i=1}^{n} \frac{1}{R_{iS} C_i}$$

where R_{iS} is resistance at terminals of ith capacitor C_i with all other capacitors replaced by short circuits. Product $R_{iS}\, C_i$ is short-circuit time constant associated with C_i.

Estimate of ω_L for C-E Amplifier





Using SCTC method, for C_1,

$$R_{1S} = R_I + (R_B \| R_{in}^{CE}) = R_2 + (R_B \| r_\pi)$$

For C_2,

$$R_{2S} = R_4 \| R_{iE} = R_4 \left\| \frac{r_\pi + R_{th}}{\beta_o + 1} \right.$$

$$= R_4 \left\| \frac{r_\pi + (R_I \| R_B)}{\beta_o + 1} \right.$$

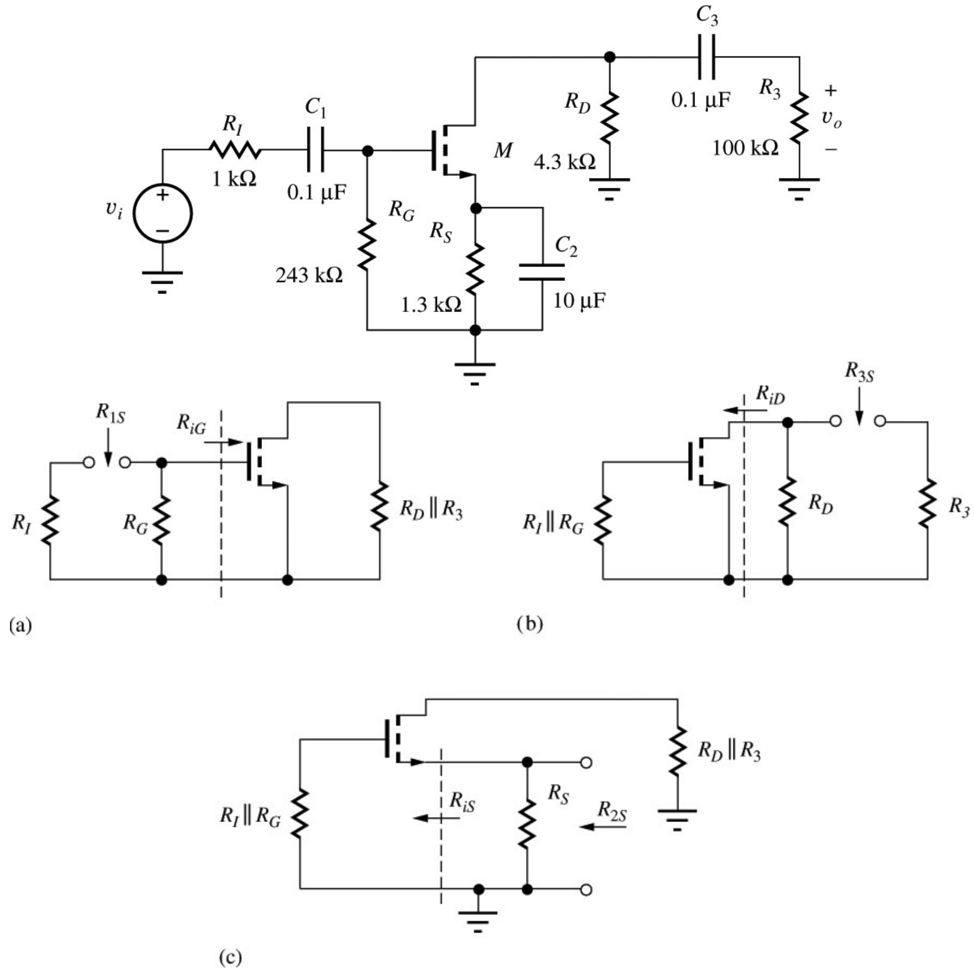
For C_3,

$$R_{3S} = R_3 + (R_C \| R_{iC}) = R_3 + (R_C \| r_o)$$

$$\cong R_3 + R_C$$

$$\omega_L \cong \sum_{i=1}^{3} \frac{1}{R_{iS} C_i} \qquad f_L = 735 Hz$$

Estimate of ω_L for C-S Amplifier



Using SCTC method,

For C_1,

$$R_{1S} = R_I + (R_G \| R_{iG}) = R_I + R_G$$

For C_2,

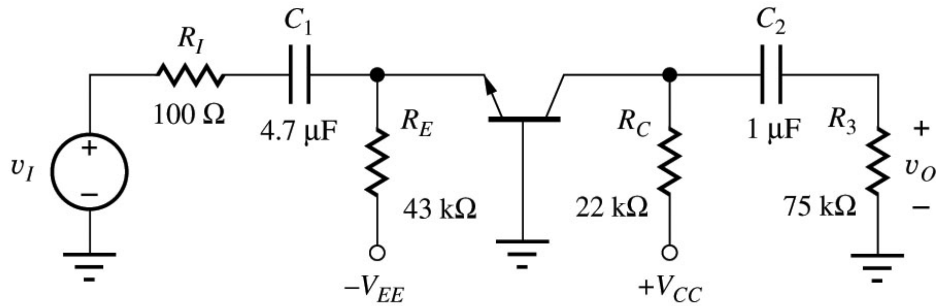
$$R_{2S} = R_S \| R_{iS} = R_S \left\| \frac{1}{g_m} \right.$$

For C_3,

$$R_{3S} = R_3 + (R_D \| R_{iD}) = R_3 + (R_D \| r_o)$$

$$\cong R_3 + R_D$$

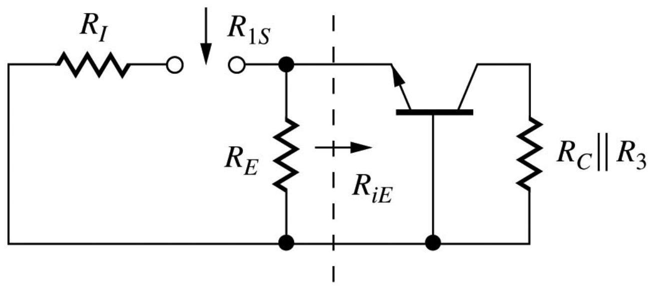
Estimate of ω_L for C-B Amplifier



Using SCTC method,

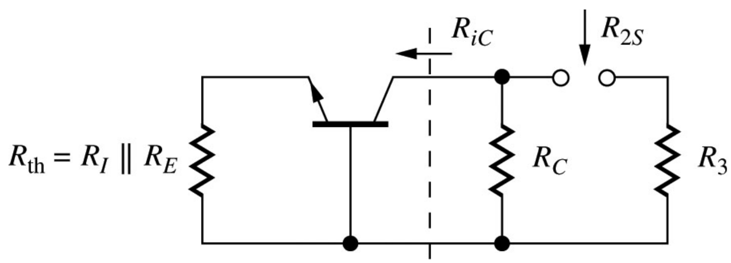
For C_1,

$$R_{1S} = R_I + (R_E \| R_{iE}) = R_I + \left(R_E \left\| \frac{1}{g_m}\right.\right)$$

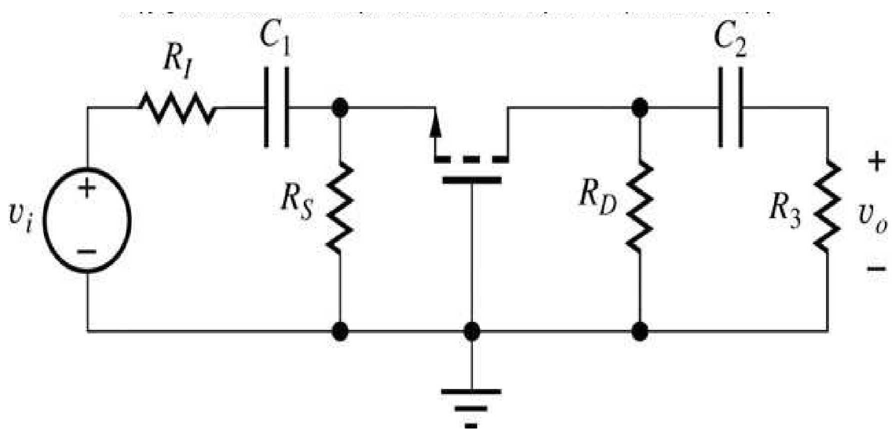


For C_2,

$$R_{2S} = R_3 + (R_C \| R_{iC}) \cong R_3 + R_C$$



Estimate of ω_L for C-G Amplifier



Using SCTC method,

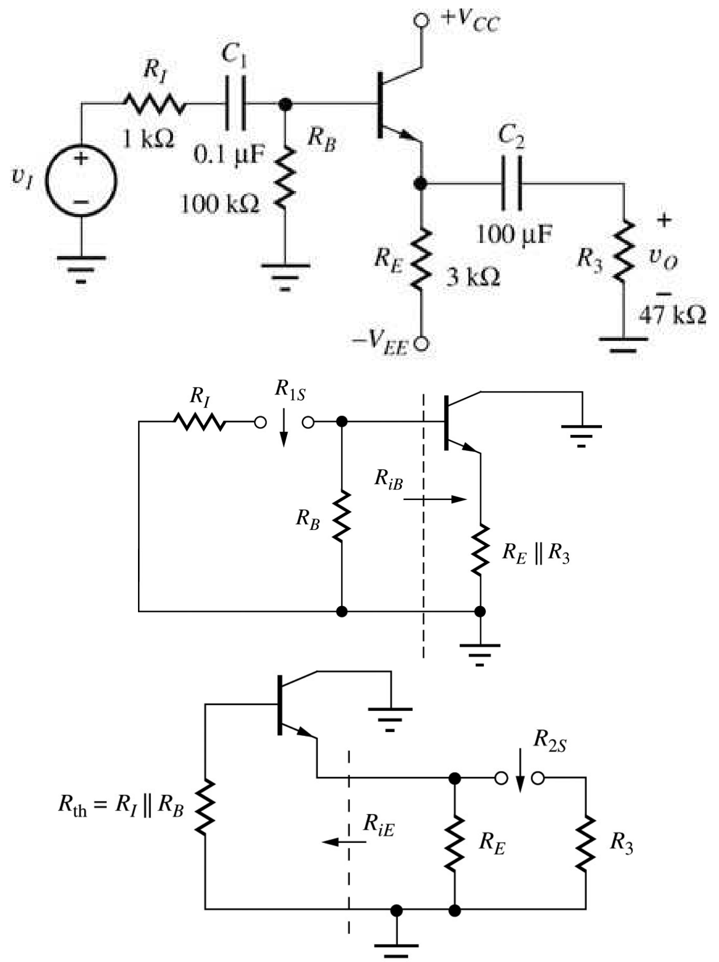
For C_1,

$$R_{1S} = R_I + (R_S \| R_{iS}) = R_I + \left(R_S \left\| \frac{1}{g_m}\right.\right)$$

For C_2,

$$R_{2S} = R_3 + (R_D \| R_{iD}) \cong R_3 + R_D$$

Estimate of ω_L for C-C Amplifier



Using SCTC method,

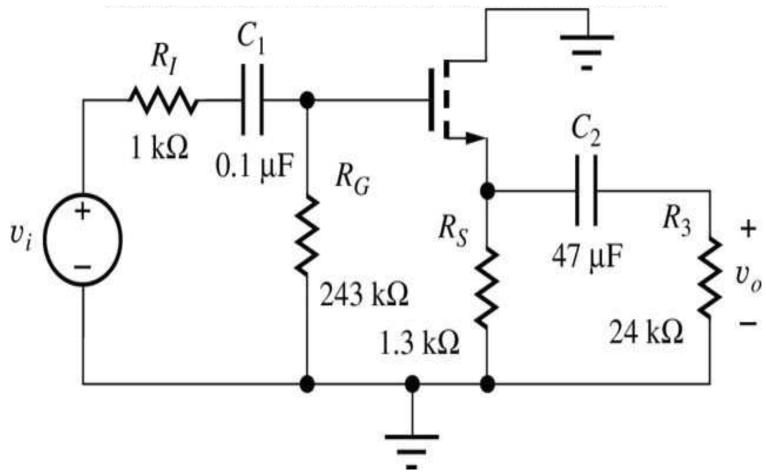
For C_1,

$$R_{1S} = R_I + (R_B \| R_{iB})$$
$$= R_I + \left(R_B \| r_\pi + (\beta_o + 1)(R_E \| R_3)\right)$$

For C_2,

$$R_{2S} = R_3 + (R_E \| R_{iE}) = R_3 + \left(R_E \left\| \frac{r_\pi + R_{th}}{\beta_o + 1}\right.\right)$$

Estimate of ω_L for C-D Amplifier



Using SCTC method,

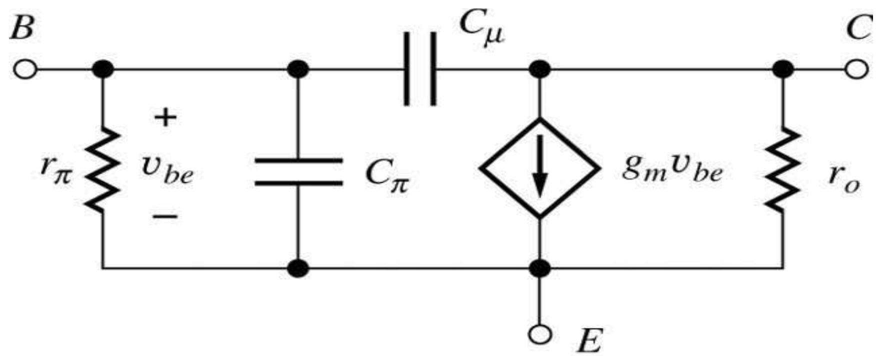
For C_1,

$$R_{1S} = R_I + (R_G \| R_{in}^{CD}) = R_I + R_G$$

For C_2,

$$R_{2S} = R_3 + R_S \| R_{out}^{CD} = R_3 + R_S \left\| \frac{1}{g_m} \right.$$

Frequency-dependent Hybrid-Pi Model for BJT



Capacitance between base and emitter terminals is:

$$C_\pi = g_m \tau_F$$

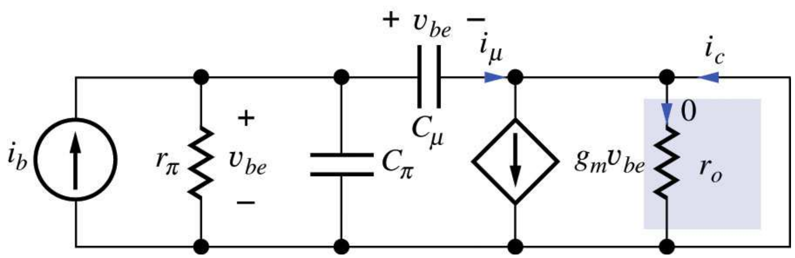
τ_F is forward transit-time of the BJT. C_π appears in parallel with r_π. As frequency increases, for a given input signal current, impedance of C_π reduces v_{be} and thus the current in the controlled source at transistor output.

Capacitance between base and collector terminals is:

$$C_\mu = \frac{C_{\mu o}}{\sqrt{1+(V_{CB}/\phi_{jc})}}$$

$C_{\mu o}$ is total collector-base junction capacitance at zero bias, Φ_{jc} is its built-in potential.

Unity-gain Frequency of BJT



The right-half plane transmission zero $\omega_Z = + g_m/C_\mu$ occurring at high frequency can be neglected.

$$\beta(s) \cong \frac{\beta_o}{s(C_\pi + C_\mu)r_\pi + 1} = \frac{\beta_o}{(s/\omega_\beta) + 1}$$

$\omega_\beta = 1/\ r_\pi(C_\mu + C_\pi)$ is the beta-cutoff frequency

$$\beta(s) \cong \frac{\beta_o \omega_\beta}{s + \omega_\beta} = \frac{\omega_T}{s + \omega_\beta}$$

where $$\omega_T = \beta_o \omega_\beta = \frac{\beta_o}{(C_\pi + C_\mu)r_\pi} = \frac{g_m}{C_\pi + C_\mu}$$

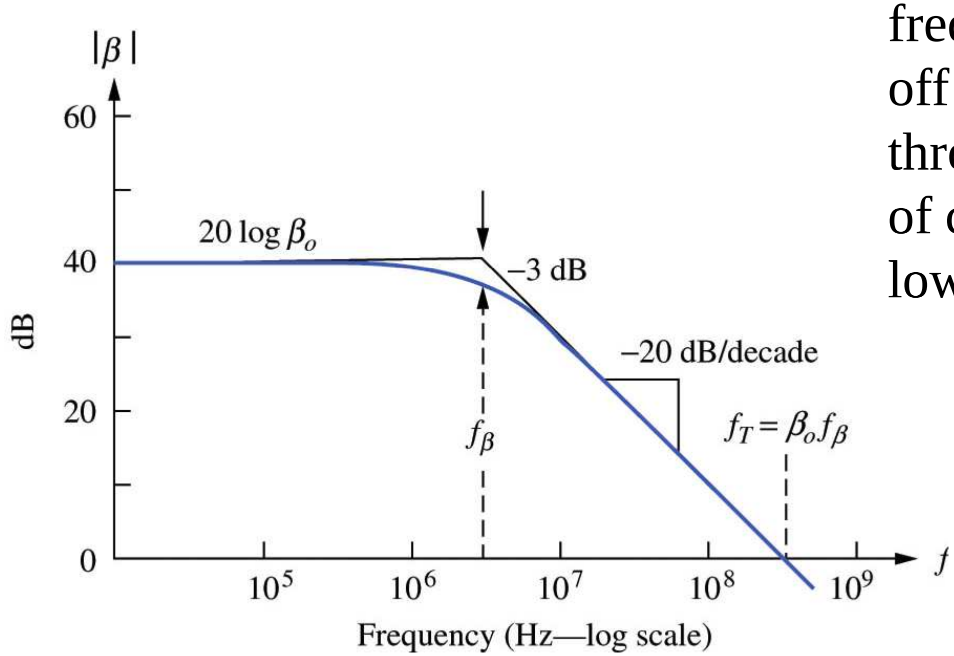
$$\mathrm{I_c(s)} = (g_m - sC_\mu)\mathrm{V_{be}(s)}$$

$$= (g_m - sC_\mu)\mathrm{I_b(s)}\frac{r_\pi}{s(C_\pi + C_\mu)r_\pi + 1}$$

$$\therefore \beta(s) = \frac{\mathrm{I_c(s)}}{\mathrm{I_b(s)}} = \frac{\beta_o\left(1 - \frac{sC_\mu}{g_m}\right)}{s(C_\pi + C_\mu)r_\pi + 1}$$

and $f_T = \omega_T/2\pi$ is the unity gain bandwidth product. Above f_T BJT has no appreciable current gain.

