Chapter 17
Amplifier Frequency Response

Microelectronic Circuit Design
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Chapter Goals

- Review transfer function analysis and dominant-pole approximations of amplifier transfer functions.
- Learn partition of ac circuits into low and high-frequency equivalents.
- Learn the short-circuit time constant method to estimate upper and lower cutoff frequencies.
- Develop bipolar and MOS small-signal models with device capacitances.
- Study unity-gain bandwidth product limitations of BJTs and MOSFETs.
- Develop expressions for upper cutoff frequency of inverting, non-inverting and follower configurations.
- Explore high-frequency limitations of single and multiple transistor circuits.
Chapter Goals (contd.)

• Understand Miller effect and design of op amp frequency compensation.
• Develop relationship between op amp unity-gain frequency and slew rate.
• Understand use of tuned circuits to design high-Q band-pass amplifiers.
• Understand concept of mixing and explore basic mixer circuits.
• Study application of Gilbert multiplier as balanced modulator and mixer.
Transfer Function Analysis

\[ A_V(s) = \frac{N(s)}{D(s)} = \frac{a_0 + a_1 s + a_2 s^2 + \ldots + a_m s^m}{b_0 + b_1 s + b_2 s^2 + \ldots + b_n s^n} \]

\[ = A_{\text{mid}} F_L(s) F_H(s) \]

\[ A_{\text{mid}} \] is midband gain between upper and lower cutoff frequencies.

\[ F_L(s) = \prod_{i=1}^{l} \left( \frac{s + \frac{L}{\omega_{Zi}}}{s + \frac{L}{\omega_{Pi}}} \right) \]

\[ F_H(s) = \prod_{j=1}^{k} \left( \frac{s + \frac{L}{\omega_{Zj}}}{s + \frac{L}{\omega_{Pj}}} \right) \]

\[ |F_H(j\omega)| \rightarrow 1 \text{ for } \omega \ll \omega_{Zi}, \omega_{Pi} \text{, } i = 1 \ldots l \]

\[ \therefore A_L(s) \equiv A_{\text{mid}} F_L(s) \]

\[ |F_L(j\omega)| \rightarrow 1 \text{ for } \omega \gg \omega_{Zj}, \omega_{Pj} \text{, } j = 1 \ldots k \]

\[ \therefore A_H(s) \equiv A_{\text{mid}} F_H(s) \]
Low-Frequency Response

\[ F_L(s) \equiv \frac{s}{s + \omega_{P2}} \]
\[ \omega_L \equiv \omega_{P2} \]

Pole \( \omega_{P2} \) is called the dominant low-frequency pole (> all other poles) and zeros are at frequencies low enough to not affect \( \omega_L \).

If there is no dominant pole at low frequencies, poles and zeros interact to determine \( \omega_L \).

\[ A_L(s) = A_{mid} F_L(s) = A_{mid} \left( \frac{s + \omega_{Z1}}{s + \omega_{P1}} \right) \left( \frac{s + \omega_{Z2}}{s + \omega_{P2}} \right) \]

For \( s = j \omega \), at \( \omega_L \),
\[ A(j \omega_L) = \frac{A_{mid}}{\sqrt{2}} \]

\[ \frac{1}{\sqrt{2}} = \frac{\left( \frac{\omega_L^2 + \omega_{Z1}^2}{\omega_L^2 + \omega_{Z2}^2} \right) \left( \frac{\omega_L^2 + \omega_{P1}^2}{\omega_L^2 + \omega_{P2}^2} \right)}{1 + \left( \frac{\omega_{Z1}^2 + \omega_{Z2}^2}{\omega_{P1}^2 + \omega_{P2}^2} \right) + \left( \frac{\omega_{Z1}^2 \omega_{Z2}^2}{\omega_{P1}^2 \omega_{P2}^2} \right)} \]

\[ \Rightarrow \frac{1}{2} = \frac{\omega_L^2}{1 + \omega_{P1}^2 + \omega_{P2}^2 + \omega_{P1}^2 \omega_{P2}^2} \]

Pole \( \omega_L > \) all other pole and zero frequencies

\[ \omega_L \equiv \sqrt{\omega_{P1}^2 + \omega_{P2}^2 - 2 \omega_{Z1}^2 - 2 \omega_{Z2}^2} \]