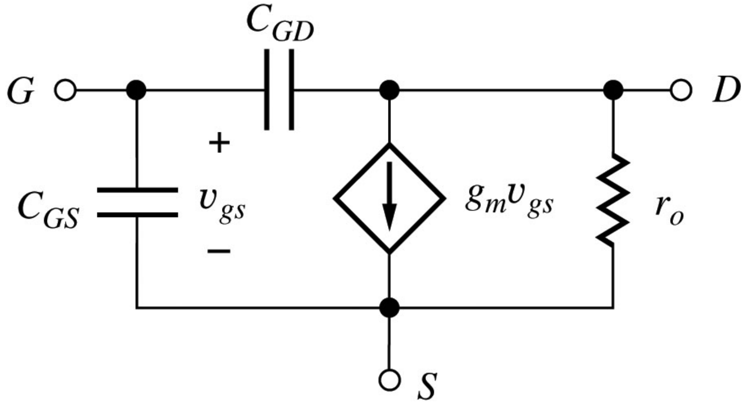
Unity-gain Frequency of BJT (contd.)

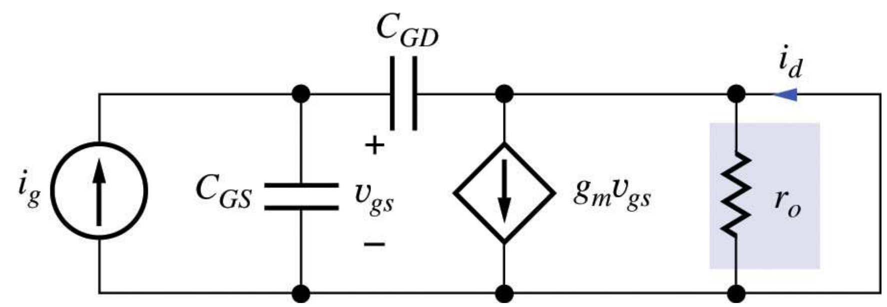


Current gain is $\beta_o = g_m r_\pi$ at low frequencies and has single pole roll-off at frequencies > f_β, crossing through unity gain at ω_T. Magnitude of current gain is 3 dB below its low-frequency value at f_β.

$$C_\pi = \frac{g_m}{\omega_T} - C_\mu = \frac{40 I_C}{\omega_T} - C_\mu$$

High-frequency Model of MOSFET





$$\omega_T = \frac{g_m}{C_{GS}+C_{GD}}$$

$$\mathrm{I_d}(\mathrm{s}) = (g_m - sC_{GD})\mathrm{V_{gs}}(\mathrm{s})$$

$$= \mathrm{I_b}(\mathrm{s}) \frac{(g_m - sC_{GD})}{s(C_{GS}+C_{GD})}$$

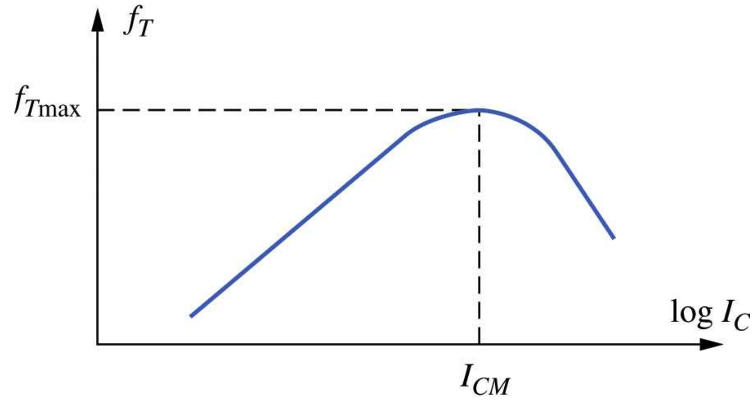
$$f_T = \frac{\mu_n C_{ox}'' \frac{W}{L}\left(V_{GS}-V_{TN}\right)}{(2/3)C_{ox}''WL} = \frac{3}{2}\frac{\mu_n\left(V_{GS}-V_{TN}\right)}{L^2}$$

$$\therefore \beta(s) = \frac{\mathrm{I_d}(\mathrm{s})}{\mathrm{I_g}(\mathrm{s})} = \frac{\omega_T}{s}\left(1 - \frac{s}{\omega_T\left(1+(C_{GS}/C_{GD})\right)}\right)$$

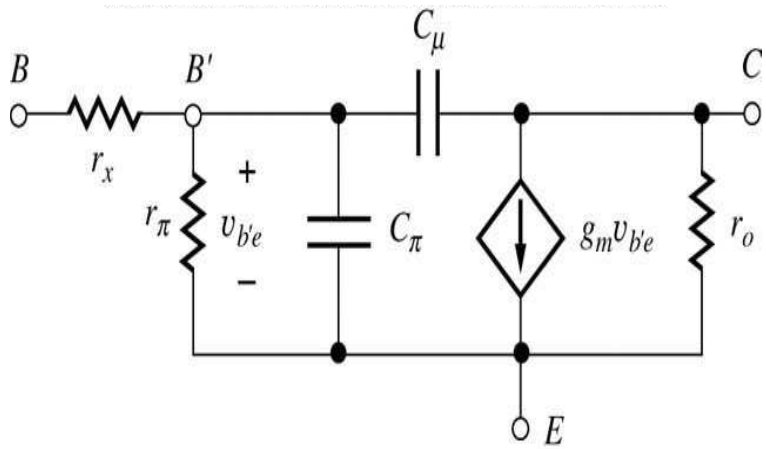
Limitations of High-frequency Models

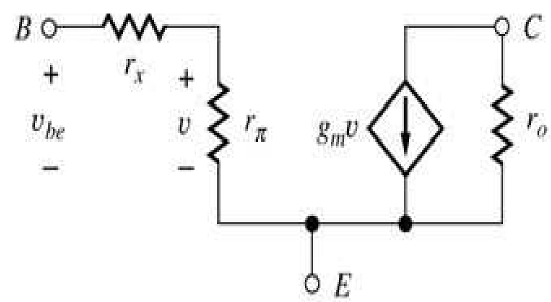
- Above 0.3 f_T, behavior of simple pi-models begins to deviate significantly from the actual device.
- Also, ω_T depends on operating current as shown and is not constant as assumed.
- For given BJT, a collector current I_{CM} exists that yield f_{Tmax}.
- For FET in saturation, C_{GS} and C_{GD} are independent of Q-point current, so

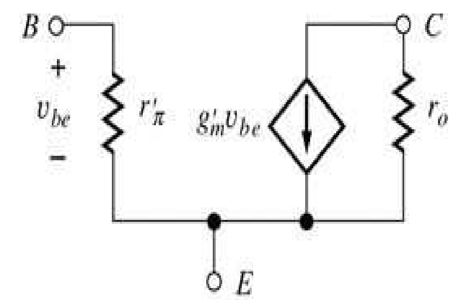
$$\omega_T \propto g_m \propto \sqrt{I_D}$$



Effect of Base Resistance on Midband Amplifiers







Base current enters the BJT through external base contact and traverses a high resistance region before entering active area. r_x models voltage drop between base contact and active area of the BJT.

To account for base resistance r_x is absorbed into equivalent pi model and can be used to transform expressions for C-E, C-C and C-B amplifiers.

$$\mathrm{i} = g_m \mathrm{v} = g_m \frac{r_\pi}{r_\pi + r_x} \mathrm{v}_{\mathrm{be}} = g_m' \mathrm{v}_{\mathrm{be}}$$

$$g_m' = g_m \frac{r_\pi}{r_\pi + r_x} = \frac{\beta_o}{r_\pi + r_x}$$

$$r_\pi' = r_\pi + r_x \qquad \beta_o' = \beta_o$$

Summary of BJT Amplifier Equations with Base Resistance

	COMMON-EMITTER AMPLIFIER	COMMON-COLLECTOR AMPLIFIER	COMMON-BASE AMPLIFIER
Terminal voltage gain $A_{vt} = \dfrac{v_o}{v_1}$	$-\dfrac{\beta_o R_L}{r'_\pi + (\beta_o + 1)R_E}$	$+\dfrac{\beta_o R_L}{r'_\pi + (\beta_o + 1)R_L}$	$+g'_m R_L$
$r'_\pi = r_x + r_\pi$	$\cong -\dfrac{g'_m R_L}{1 + g'_m R_E}$	$\cong +\dfrac{g'_m R_L}{1 + g'_m R_L} \cong +1$	
$g'_m = \dfrac{\beta_o}{r'_\pi}$			
Signal-source voltage gain $A_v = \dfrac{\mathbf{v_o}}{\mathbf{v_i}}$	$-\dfrac{g'_m R_L}{1 + g'_m R_E}\left(\dfrac{R_B \Vert R_{iB}}{R_I + R_B \Vert R_{iB}}\right)$	$+\dfrac{g'_m R_L}{1 + g'_m R_L}\left(\dfrac{R_B \Vert R_{iE}}{R_I + R_B \Vert R_{iE}}\right) \cong +1$	$+\dfrac{g'_m R_L}{1 + g'_m (R_I \Vert R_E)}\left(\dfrac{R_E}{R_I + R_E}\right)$
Input resistance	$r'_\pi + (\beta_o + 1)R_E$	$r'_\pi + (\beta_o + 1)R_L$	$\dfrac{1}{g'_m}$
Output resistance	$r_o(1 + g'_m R_E)$	$\dfrac{1}{g'_m} + \dfrac{R_{\text{th}}}{\beta_o + 1}$	$r_o[1 + g'_m (R_I \Vert R_E)]$
Input signal range	$\cong 0.005(1 + g'_m R_E)$	$\cong 0.005(1 + g'_m R_L)$	$\cong 0.005[1 + g'_m (R_I \Vert R_E)]$
Current gain	$-\beta_o$	$\beta_o + 1$	$\alpha_o \cong +1$

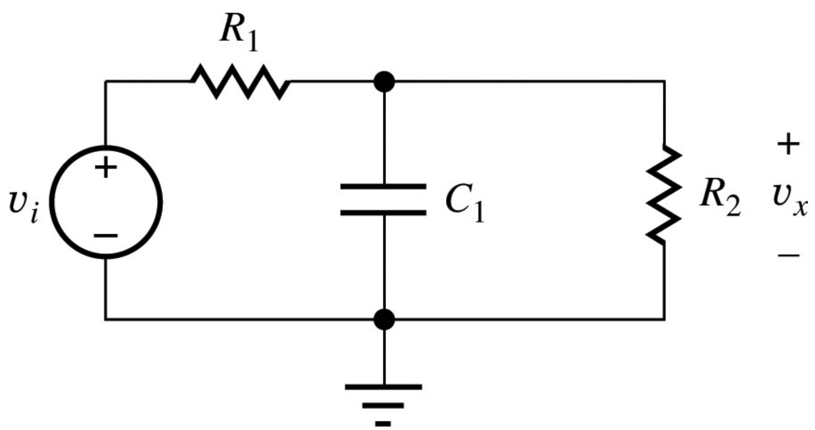
Single-Pole High Frequency Response

Let's first start with a simple two resistor, one capacitor network.

$$\frac{v_x}{v_i} = \frac{R_2 \left\| \frac{1}{sC_1} \right.}{R_1 + R_2 \left\| \frac{1}{sC_1} \right.} = \frac{\dfrac{R_2}{1+sR_2C_1}}{R_1 + \dfrac{R_2}{1+sR_2C_1}}$$

$$= \frac{R_2}{R_1+R_2} \frac{1}{\left(1 + s\dfrac{R_1R_2}{R_1+R_2}C_1\right)}$$

$$= \frac{R_2}{R_1+R_2} \frac{1}{\left(1+s[R_1 \parallel R_2]C_1\right)}$$

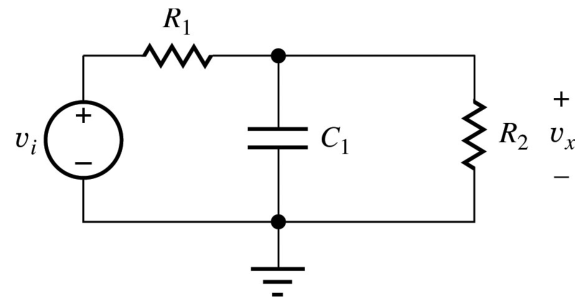


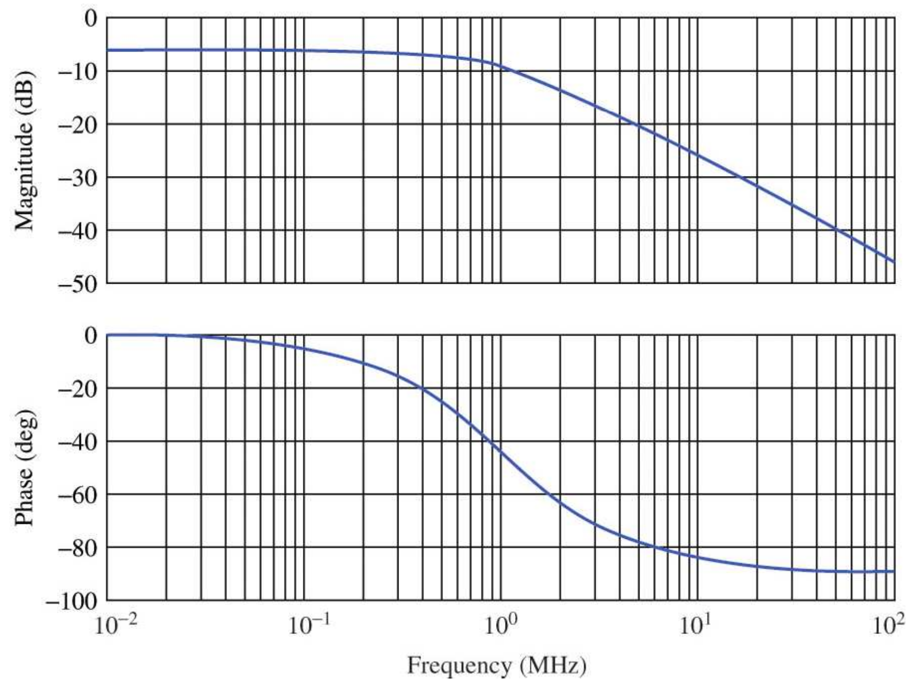
Single-Pole High Frequency Response (cont.)

Substituting s=j2πf and using $f_p = 1/(2\pi[R_1 \| R_2]C_1)$

$$\frac{v_x}{v_i} = \frac{R_2}{R_1 + R_2}\frac{1}{\left(1 + j\frac{f}{f_p}\right)}$$

This expression has two parts, the midband gain, $R_2/(R_2+R_1)$, and the high frequency characteristic, $1/(1+jf/f_p)$.

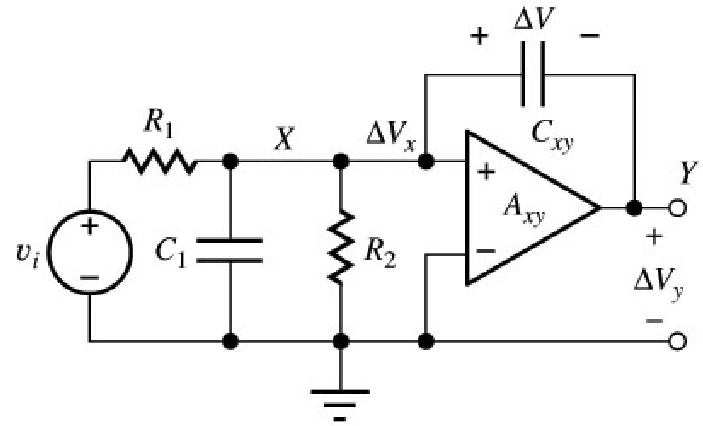




Miller Effect

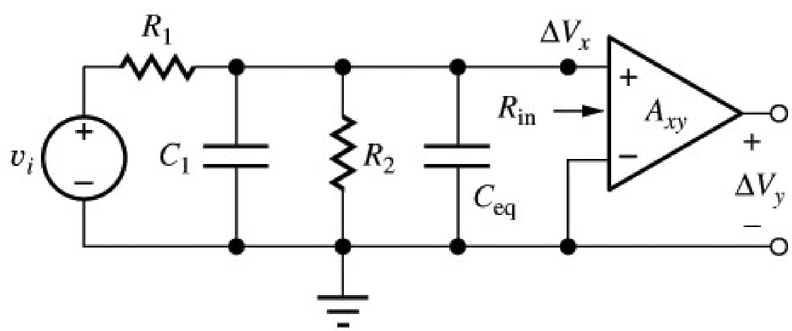
We desire to replace C_{xy} with C_{eq} to ground. Starting with the definition of small-signal capacitance:

$$C = \frac{\Delta Q}{\Delta V}$$



Now write an expression for the change in charge for Cxy:

$$\Delta Q = C_{phy}(\Delta V_x - \Delta V_y) = C_{xy}(\Delta V_x - A_{xy}\Delta V_x)$$
$$= C_{xy}\Delta V_x(1 - A_{xy})$$

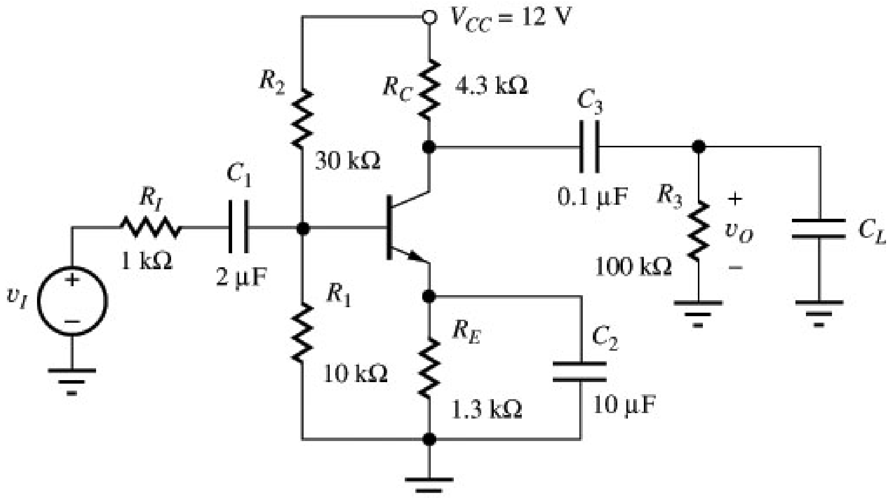


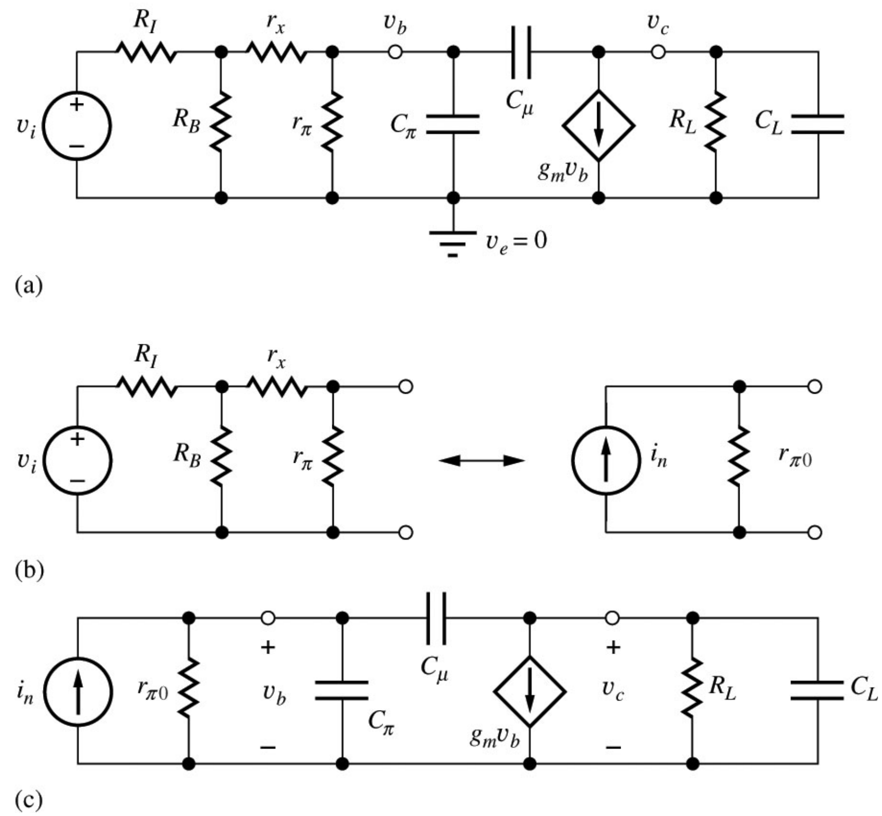
We can now find and equivalent capacitance, C_{eq}:

$$C_{eq} = \frac{C_{xy}\Delta V_x(1 - A_{xy})}{\Delta V_x} = C_{xy}(1 - A_{xy})$$

C-E Amplifier High Frequency Response using Miller Effect

First, find the simplifed small-signal model of the C-A amp.





C-E Amplifier High Frequency Response using Miller Effect (cont.)

Input gain is found as

$$A_i = \frac{v_b}{v_i} = \frac{R_{in}}{R_i + R_{in}} \cdot \frac{r_\pi}{r_x + r_\pi}$$

$$= \frac{R_1 \| R_2 \| (r_x + r_\pi)}{R_i + R_1 \| R_2 \| (r_x + r_\pi)} \cdot \frac{r_\pi}{r_x + r_\pi}$$

Terminal gain is $A_{bc} = \dfrac{v_c}{v_b} = -g_m r_o \| R_C \| R_3 \cong -g_m R_C \| R_3 \cong -g_m R_L$

Using the Miller effect, we find the equivalent capacitance at the base as:

$$C_{eqB} = C_\mu (1 - A_{bc}) + C_\pi (1 - A_{be})$$

$$= C_\mu (1 - [-g_m R_L]) + C_\pi (1 - 0)$$

$$= C_\mu (1 + g_m R_L) + C_\pi$$

C-E Amplifier High Frequency Response using Miller Effect (cont.)

The total equivalent resistance at the base is

$$R_{eqB} = (R_{th} + r_x) \parallel R_{inB}$$

$$= (R_i \parallel R_1 \parallel R_2 + r_x) \parallel r_\pi = r_{\pi 0}$$

The total capacitance and resistance at the collector is

$$C_{eqC} = C_\mu + C_L$$

$$R_{eqC} = r_o \parallel R_C \parallel R_3 \cong R_C \parallel R_3 \cong R_L$$

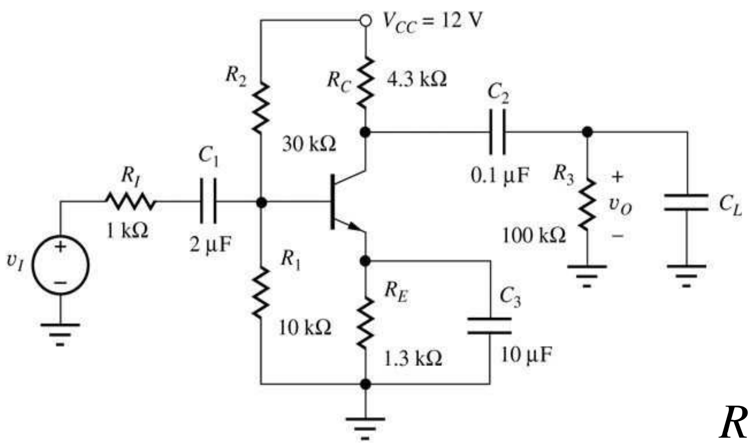
Because of interaction through C_μ, the two RC time constants interact, giving rise to a dominant pole

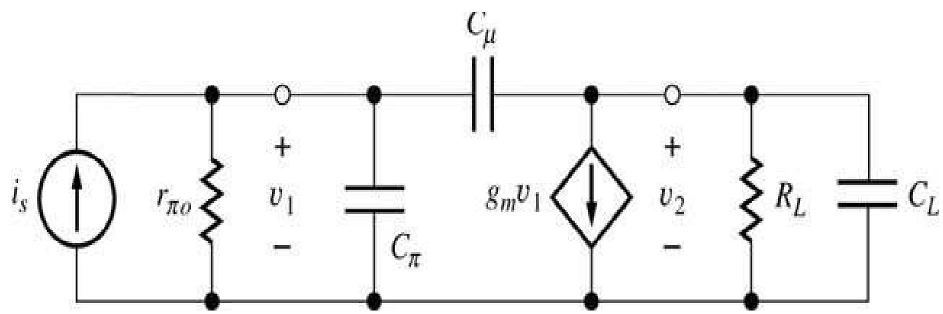
$$\omega_{p1} = \frac{1}{r_{\pi 0}[C_\mu(1 + g_m R_L) + C_\pi] + R_L[C_\mu + C_L]}$$

$$C_T = [C_\mu(1 + g_m R_L) + C_\pi] + \frac{R_L}{r_{\pi 0}}[C_\mu + C_L]$$

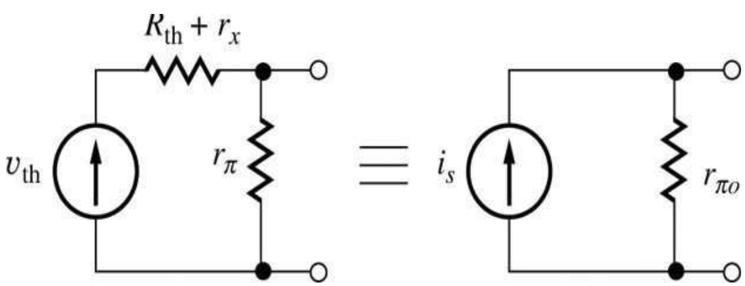
$$\omega_{p1} = \frac{1}{r_{\pi 0} C_T} = \frac{1}{r_{\pi 0}\left([C_\mu(1 + g_m R_L) + C_\pi] + \frac{R_L}{r_{\pi 0}}[C_\mu + C_L]\right)}$$

Direct High-Frequency Analysis: C-E Amplifier





$$R_L = R_3 \| R_C = 100\text{k}\Omega \| 4.3\text{k}\Omega \qquad R_B = R_1 \| R_2 = 30\text{k}\Omega \| 10\text{k}\Omega$$



The small-signal model can be simplified by using Norton source transformation.

$$v_{th} = v_i \frac{R_B}{R_I + R_B} \qquad R_{th} = \frac{R_I R_B}{R_I + R_B}$$

$$i_s = \frac{v_{th}}{R_{th} + r_x} \qquad r_{\pi o} = r_\pi \| (R_{th} + r_x)$$

Direct High-Frequency Analysis: C-E Amplifier (Pole Determination)

From nodal equations for the circuit in frequency domain,

$$V_C(s) = I_S(s)\frac{(sC_\mu - g_m)}{\Delta}$$

$$\Delta = s^2\left(C_\pi\left(C_\mu + C_L\right) + C_\pi C_L\right)$$

$$+ s\left(C_\pi g_L + C_\mu\left(g_L + g_\mu + g_\pi\right) + C_L g_{\pi o}\right) + g_L g_{\pi o}$$

High-frequency response is given by 2 poles, one finite zero and one zero at infinity. Finite right-half plane zero, $\omega_Z = +\, g_m/C_\mu > \omega_T$ can be important in FET amplifiers.

For a polynomial $s^2 + sA_1 + A_0$ with roots a and b, $a = A_1$ and $b = A_0/A_1$.

$$C_T = C_\pi + C_\mu\left(1 + g_m R_L + \frac{R_L}{r_{\pi o}}\right) + C_L\frac{R_L}{r_{\pi o}}$$

$$\omega_{P1} = \frac{A_0}{A_1} \cong \frac{1}{r_{\pi o} C_T}$$

$$\omega_{P2} \cong \frac{g_m}{C_\pi\left(1 + (C_L/C_\mu)\right) + C_L} \cong \frac{g_m}{C_\pi + C_L}$$

Smallest root that gives first pole limits frequency response and determines ω_H. Second pole is important in frequency compensation as it can degrade phase margin of feedback amplifiers.

Direct High-Frequency Analysis: C-E Amplifier (Overall Transfer Function)

$$V_o(s) = \frac{V_{th}(s)}{R_{th} + r_x} \frac{(sC_\mu - g_m)}{g_L g_{\pi o}\left(1+(s/\omega_{P1})\right)\left(1+(s/\omega_{P2})\right)}$$

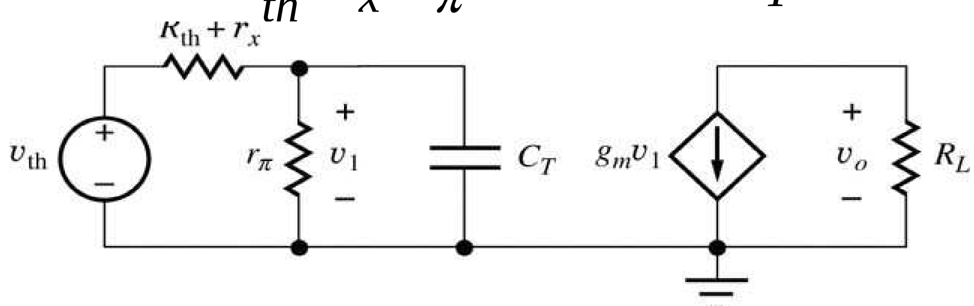
$$V_o(s) = \frac{V_{th}(s)}{R_{th} + r_x}(-g_m R_L r_{\pi o}) \frac{\left(1-(s/\omega_Z)\right)}{g_L g_{\pi o}\left(1+(s/\omega_{P1})\right)\left(1+(s/\omega_{P2})\right)}$$

$$\therefore V_o(s) \cong -\frac{V_{th}(s)}{R_{th} + r_x} \frac{\left(g_m R_L r_{\pi o}\right)}{\left(1+(s/\omega_{P1})\right)}$$

$$A_{vth}(s) = \frac{V_o(s)}{V_{th}(s)} \cong \frac{A_{mid}}{\left(1+(s/\omega_{P1})\right)}$$

$$A_{mid} = -\frac{\beta_o R_L}{R_{th} + r_x + r_\pi} \qquad \omega_{P1} = \frac{1}{r_{\pi o} C_T}$$

Dominant pole model at high frequencies for C-E amplifier is as shown.



Direct High-Frequency Analysis: C-E Amplifier (Example)

- **Problem:** Find midband gain, poles, zeros and f_L.
- Given data: Q-point= (1.60 mA, 3.00V), f_T =500 MHz, β_o =100, C_μ =0.5 pF, r_x =250Ω, C_L =0
- **Analysis:** $g_m = 40 I_C = 40(0.0016) = 64$ mS, $r_\pi = \beta_o / g_m = 1.56$ kΩ.

$$C_\pi = \frac{g_m}{2\pi f_T} - C_\mu = 19.9\text{pF}$$

$$R_L = R_3 \| R_C = 100\text{k}\Omega \| 4.3\text{k}\Omega = 4.12\text{k}\Omega$$

$$R_{th} = R_B \| R_I = 7.5\text{k}\Omega \| 1\text{k}\Omega = 882\Omega$$

$$r_{\pi o} = r_\pi \| (R_{th} + r_x) = 656\Omega$$

$$C_T = C_\pi + C_\mu \left(1 + g_m R_L\right) + \frac{R_L}{r_{\pi o}}(C_\mu + C_L) = 156\text{pF}$$

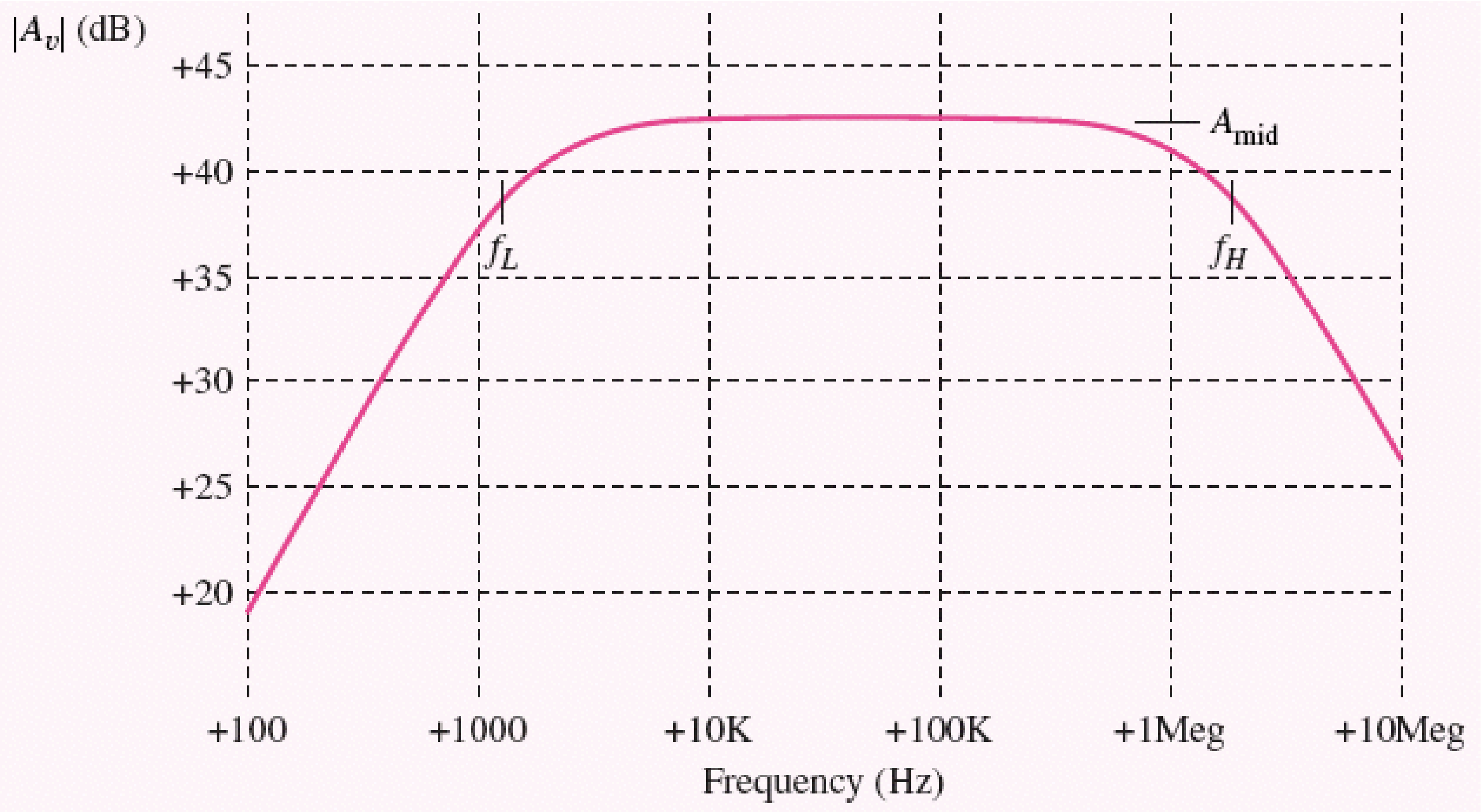
$$f_{P1} = \frac{1}{2\pi r_{\pi o} C_T} = 1.56\text{MHz}$$

$$f_{P2} = \frac{g_m}{2\pi(C_\pi + C_L)} = 512\ MHz$$

$$f_Z = \frac{g_m}{2\pi C_\mu} = 20.4\text{GHz}$$

$$A_{vth} = A_i A_{bc} = 0.512(-264) = -135$$

Spice Simulation of Example C-E Amplifier



Estimation of ω_H using the Open-Circuit Time Constant Method

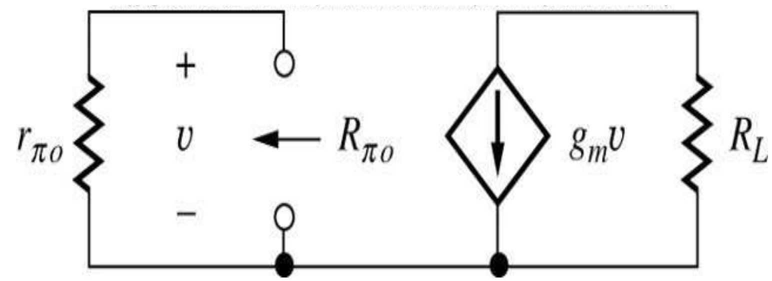
At high frequencies, impedances of coupling and bypass capacitors are small enough to be considered short circuits. Open-circuit time constants associated with impedances of device capacitances are considered instead.

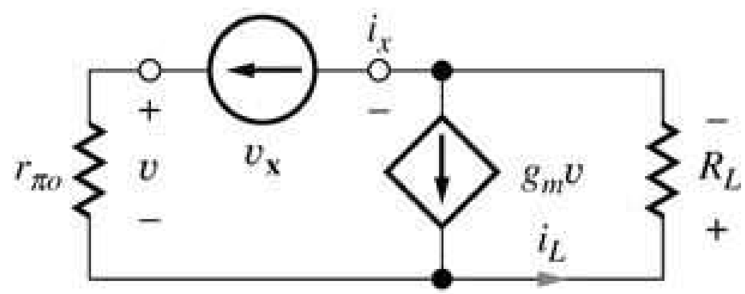
$$\omega_H \cong \frac{1}{\sum_{i=1}^{m} R_{io} C_i}$$

where R_{io} is resistance at terminals of ith capacitor C_i with all other capacitors open-circuited.

For a C-E amplifier, assuming $C_L = 0$

$$R_{\pi o} = r_{\pi o}$$



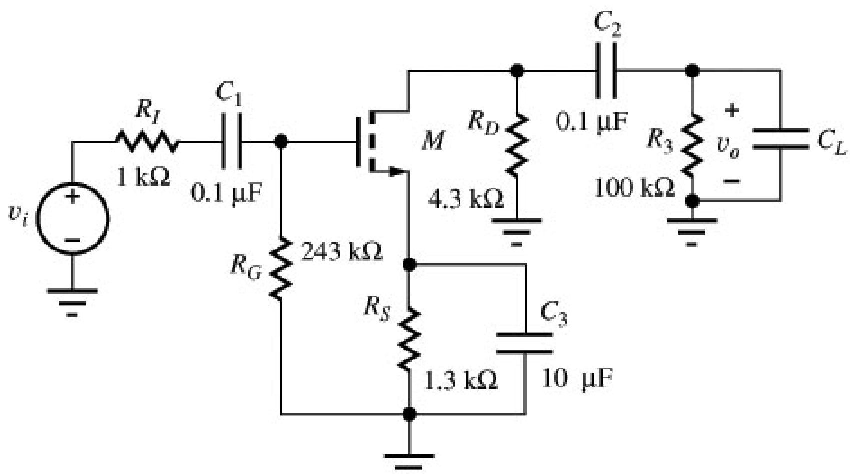


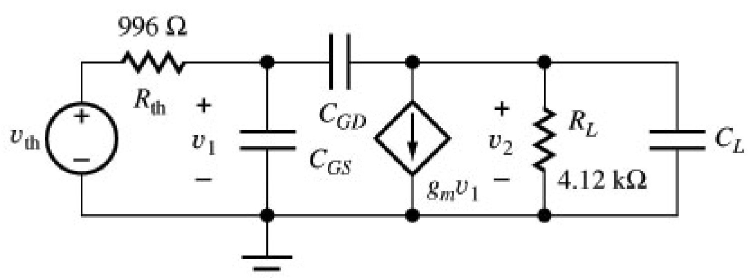
$$R_{\mu o} = \frac{v_x}{i_x} = r_{\pi o} + \left(1 + g_m R_L + \frac{R_L}{r_{\pi o}}\right)$$

$$\omega_H \cong \frac{1}{R_{\pi o} C_\pi + R_{\mu o} C_\mu} = \frac{1}{r_{\pi o} C_T}$$

High-Frequency Analysis: C-S Amplifier

Analysis similar to the C-E case yields the following equations:





$$R_{th} = R_I \| R_G$$

$$R_L = R_D \| R_3$$

$$v_{th} = v_i \frac{R_G}{R_I + R_G}$$

$$C_T = C_{GS} + C_{GD}\left(1 + g_m R_L\right) + \frac{R_L}{R_{th}}(C_{GD} + C_L)$$

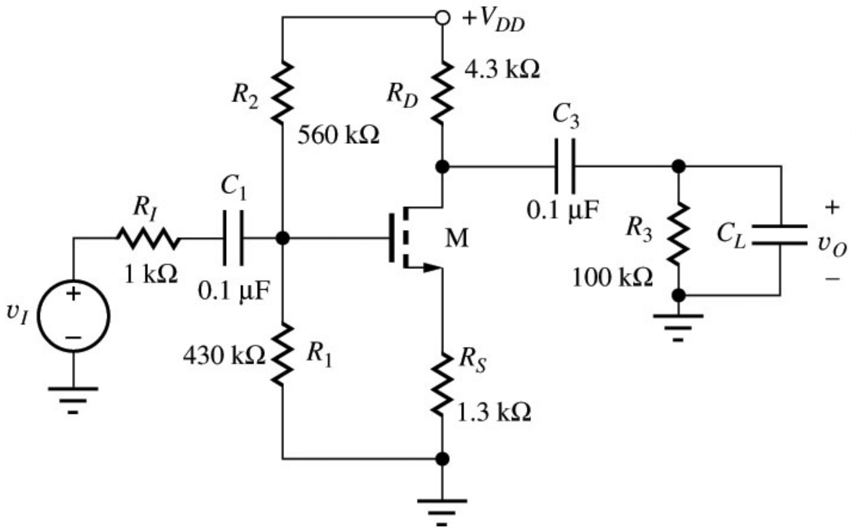
$$\omega_{P1} = \frac{1}{R_{th} C_T}$$

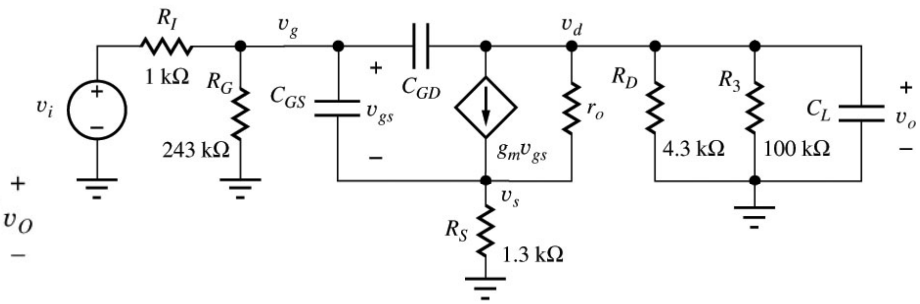
$$\omega_{P2} = \frac{g_m}{C_{GS} + C_L}$$

$$\omega_Z = \frac{+g_m}{C_{GD}}$$

C-S Amplifier High Frequency Response with Source Degeneration Resistance

First, find the simplifed small-signal model of the C-A amp.





Recall that we can define an effective g_m to account for the unbypassed source resistance.

$$g_m' = \frac{g_m}{1 + g_m R_S}$$

C-S Amplifier High Frequency Response with Source Degeneration Resistance (cont.)

Input gain is found as

$$A_i = \frac{v_g}{v_i} = \frac{R_G}{R_i + R_G} = \frac{R_1 \| R_2}{R_i + R_1 \| R_2}$$

Terminal gain is

$$A_{gd} = \frac{v_d}{v_g} = -g_m'(R_{iD} \| R_D \| R_3) \cong \frac{-g_m R_D \| R_3}{1 + g_m R_S}$$

Again, we use the Miller effect to find the equivalent capacitance at the gate as:

$$C_{eqG} = C_{GD}(1 - A_{gd}) + C_{GS}(1 - A_{gs})$$

$$= C_{GD}\left(1 - \frac{[-g_m R_L]}{1 + g_m R_S}\right) + C_{GS}\left(1 - \frac{g_m R_S}{1 + g_m R_S}\right)$$

$$= C_{GD}\left(1 + \frac{g_m R_D \| R_L}{1 + g_m R_S}\right) + \frac{C_{GS}}{1 + g_m R_3}$$

C-S Amplifier High Frequency Response with Source Degeneration Resistance (cont.)

The total equivalent resistance at the gate is

$$R_{eqG} = R_G \| R_I = R_{th}$$

The total capacitance and resistance at the collector is

$$C_{eqD} = C_{GD} + C_L$$

$$R_{eqD} = R_{iD} \| R_D \| R_3 \cong R_D \| R_3 = R_L$$

Because of interaction through C_{GD}, the two RC time constants interact, giving rise to the dominant pole:

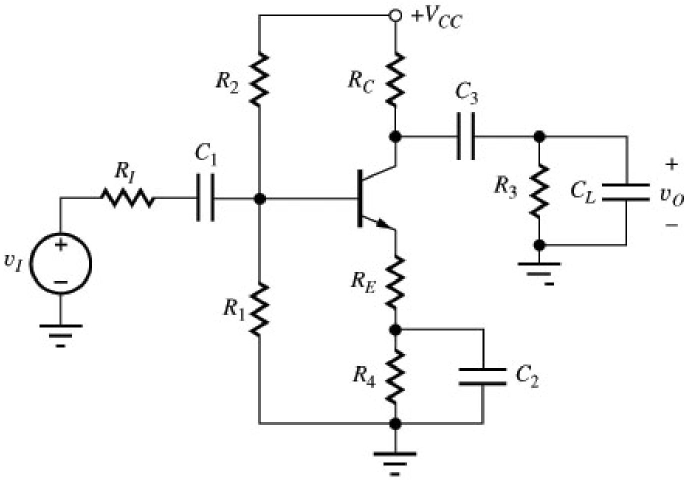
$$\omega_{p1} = \frac{1}{R_{th}[C_{GD}(1 + \frac{g_m R_L}{1 + g_m R_S}) + \frac{C_{GS}}{1 + g_m R_S} + \frac{R_L}{R_{th}}(C_{GD} + C_L)]}$$

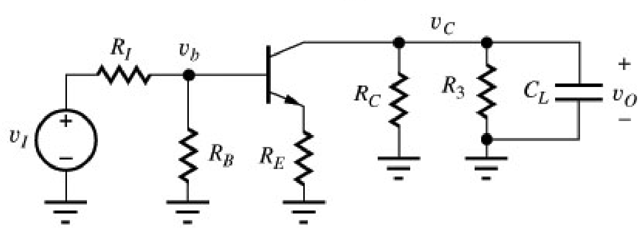
And from previous analysis:

$$\omega_{p2} = \frac{g_m'}{(C_{GS} + C_L)} = \frac{g_m}{(1 + g_m R_S)(C_{GS} + C_L)} \qquad \omega_z = \frac{+g_m'}{C_{GD}} = \frac{+g_m}{(1 + g_m R_S)(C_{GD})}$$

C-E Amplifier with Emitter Degeneration Resistance

Analysis similar to the C-S case yields the following equations:





$$r_{\pi 0} = R_{eqB} = (R_{th} + r_x) \| [r_\pi + (\beta + 1)R_E]$$

$$R_L = R_C \| R_3$$

$$\omega_{p1} = \frac{1}{r_{\pi 0} C_T}$$

$$= \frac{1}{r_{\pi 0}([C_\mu(1 + \frac{g_m R_L}{1 + g_m R_E}) + \frac{C_\pi}{1 + g_m R_E}] + \frac{R_L}{r_{\pi 0}}[C_\mu + C_L])}$$

$$\omega_{p2} \cong \frac{g_m}{2\pi(1 + g_m R_E)(C_\pi + C_L)}$$

$$\omega_z = \frac{+g_m}{2\pi[1 + g_m R_E][C_\mu]}$$

Gain-Bandwidth Trade-offs Using Source/Emitter Degeneration Resistor

Adding source resistance to the CS amp caused gain to decrease and dominant pole frequency to increase.

$$A_{gd} = \frac{v_d}{v_g} = \frac{-g_m R_D \| R_3}{1 + g_m R_S}$$

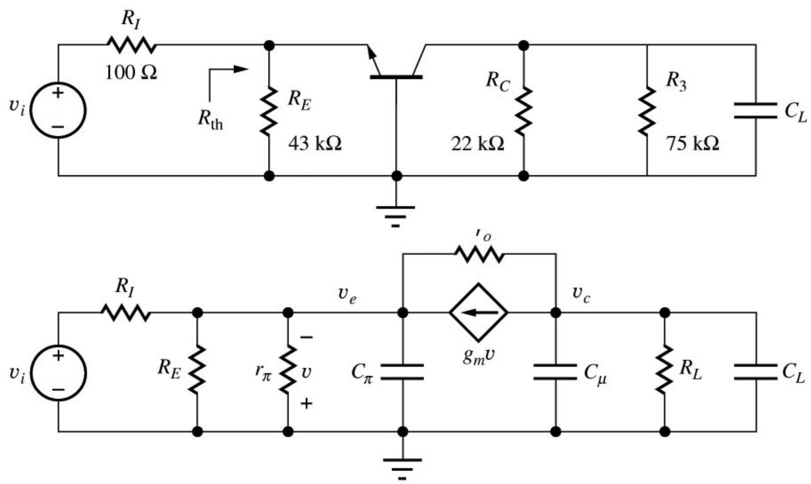
$$\omega_{p1} = \frac{1}{R_{th}[C_{GD}(1 + \frac{g_m R_L}{1 + g_m R_S}) + \frac{C_{GS}}{1 + g_m R_S} + \frac{R_L}{R_{th}}(C_{GD} + C_L)]}$$

However, decreasing the gain also decreased the frequency of the second pole.

$$\omega_{p2} = \frac{g_m}{(1 + g_m R_S)(C_{GS} + C_L)}$$

Increasing the gain of the C-E/C-S stage causes **pole-splitting**, or increase of the difference in frequency between the first and second poles.

High Frequency Poles for the C-B Amplifier



$$A_i \cong \frac{1}{1+g_m R_i}$$

$$A_{ec} = \frac{v_c}{v_e} = g_m R_{iC} \parallel R_L \cong g_m R_L$$

$$R_{iC} = r_o(1+g_m r_\pi \parallel R_I)$$

Since C_μ does not couple input and output, input and output poles can be found directly.

$$C_{eqE} = C_\pi$$

$$R_{eqE} = \frac{1}{g_m} \parallel R_E \parallel R_i$$

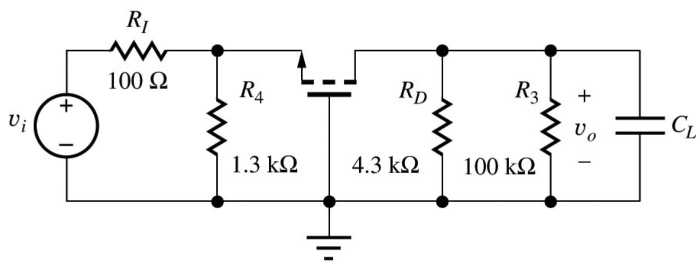
$$\omega_{p1} = \frac{1}{(\frac{1}{g_m} \parallel R_E \parallel R_i)C_\pi} \cong \frac{g_m}{C_\pi}$$

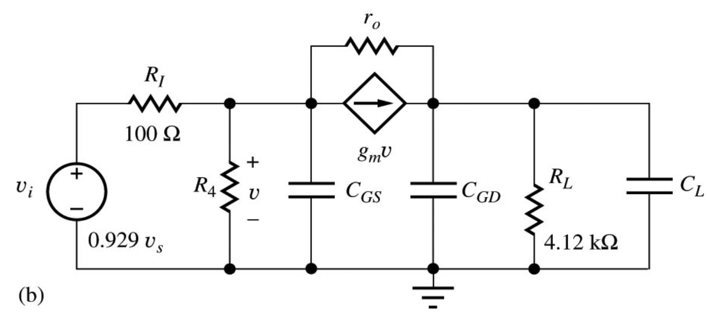
$$C_{eqC} = C_\mu + C_L$$

$$R_{eqC} = R_{iC} \parallel R_L \cong R_L$$

$$\omega_{p2} = \frac{1}{(R_{iC} \parallel R_L)(C_\mu + C_L)} \cong \frac{1}{R_L(C_\mu + C_L)}$$

High Frequency Poles for the C-G Amplifier





Similar to the C-B, since C_{GD} does not couple the input and output, input and output poles can be found directly.

$$C_{eqS} = C_{GS}$$

$$R_{eqS} = \frac{1}{g_m} \| R_4 \| R_I$$

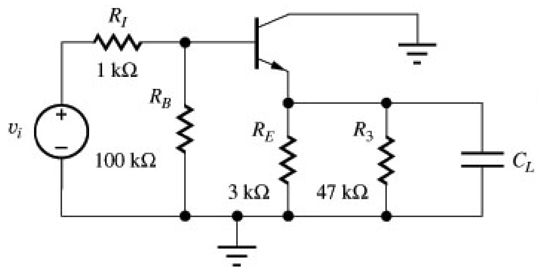
$$\omega_{p1} = \frac{1}{(\frac{1}{g_m} \| R_4 \| R_I)C_{GD}} \cong \frac{g_m}{C_{GD}}$$

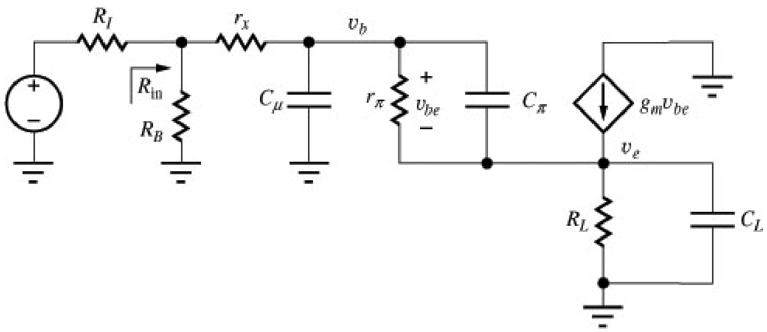
$$C_{eqD} = C_{GD} + C_L$$

$$R_{eqD} = R_{iD} \| R_L \cong R_L$$

$$\omega_{p2} = \frac{1}{(R_{iD} \| R_L)(C_{GD} + C_L)} \cong \frac{1}{R_L(C_{GD} + C_L)}$$

High Frequency Poles for the C-C Amplifier





$$A_i = \frac{v_b}{v_i} = \frac{R_{in}}{R_i + R_{in}}$$

$$A_{be} = \frac{v_e}{v_b} = \frac{g_m R_L}{1 + g_m R_L}$$

$$C_{eqB} = C_\mu(1 - A_{bc}) + C_\pi(1 - A_{be}) = C_\mu(1 - 0) + C_\pi(1 - \frac{g_m R_L}{1 + g_m R_L})$$

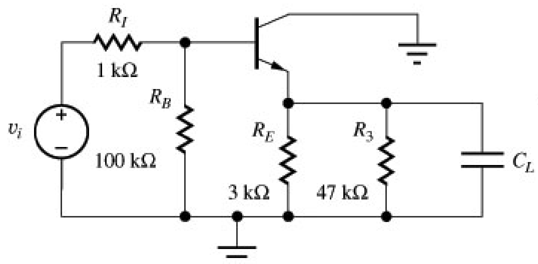
$$= C_\mu + \frac{C_\pi}{1 + g_m R_L}$$

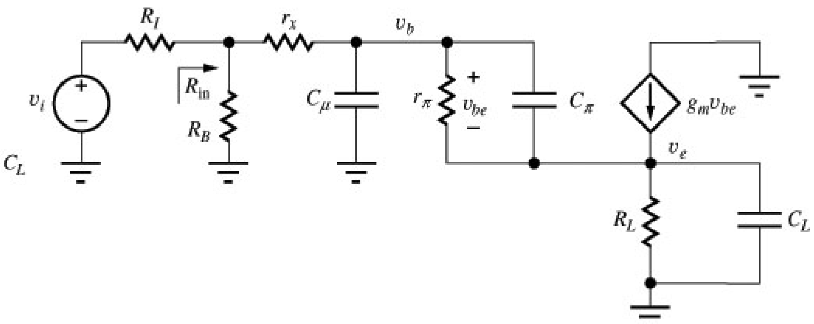
$$R_{eqB} = R_i \parallel R_{in} = [(R_i \parallel R_B) + r_x] \parallel [r_\pi + (\beta + 1)R_L] = (R_{th} + r_x) \parallel [r_\pi + (\beta + 1)R_L]$$

$$C_{eqE} = C_\pi + C_L$$

$$R_{eqE} = R_{iE} \parallel R_L \cong [1/g_m + \frac{(R_{th} + r_x)}{\beta + 1}] \parallel R_L$$

High Frequency Poles for the C-C Amplifier (cont.)





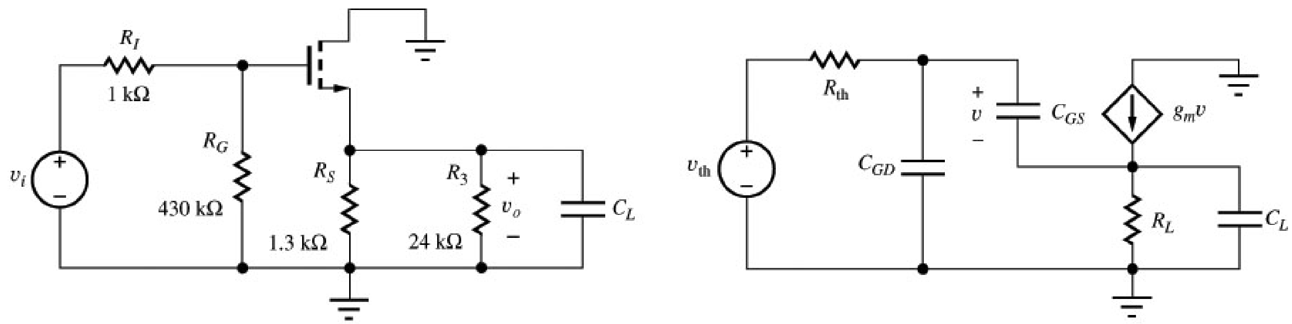
The low impedance at the output makes the input and output time constants relatively well decoupled, leading to two poles.

$$\omega_{p1} = \frac{1}{([R_{th} + r_x] \| [r_\pi + (\beta+1)R_L])(C_\mu + \dfrac{C_\pi}{1+g_m R_L})} \qquad \omega_{p2} = \frac{1}{[R_{iE} \| R_L][C_\pi + C_L]} \cong \frac{1}{[(1/g_m + \dfrac{R_{th}+r_x}{\beta+1}) \| R_L][C_\pi + C_L]}$$

The feed-forward high-frequency path through Cp leads to a zero in the C-C response. Both the zero and the second pole are quite high frequency and are often neglected, although their effect can be significant with large load capacitances.

$$\omega_z \cong \frac{g_m}{C_\pi}$$

High Frequency Poles for the C-D Amplifier



Similar the the C-C amplifier, the high frequency response is dominated by the first pole due to the low impedance at the output of the C-C amplifier.

$$\omega_{p1} = \frac{1}{R_{th}\left(C_{GD} + \frac{C_{GS}}{1 + g_m R_L}\right)}$$

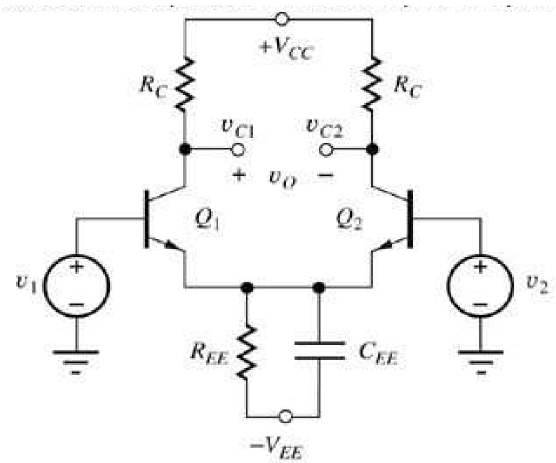
$$\omega_{p2} = \frac{1}{[R_{iS} \| R_L][C_{GS} + C_L]} \cong \frac{1}{[1/g_m \| R_L][C_{GS} + C_L]}$$

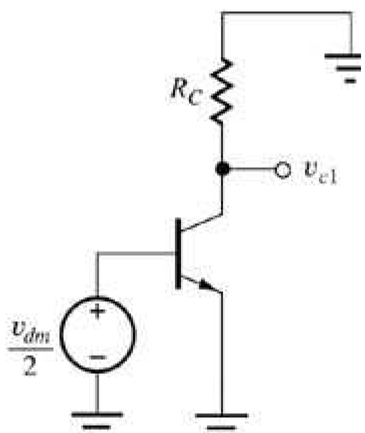
$$\omega_z \cong \frac{g_m}{C_{GS}}$$

Summary of the Upper-Cutoff Frequencies of the Single-Stage Amplifiers (pg.1037)

	ω_H	
Common-emitter	$\dfrac{1}{r_{\pi 0}C_T} = \dfrac{1}{r_{\pi 0}\left[C_\pi + C_\mu(1+g_m R_L) + (C_u + C_L)\dfrac{R_L}{r_{\pi 0}}\right]}$	$r_{\pi 0} = r_\pi \parallel [r_x + (R_I \parallel R_B)]$
Common-source	$\dfrac{1}{R_{\text{th}}C_T} = \dfrac{1}{R_{\text{th}}\left[C_{GS} + C_{GD}\,(1+g_m R_L) + (C_{GD} + C_L)\,\dfrac{R_L}{R_{\text{th}}}\right]}$	$R_{\text{th}} = R_I \parallel R_G$
Common-emitter with emitter resistor R_E	$\dfrac{1}{r_{\pi 0}\left[\dfrac{C_\pi}{1+g_m R_E} + C_\mu\left(1+\dfrac{g_m R_L}{1+g_m R_E}\right) + (C_u + C_L)\dfrac{R_L}{r_{\pi 0}}\right]}$	$r_{\pi 0} = r_\pi \parallel [r_x + (R_1 \parallel R_B)]$
Common-source with source resistor R_S	$\dfrac{1}{R_{\text{th}}\left[\dfrac{C_{GS}}{1+g_m R_S} + C_{GD}\left(1+\dfrac{g_m R_L}{1+g_m R_S}\right) + (C_{GD} + C_L)\dfrac{R_L}{R_{\text{th}}}\right]}$	$R_{\text{th}} = R_I \parallel R_G$
Common-base	$\dfrac{1}{R_L(C_\mu + C_L)}$	
Common-gate	$\dfrac{1}{R_L(C_{GD} + C_L)}$	
Common-collector	$\dfrac{1}{[(R_I \parallel R_B) + r_x]\left(\dfrac{C_\pi}{1+g_m R_L} + C_\mu\right)}$	
Common-drain	$\dfrac{1}{(R_I \parallel R_G)\left(\dfrac{C_{GS}}{1+g_m R_L} + C_{GD}\right)}$	

Frequency Response: Differential Amplifier





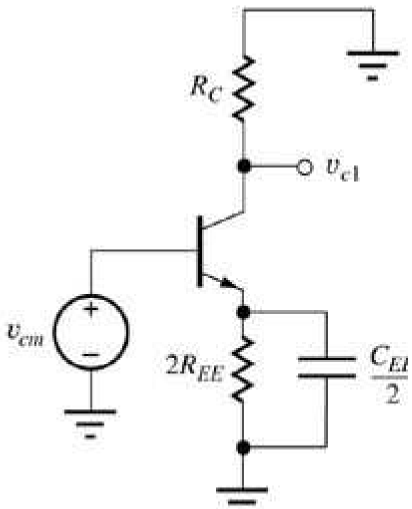
C_{EE} is total capacitance at emitter node of the differential pair.

Differential mode half-circuit is similar to a C-E stage. Bandwidth is determined by the $r_{\pi o}C_T$ product. As emitter is a virtual ground, C_{EE} has no effect on differential-mode signals.

For common-mode signals, at very low frequencies, $$|A_{cc}(0)| \cong \frac{R_C}{2R_{EE}} << 1$$

Transmission zero due to C_{EE} is
$$s = -\omega_Z = -\frac{1}{C_{EE}R_{EE}}$$

Frequency Response: Differential Amplifier (contd.)



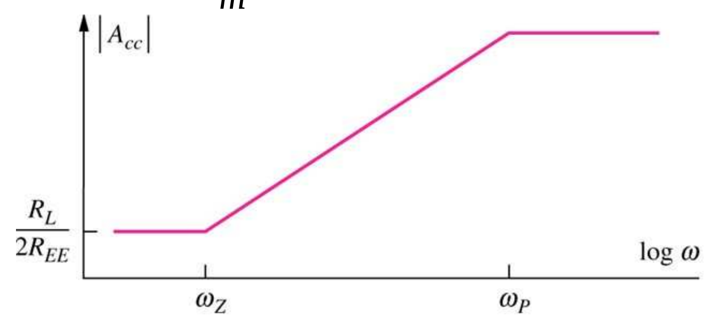
$$R_{EEO} = 2R_{EE} \left\| \frac{r_\pi + r_x}{\beta_o + 1} \cong \frac{1}{g_m} \right.$$

$$\omega_P \cong \frac{1}{r_x\left(\frac{C_\pi}{1+2g_mR_{EE}}\left(1+\frac{2R_{EE}}{r_x}\right)+C_\mu\left(1+\frac{g_mR_C}{1+2g_mR_{EE}}+\frac{R_C}{r_x}\right)\right)+\frac{C_{EE}}{2g_m}}$$

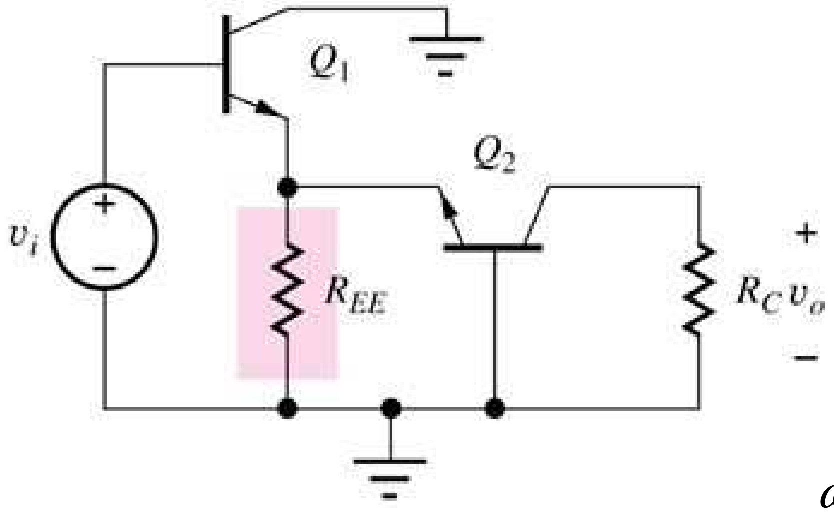
As R_{EE} is usually designed to be large,

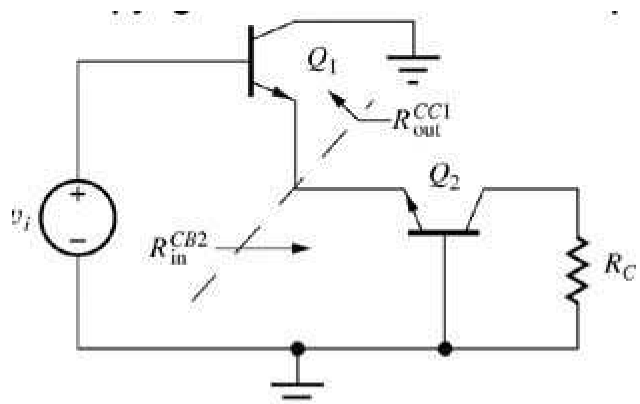
$$\omega_P \cong \frac{1}{\frac{C_\pi + C_{EE}}{2g_m} + C_\mu(R_C + r_x)} \cong \frac{1}{C_\mu(R_C + r_x)}$$

Common-mode half-circuit is similar to a C-E stage with emitter resistor $2R_{EE}$. OCTC for C_π and C_μ is similar to the C-E stage. OCTC for $C_{EE}/2$ is:



Frequency Response: Common-Collector/ Common-Base Cascade





R_{EE} is assumed to be large and neglected.

$$R_{out}^{CC1} = \frac{r_{\pi 1} + r_{x1}}{\beta_{o1} + 1} \cong \frac{1}{g_{m1}} \qquad R_{in}^{CB2} = \frac{r_{\pi 2} + r_{x2}}{\beta_{o2} + 1} \cong \frac{1}{g_{m2}}$$

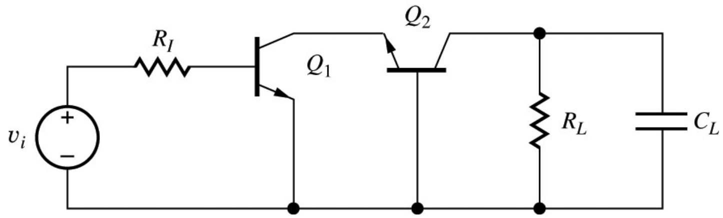
The intermediate node pole is neglected since the impedance is quite low. We are left with the input pole for a C-D and the output pole of a C-B stage.

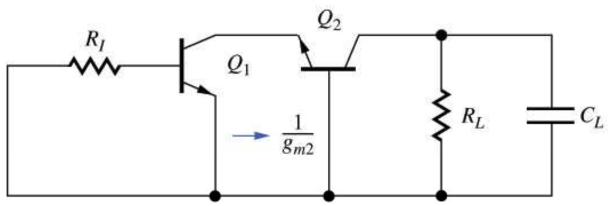
$$\omega_{pB1} = \frac{1}{([R_{th} + r_{x1}] \,||\, [r_{\pi 1} + (\beta + 1)R_{L1}])(C_{\mu 1} + \frac{C_{\pi 1}}{1 + g_m R_L})}$$

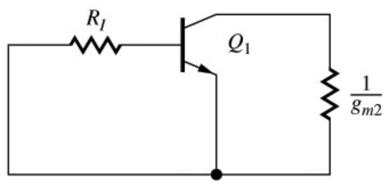
$$= \frac{1}{([R_{th} + r_{x1}] \,||\, [2r_{\pi 1}])(C_{\mu 1} + \frac{C_{\pi 2}}{2})}$$

$$\omega_{pC2} \cong \frac{1}{R_C (C_\mu + C_L)}$$

Frequency Response: Cascode Amplifier







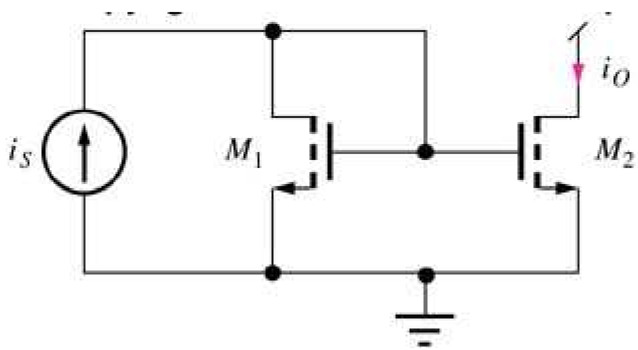
There are two important poles, the input pole for the C-E and the output pole for the C-B stage. The intermediate node pole can usually be neglected because of the low impedance at the input of the C-B stage. R_{L1} is small, so the second term in the first pole can be neglected. Also note the R_{L1} is equal to $1/g_{m2}$.

$$\omega_{pB1} = \frac{1}{r_{\pi 0} C_T} = \frac{1}{r_{\pi 01}\left([C_{\mu 1}(1+\frac{g_{m1}R_{L1}}{1+g_{m1}R_{E1}}) + C_{\pi 1}] + \frac{R_{L1}}{r_{\pi 0}}[C_{\mu 1} + C_{L1}]\right)}$$

$$= \frac{1}{r_{\pi 01}\left([C_{\mu 1}(1+\frac{g_{m1}}{g_{m2}}) + C_{\pi 1}] + \frac{1/g_{m2}}{r_{\pi 01}}[C_{\mu 1} + C_{\pi 2}]\right)} \cong \frac{1}{r_{\pi 01}(2C_{\mu 1} + C_{\pi 1})}$$

$$\omega_{pC2} \cong \frac{1}{R_L(C_{\mu 2} + C_L)}$$

Frequency Response: MOS Current Mirror

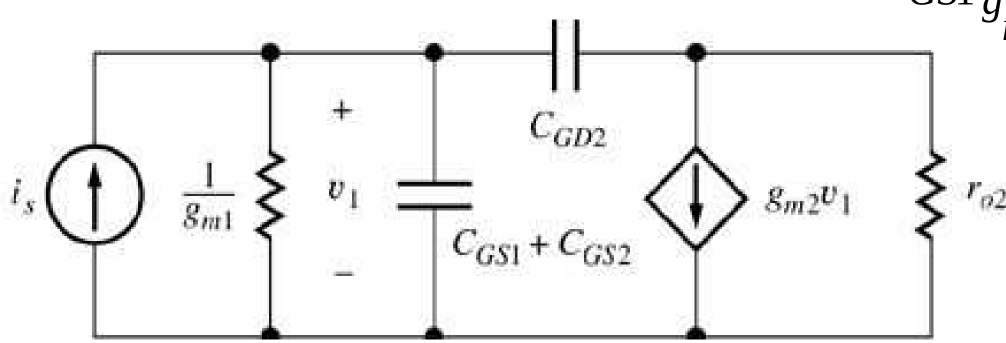


This is very similar to the C-S stage simplified model, so we will apply the C-S equations with relevant changes.

$$\omega_{P1} = \frac{1}{(1/g_{m1})C_T}$$

$$= \frac{}{1/g_{m1}\left(C_{GS1} + C_{GS2} + C_{GD2}\left(1+g_{m2}r_{o2}\right) + \frac{r_{o2}}{1/g_{m1}}C_{GD2}\right)}$$

$$= \frac{}{2C_{GS1}\frac{1}{g_{m1}} + 2C_{GD2}r_{o2}}$$

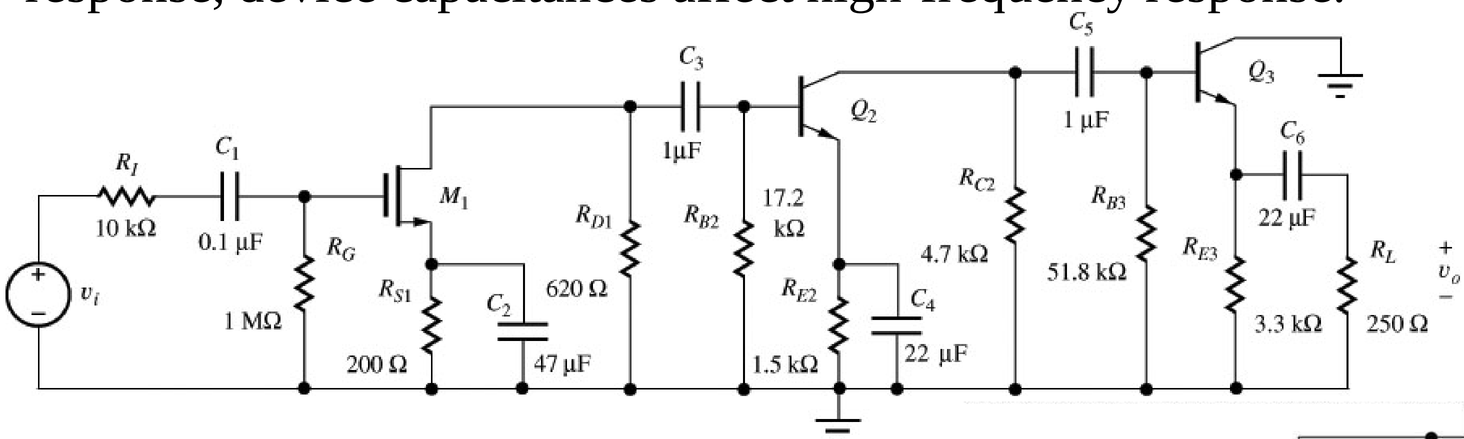


Assumes matched transistors.

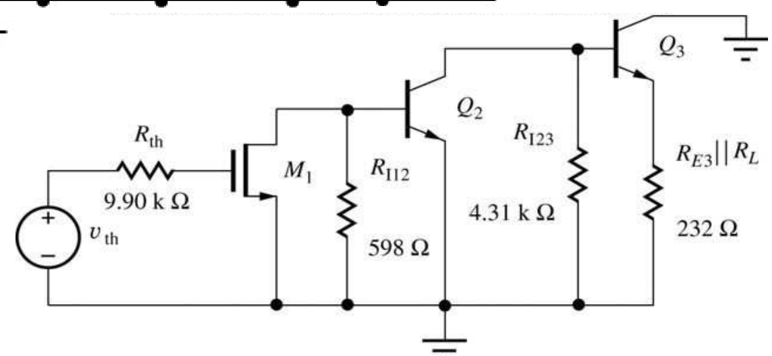
Frequency Response: Multistage Amplifier

•**Problem:** Use open-circuit and short-circuit time constant methods to estimate upper and lower cutoff frequencies and bandwidth.

•**Approach:** Coupling and bypass capacitors determine low-frequency response, device capacitances affect high-frequency response.



At high frequencies, ac model for multi-stage amplifier is as shown.



Frequency Response: Multistage Amplifier Parameters

Parameters and operation point information for the example multistage amplifier.

Transistor Parameters

	g_m	r_π	r_o	β_o	C_{GS}/C_π	C_{GD}/C_μ	r_x
M_1	10 mS	∞	12.2 kΩ	∞	5 pF	1 pF	0 Ω
Q_2	67.8 mS	2.39 kΩ	54.2 kΩ	150	39 pF	1 pF	250 Ω
Q_3	79.6 mS	1.00 kΩ	34.4 kΩ	80	50 pF	1 pF	250 Ω

Frequency Response: Multistage Amplifier (SCTC Estimate of ω_L)

SCTC for each of the six independent coupling and bypass capacitors are calculated as follows:

$$R_{1S} = R_I + (R_G \| R_{in1}) = 10\text{k}\Omega + 1\text{M}\Omega \| \infty = 1.01\text{M}\Omega$$

$$R_{2S} = R_{S1} \left\| \frac{1}{g_{m1}} \right. = 200\Omega \left\| \frac{1}{0.01\text{S}} \right. = 66.7\Omega$$

$$R_{3S} = \left(R_{D1} \| R_{O1}\right) + \left(R_{B2} \| R_{in2}\right) = \left(R_{D1} \| r_{o1}\right) + \left(R_{B2} \| r_{\pi 2}\right) = 2.69\text{k}\Omega$$

$$R_{th2} = R_{B2} \| R_{D1} \| r_{o1} = 571\Omega$$

$$R_{4S} = R_{E2} \left\| \frac{R_{th2} + r_{\pi 2}}{\beta_{o2} + 1} \right. = 19.4\Omega$$

$$R_{5S} = \left(R_{C2} \| R_{O2}\right) + \left(R_{B3} \| R_{in3}\right) = \left(R_{C2} \| r_{o2}\right) + \left(R_{B3} \| \left(r_{\pi 3} + (\beta_{o3} + 1)(R_{E3} \| R_L)\right)\right) = 18.4\text{k}\Omega$$

$$R_{th3} = R_{B3} \| R_{C2} \| r_{o2} = 3.99\text{k}\Omega$$

$$R_{6S} = R_L + R_{E3} \left\| \frac{R_{th3} + r_{\pi 3}}{\beta_{o3} + 1} \right. = 311\Omega$$

$$\omega_L \cong \sum_{i=1}^{n} \frac{1}{R_{iS} C_i} = 3330\text{rad/s}$$

$$f_L = \frac{\omega_L}{2\pi} = 530\text{Hz}$$

Frequency Response: Multistage Amplifier (High-Frequency Poles)

High-frequency pole at the gate of M1: Using our equation for the C-S input pole:

$$f_{p1} = \frac{1}{2\pi R_{th1}\left[C_{GD1}(1+g_{m1}R_{L1}) + C_{GS1} + \dfrac{R_{L1}}{R_{th1}}(C_{GD1}+C_{L1})\right]}$$

$$R_{L1} = R_{I12} \parallel r_{\pi 2} \parallel r_{o1} = 598\Omega \parallel (2.39\text{k}\Omega + 250\Omega) \parallel 12.2\text{k}\Omega = 469\ \ \Omega$$

$$C_{L1} = C_{\pi 2} + C_{\mu 2}(1+g_{m2}R_{L2})$$

$$R_{L2} = R_{I23} \parallel R_{in3} \parallel r_{o2} = R_{I23} \parallel [r_{x3} + r_{\pi 3} + (\beta_{03}+1)(R_{E3} \parallel R_L)] \parallel r_{o2} = 3.33k\Omega$$

$$C_{L1} = 39\text{pF} + 1\text{pF}[1 + 67.8\text{mS}(3.33\text{k}\Omega)] = 266\text{pF}$$

$$f_{p1} = \frac{1}{2\pi\, 9.9\text{k}\Omega\left[1\text{pF}(1+0.01S(469\Omega))\right] + 5\text{pF} + \dfrac{469\Omega}{9.9\text{k}\Omega}(1\text{pF}+266\text{pF})]} = 689\ \ \text{KHz}$$

Frequency Response: Multistage Amplifier (High-Frequency Poles cont.)

High-frequency pole at the base of Q2: From the detailed analysis of the C-S amp, we find the following expression for the pole at the output of the M1 C-S stage:

$$f_{p2} = \frac{C_{GS1}g_{L1} + C_{GD1}(g_{m1} + g_{th1} + g_{L1}) + C_{L1}g_{th1}}{2\pi[C_{GS1}(C_{GD1} + C_{L1}) + C_{GD1}C_{L1}]}$$

For this particular case, C_{L1} (Q2 input capacitance) is much larger than the other capacitances, so f_{p2} simplifies to:

$$f_{p2} \cong \frac{C_{L1}g_{th1}}{2\pi[C_{GS1}C_{L1} + C_{GD1}C_{L1}]} \cong \frac{1}{2\pi R_{th1}(C_{GS1} + C_{GD1})}$$

$$f_{p2} = \frac{1}{2\pi\, 9.9\text{k}\Omega(5\text{pF} + 1\text{pF})} = 2.68 \ \text{MHz}$$

Frequency Response: Multistage Amplifier (High-Frequency Poles cont.)

High-frequency pole at the base of Q3: Again, due to the pole-splitting behavior of the C-E second stage, we expect that the pole at the base of Q3 will be set by equation 16.95:

$$f_{p3} \cong \frac{g_{m2}}{2\pi\left[C_{\pi 2}\left(1+\frac{C_{L2}}{C_{\mu 2}}\right)+C_{L2}\right]}$$

The load capacitance of Q2 is the input capacitance of the C-C stage.

$$C_{L2} = C_{\mu 3} + \frac{C_{\pi 3}}{1+g_{m3}R_{E3} \parallel R_L} = 1\text{pF} + \frac{50\text{pF}}{1+79.6\text{mS}(3.3\text{k}\Omega \parallel 250\Omega)} = 3.55 \ \text{pF}$$

$$f_{p3} \cong \frac{67.8\text{mS}[1\text{k}\Omega/(1\text{k}\Omega+250\Omega)]}{2\pi\left[39\text{pF}\left(1+\frac{3.55\text{pF}}{1\text{pF}}\right)+3.55\text{pF}\right]} = 47.7 \ \text{MHz}$$

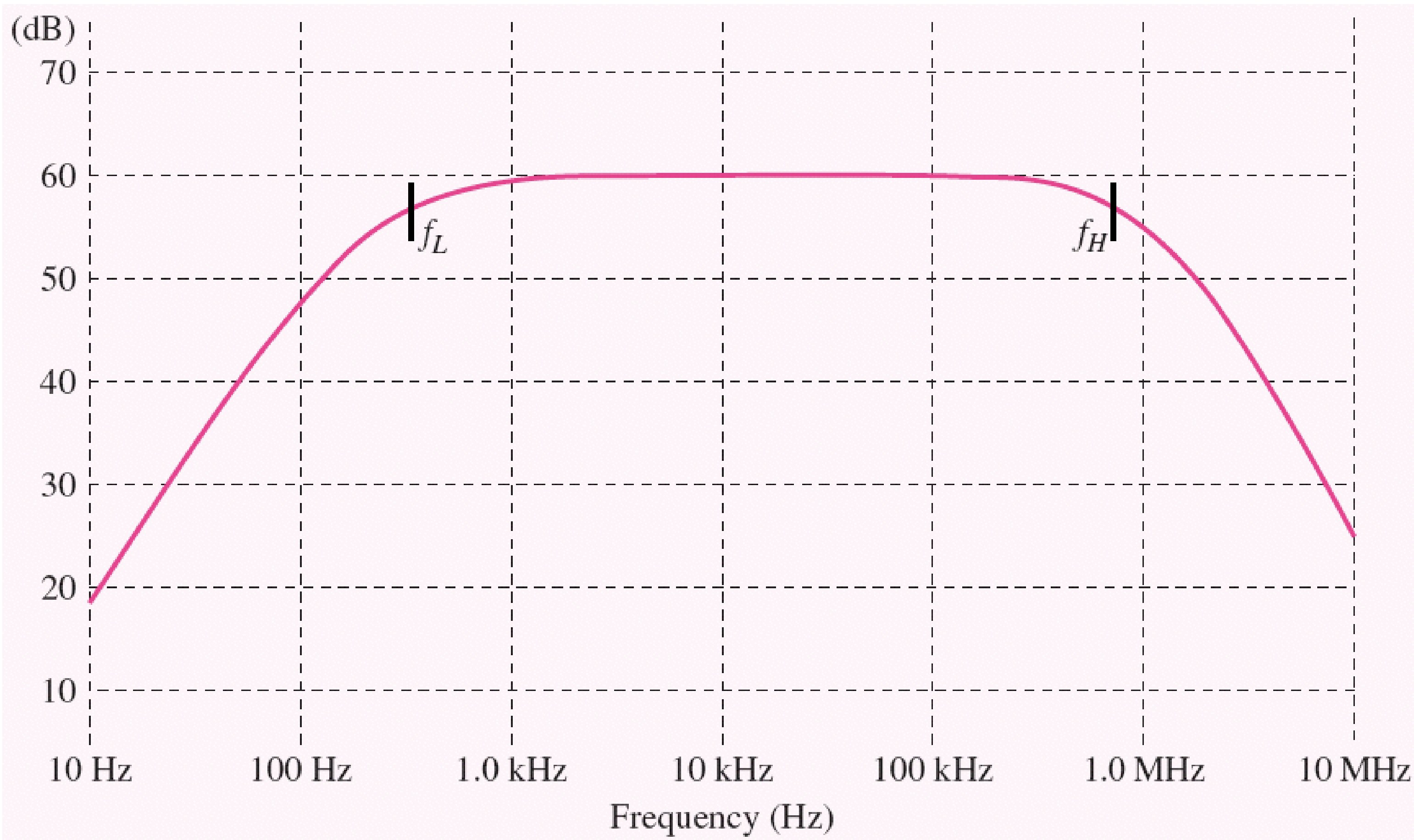
Frequency Response: Multistage Amplifier (f_H estimate)

There is an additional pole at the output of Q3, but it is expected to be at a very high frequency due to the low output impedance of the C-C stage. We can estimate f_H from eq. 16.23 using the calculated pole frequencies.

$$f_H = \frac{1}{\sqrt{\frac{1}{f_{p1}^2} + \frac{1}{f_{p2}^2} + \frac{1}{f_{p3}^2}}} = 667 \ \text{kHz}$$

The SPICE simulation of the circuit on the next slide shows an f_H of 667 KHz and an f_L of 530 Hz. The phase and gain characteristics of our calculated high frequency response is quite close to that of the SPICE simulation. It was quite important to take into account the pole-splitting behavior of the C-S and C-E stages. Not doing so would have resulted in a calculated f_H of less than 550 KHz.

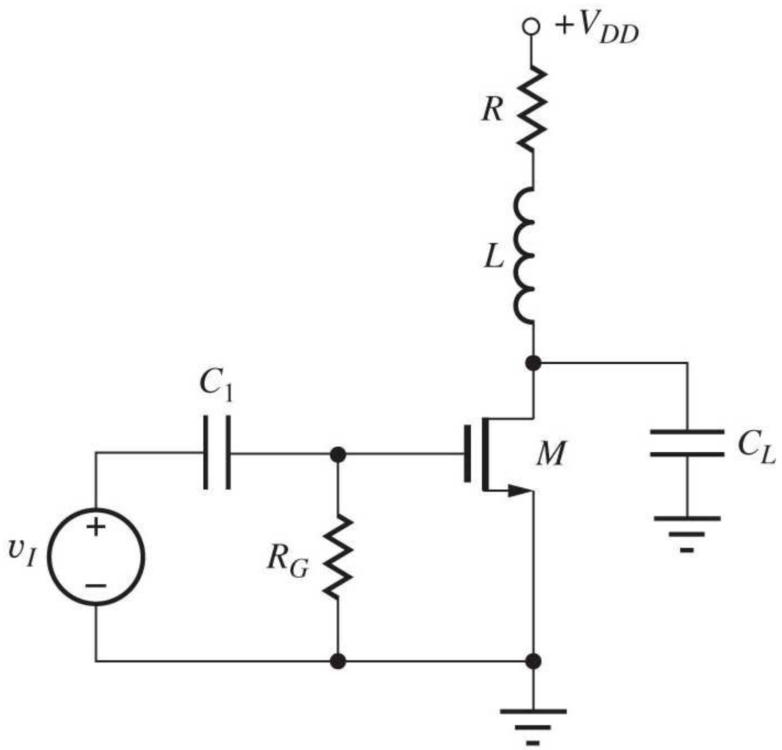
Frequency Response: Multistage Amplifier (SPICE Simulation)



Intro to RF Amplifiers

- Amplifiers with narrow bandwidth are often required in radio frequency (RF) applications to be able to select one signal from a large number of signals.
- Frequencies of interest > unity gain frequency of op amps, so active RC filters can't be used.
- These amplifiers have high Q (f_H and f_L close together relative to center frequency)
- These applications use resonant RLC circuits to form frequency selective tuned amplifiers.

The Shunt-Peaked Amplifier



- As the frequency goes up, the gain is enhanced by the increasing impedance of the inductor.

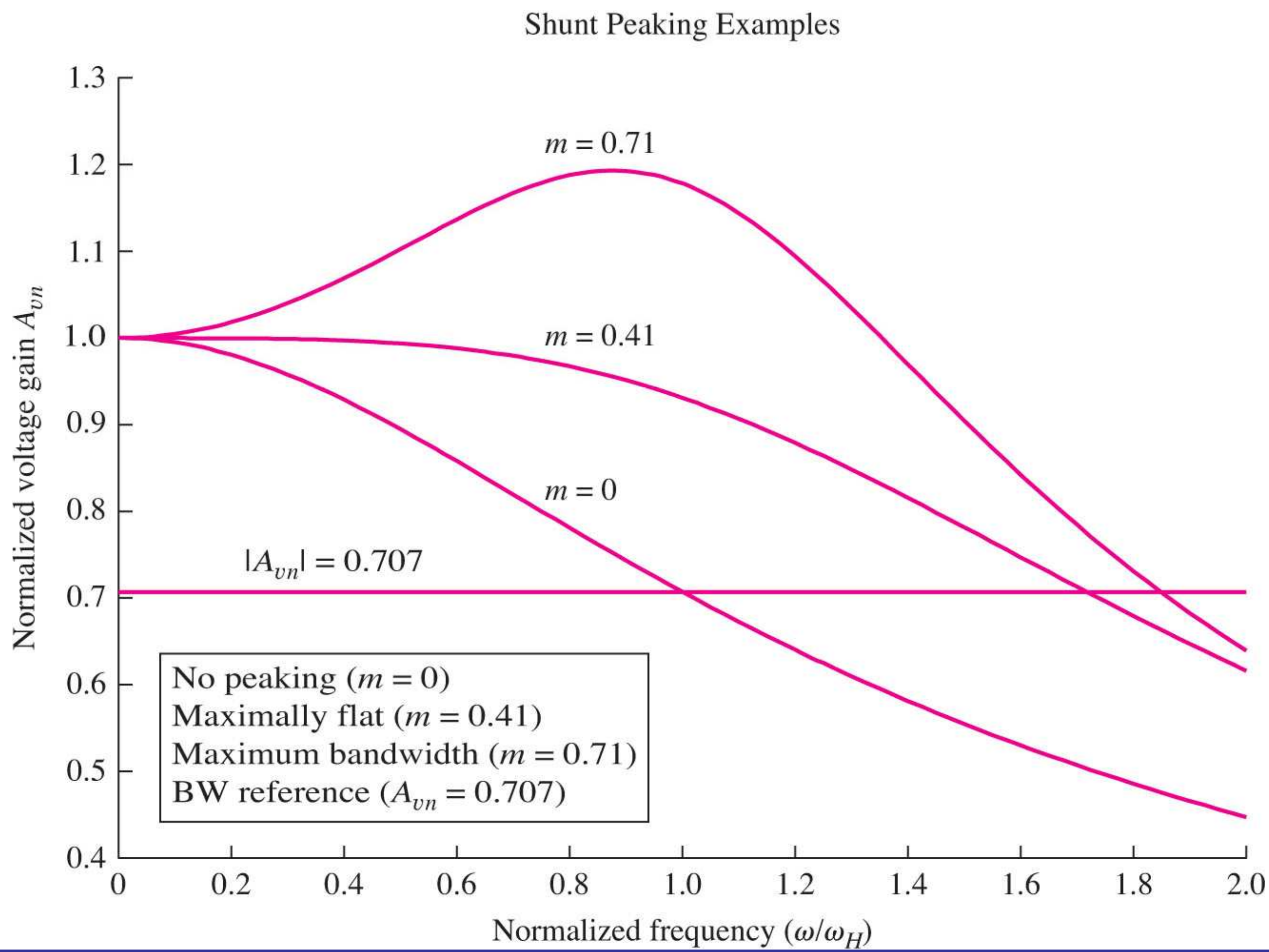
$$A_v(s) = \frac{(-g_m R)(1 + sL/R)}{1 + sRC + s^2 LC} \quad \text{where} \quad C = C_L + C_{GD}$$

- The gain improvement can be plotted as a function of parameter, m, defined below:

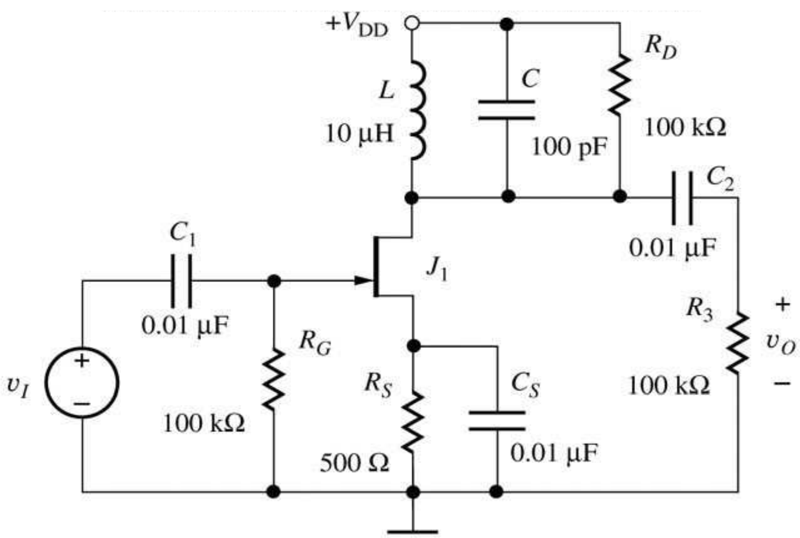
$$A_{vn}(s) = \frac{1 + ms}{1 + s + ms^2}$$

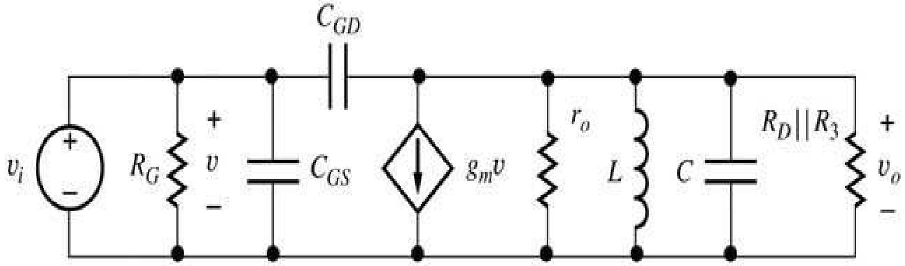
where $L = mR^2C$

The Shunt-Peaked Amplifier



Single-Tuned Amplifiers





- RLC network selects the frequency, parallel combination of R_D, R_3 and r_o set the Q and bandwidth.

$$A_v(s) = \frac{V_o(s)}{V_i(s)} = \frac{sC_{GD} - g_m}{G_P + s(C + C_{GD}) + (1/sL)}$$

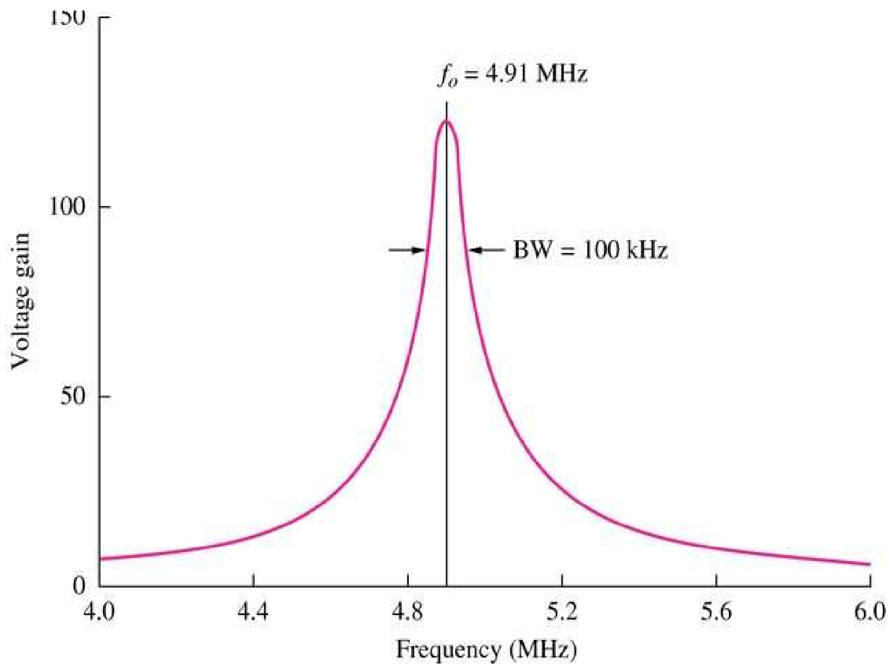
$$G_P = g_o + G_D + G_3$$

- Neglecting right-half plane zero,

$$A_v(s) = A_{mid} \frac{s\frac{\omega_o}{Q}}{s^2 + s\frac{\omega_o}{Q} + \omega_o^2}$$

$$\omega_o = \frac{1}{\sqrt{L(C + C_{GD})}} \qquad Q = \omega_o R_P (C + C_{GD}) = \frac{R_P}{\omega_o L}$$

Single-Tuned Amplifiers (contd.)

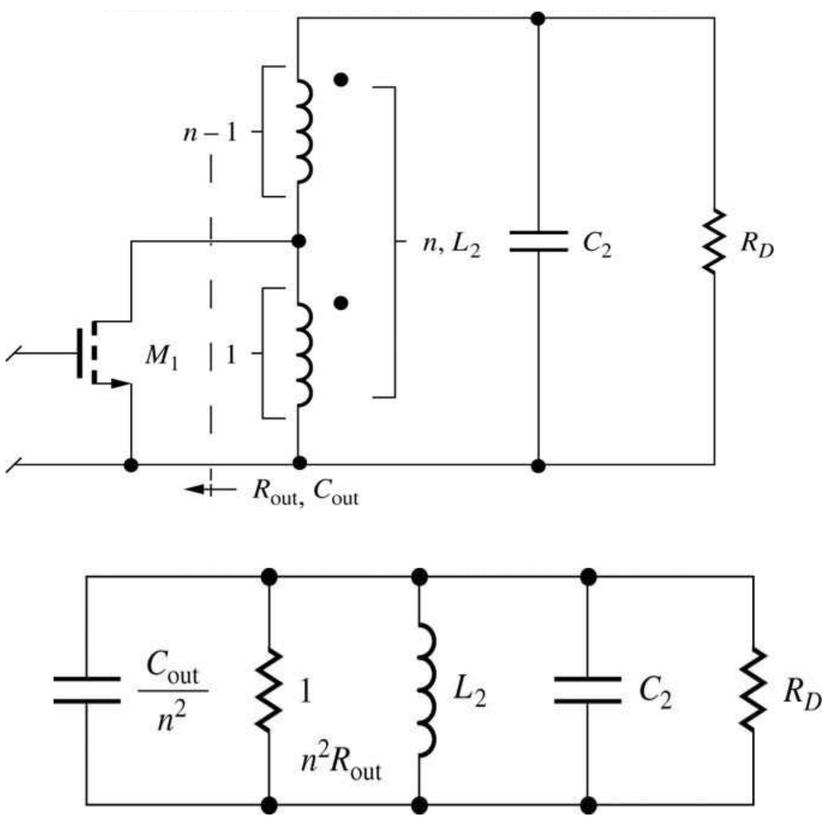


- At center frequency, $s = j\omega_o$, $A_v = A_{mid}$.

$$A_{mid} = -g_m R_P = -g_m (r_o \| R_D \| R_3)$$

$$\text{BW} = \frac{\omega_o}{Q} = \frac{1}{R_P(C + C_{GD})} = \frac{\omega_o^2 L}{R_P}$$

Use of tapped Inductor- Auto Transformer



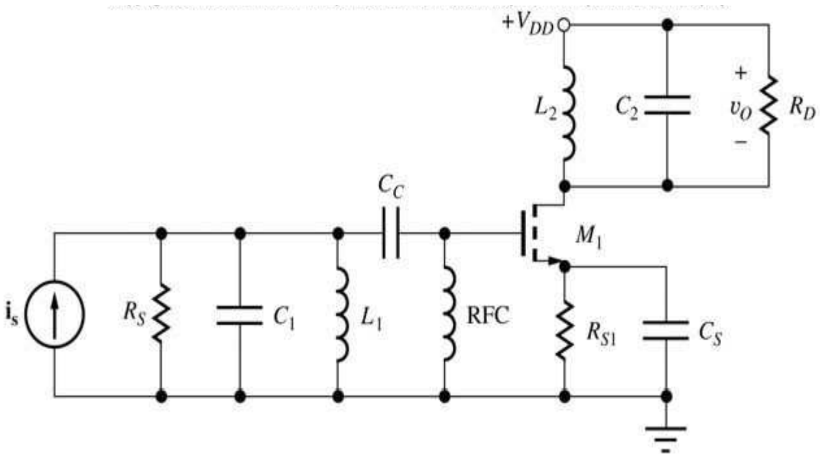
C_{GD} and r_o can often be small enough to degrade characteristics of the tuned amplifier. Inductor can be made to work as an auto transformer to solve this problem.

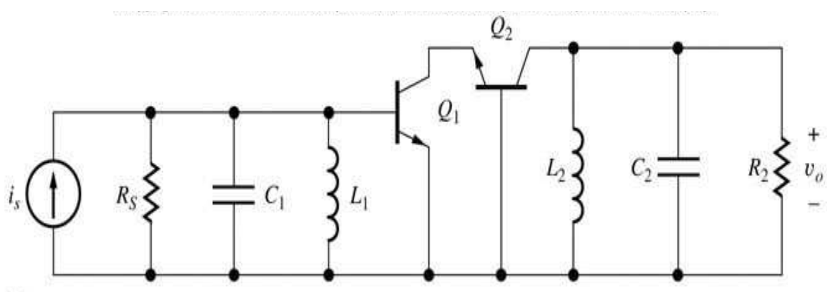
$$\frac{V_o(s)}{I_2(s)} = \frac{nV_1(s)}{I_S(s)/n} = n^2 \frac{V_1(s)}{I_S(s)} \qquad Z_S(s) = n^2 Z_p(s)$$

These results can be used to transform the resonant circuit and higher Q can be obtained and center frequency doesn't shift significantly due to changes in C_{GD}.

Similar solution can be used if tuned circuit is placed at amplifier input instead of output

Multiple Tuned Circuits





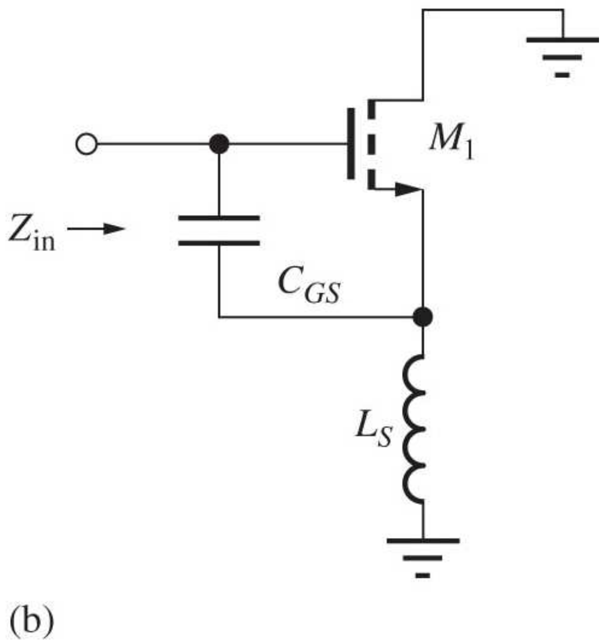
- Tuned circuits can be placed at both input and output to tailor frequency response.
- Radio-frequency choke(an open circuit at operating frequency) is used for biasing.
- Synchronous tuning uses two circuits tuned to same center frequency for high Q.

$$BW_n = BW_1\sqrt{2^{1/n} - 1}$$

- Stagger tuning uses two circuits tuned to slightly different center frequencies to realize broader band amplifiers.

Cascode stage is used to provide isolation between the two tuned circuits and eliminate feedback path between them due to Miller multiplication.

CS Amp with Inductive Degeneration



- Typically need to match input resistance to antenna impedance at center frequency, usually 50 ohms.
- Using our follower analyses, the input impedance is found as:

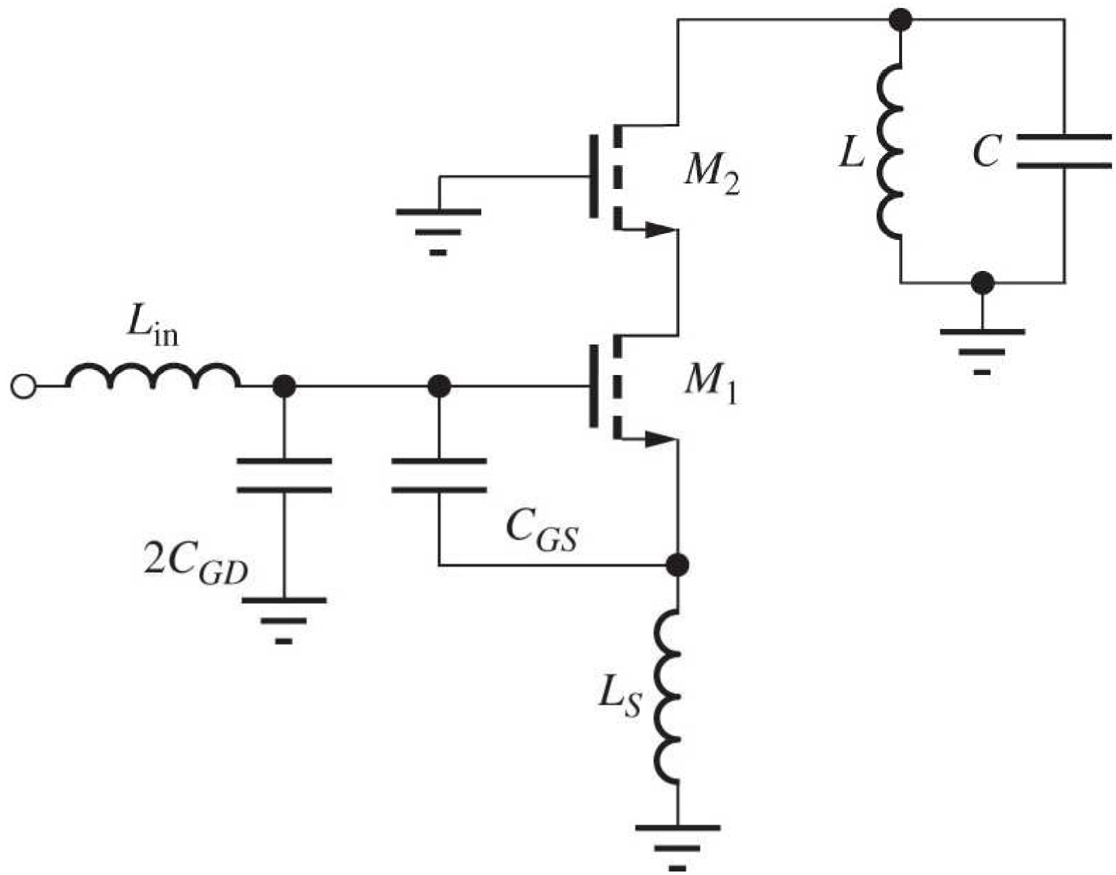
$$Z_{in}(s) = Z_{gs} + Z_s + (g_m Z_{gs}) Z_s$$

$$Z_{in}(s) = 1/sC_{GS} + sL_s + R_{eq}$$

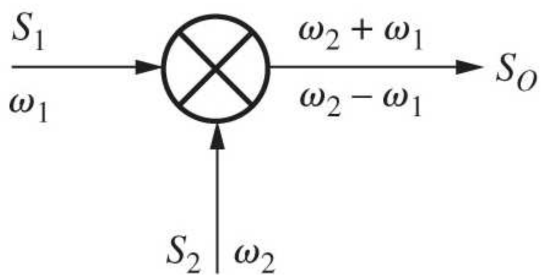
where $R_{eq} = +g_m L_S / C_{GS}$

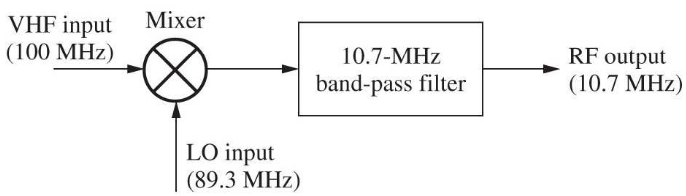
- The following slide shows a complete low noise CS amp where a series inductor resonates with the input capacitance to leave only the r esistance at the center frequency.

Complete Cascode LNA

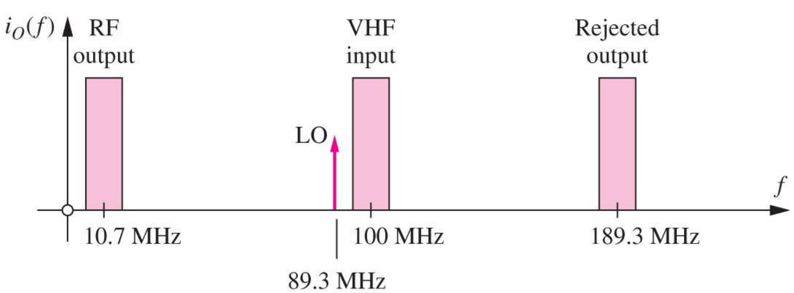


Mixer Introduction





(a)



(b)

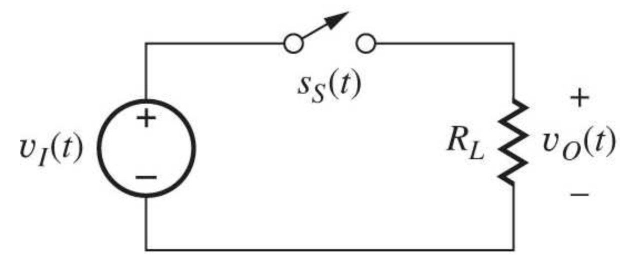
- A mixer is a circuit that multiplies two signals to produce sum and difference frequencies:

$$S0 = S_2 \bullet S_1 = \sin\omega_2 t \bullet \sin\omega_1 t$$

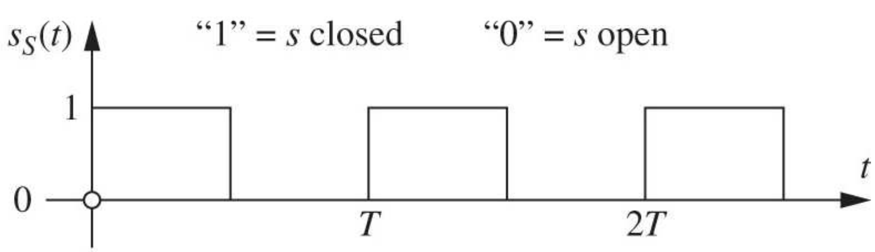
$$= \frac{\cos(\omega_2 - \omega_1)t - \cos(\omega_2 - \omega_1)t}{2}$$

- A filter is usually used to reject either the sum or difference frequency to implement up-conversion or down-conversion.

Single-Balanced Mixer

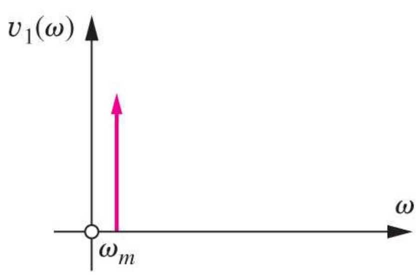


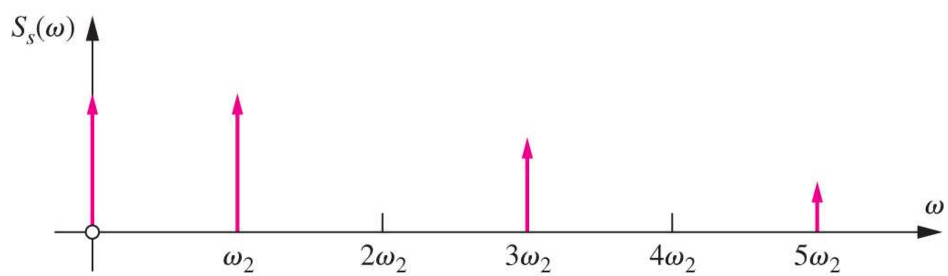
- This basic mixer form is essentially a switched circuit that 'chops' the sine wave input with a square wave function



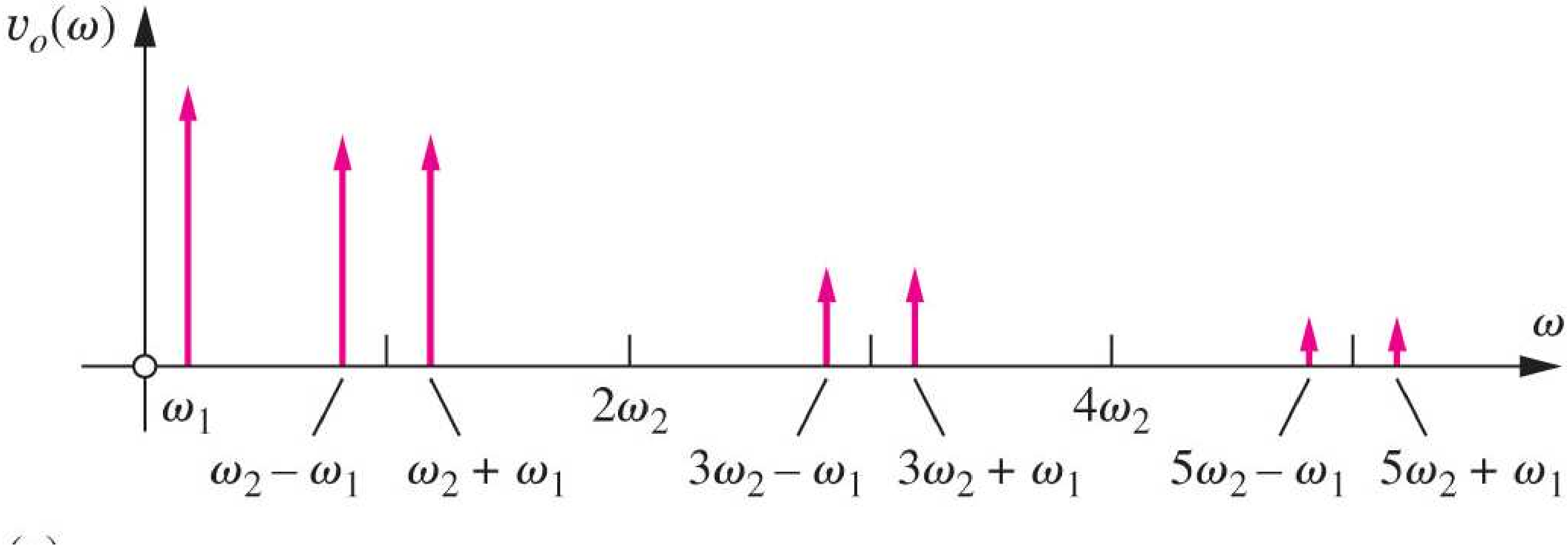
$$v_1(t) = A\sin\omega_1 t$$

$$s_S(t) = \frac{1}{2} + \frac{2}{\pi}\sum_{n\ odd}\frac{1}{n}\sin n\omega_2 t$$



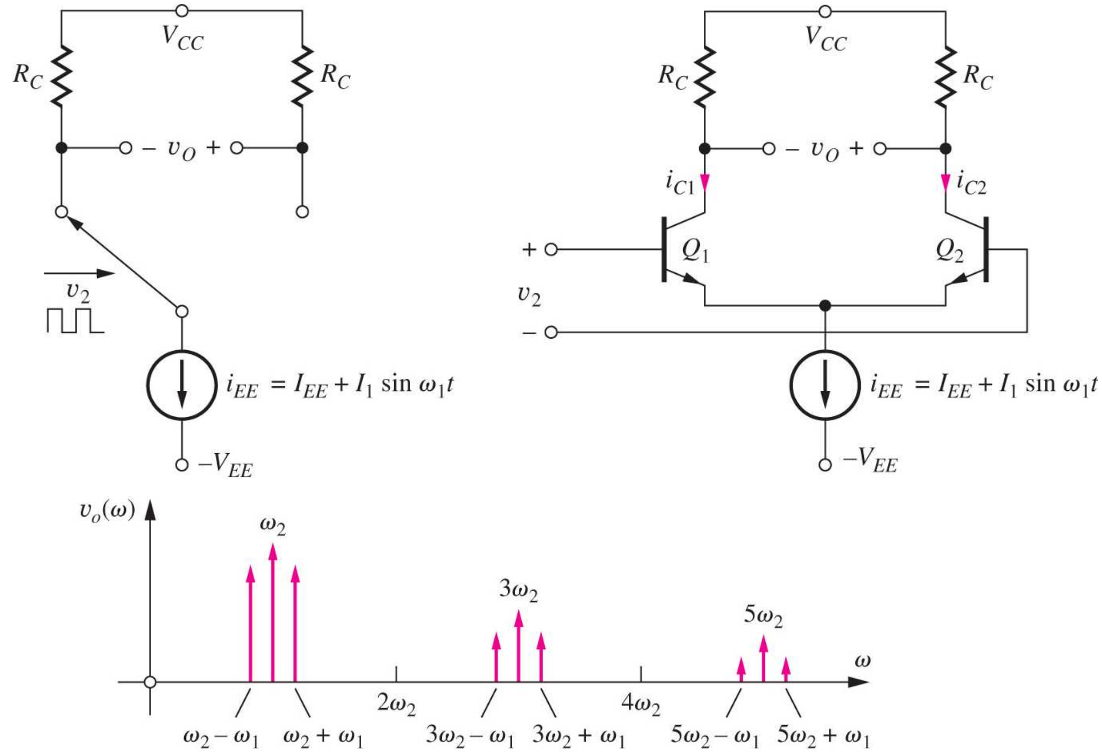


Single-Balanced Mixer Output Spectra



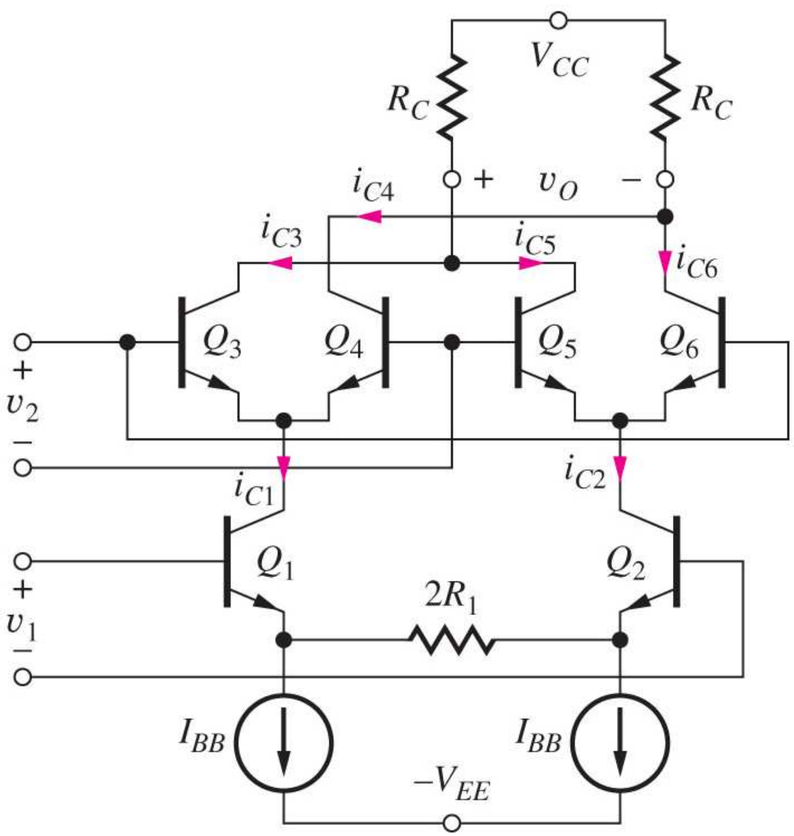
$$v_O(t) = \frac{A}{2}\sin\omega_1 t + \frac{A}{\pi}\sum_{n\ odd}\frac{\cos(n\omega_2 - \omega_1)t - \cos(n\omega_2 + \omega_1)t}{n}$$

Differential Pair as Single-Balanced Mixer



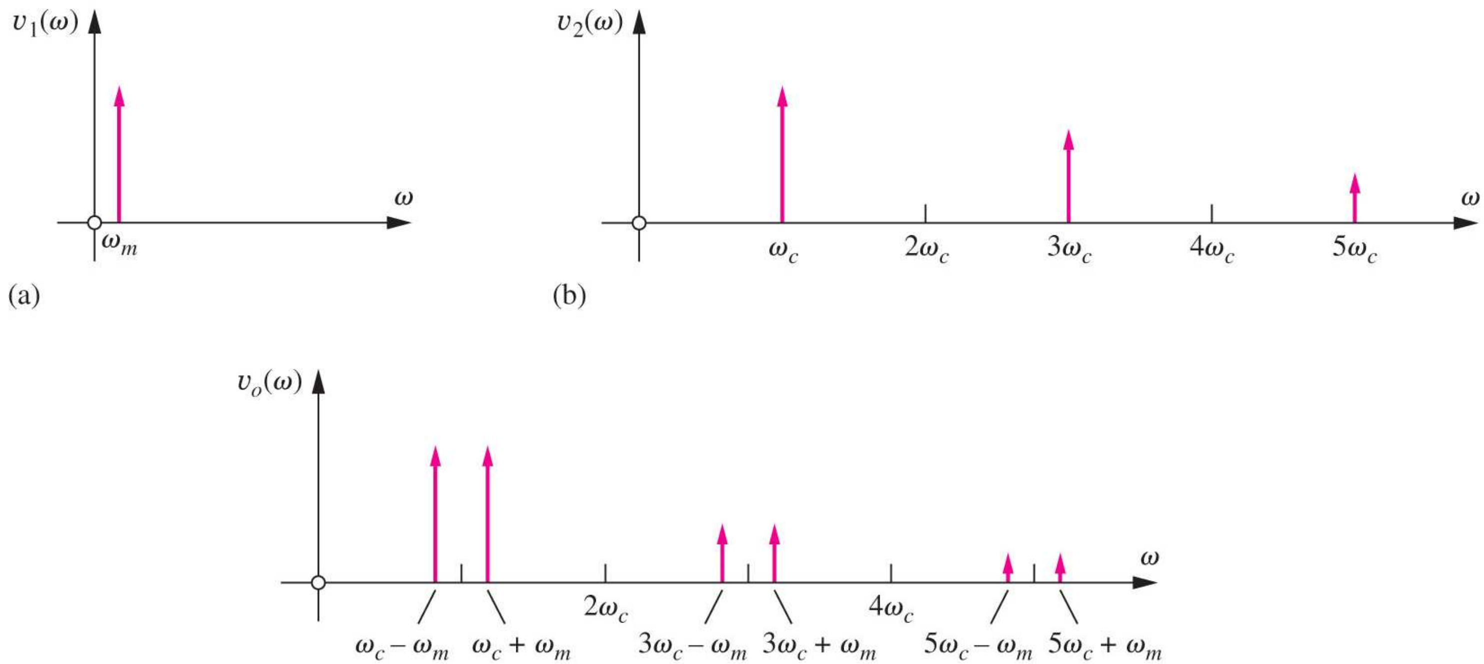
$$v_O(t) = \sum_{n\ odd} \frac{4}{n\pi}\left[I_{EE} R_C \sin n\omega_2 t + I_1 R_C \frac{\cos(n\omega_2 - \omega_1)t - \cos(n\omega_2 + \omega_1)t}{2} \right]$$

Gilbert Multiplier as a Double-Balanced Mixer



- The Gilbert Multiplier is an extension of the differential single-balanced mixer.
- The input polarity is reversed on the second diff pair and the signal v1 selects between the two diff pairs.
- The currents are summed in the load resistors and the DC component is zero.
- Only sum and difference frequencies are present at the output.

Gilbert Multiplier Mixer Spectra



$$v_o(t) = V_m \frac{R_C}{R_1} \sum_{n\ odd} \frac{2}{n\pi} \left[\cos(n\omega_c - \omega_m)t - \cos(n\omega_c + \omega_m)t \right]$$

End of Chapter 17