In general, for \( n \) poles and \( n \) zeros,

\[ \omega_L \equiv \sqrt{\frac{\sum \omega_{Pn}^2 - 2 \sum \omega_{Zn}^2}{n}} \]
Low-Frequency Response

\[ |A_v(j\omega)| \]

| dB |

Actual

Dominant pole approximation (dashed)

\[ A_{mid} \]

46 dB

\[ \omega (\text{log scale}) \]

10 100 1000
Transfer Function Analysis and Dominant Pole Approximation Example

- **Problem:** Find midband gain, $F_L(s)$ and $f_L$ for

$$A_L(s) = \frac{s}{\frac{s}{100} + 1} \frac{s(s+100)}{(s+10)(s+1000)}$$

- **Analysis:** Rearranging the given transfer function to get it in standard form,

$$A_L(s) = 200 \frac{s(s+100)}{(s+10)(s+1000)}$$

Now,

$$A_L(s) = A_{\text{mid}} F_L(s)$$

$$F_L(s) = \frac{s(s+100)}{(s+10)(s+1000)}$$

and

$$A_{\text{mid}} = 200$$

Zeros are at $s=0$ and $s=-100$. Poles are at $s= -10$, $s=-1000$

$$f_L = \frac{1}{2\pi} \sqrt{10^2 + 1000^2} - 2(0^2 + 100^2) = 158 \text{ Hz}$$

All pole and zero frequencies are low and separated by at least a decade. Dominant pole is at $\omega=1000$ and $f_L = 1000/2\pi = 159 \text{ Hz}$. For frequencies $> a$ few rad/s: $A_L(s) = 200 \frac{s}{(s+1000)}$
High-Frequency Response

\[ F_L(s) \equiv \frac{s}{1 + (s/\omega_{P3})} \]

\[ \omega_H \equiv \omega_{P3} \]

Pole \( \omega_{P3} \) is called the dominant high-frequency pole (< all other poles).

If there is no dominant pole at low frequencies, poles and zeros interact to determine \( \omega_H \).

\[ A_H(s) = A_{mid} F_H(s) \]

\[ = A_{mid} \left[ \frac{1+(s/\omega_{Z1})}{1+(s/\omega_{Z2})} \right] \left[ \frac{1+(s/\omega_{P1})}{1+(s/\omega_{P2})} \right] \]

For \( s=j\omega \), at \( \omega_H \), \( |A(j\omega_H)| = \frac{A_{mid}}{\sqrt{2}} \)

\[ \frac{1}{\sqrt{2}} = \sqrt{\left[\frac{1+(\omega_H^2/\omega_{Z1}^2)}{1+(\omega_H^2/\omega_{P1}^2)}\right] \left[\frac{1+(\omega_H^2/\omega_{Z2}^2)}{1+(\omega_H^2/\omega_{P2}^2)}\right]} \]

\[ \Rightarrow \frac{1}{2} = \frac{1 + \frac{\omega_H^2}{\omega_{Z1}^2} + \frac{\omega_H^2}{\omega_{Z2}^2} + \frac{\omega_H^4}{\omega_{Z1}^2\omega_{Z2}^2}}{1 + \frac{\omega_H^2}{\omega_{P1}^2} + \frac{\omega_H^2}{\omega_{P2}^2} + \frac{\omega_H^4}{\omega_{P1}^2\omega_{P2}^2}} \]

Pole \( \omega_H \) < all other pole and zero frequencies

\[ \omega_H \equiv \sqrt{\frac{1}{\omega_{P1}^2 + \omega_{P2}^2 - \frac{2}{\omega_{Z1}^2} - \frac{2}{\omega_{Z2}^2}}} \]

In general, \( \omega_H \equiv \sqrt{\frac{1}{\sum \frac{1}{\omega_{Pn}^2} - 2\sum \frac{1}{\omega_{Zn}^2}}} \)
High-Frequency Response

![Graph showing high-frequency response with $|A(j\omega)|$ in dB on the y-axis and $\omega$ (log scale) on the x-axis. The graph includes a dashed line representing the dominant-pole approximation, with a point marked $\omega_H$ at 34 dB. There are also markers at $10^6$, $10^8$, and $10^9$.](image)
Direct Determination of Low-Frequency Poles and Zeros: C-S Amplifier

\[ V_0(s) = I_0(s)R_3 = -g_m V_{gs}(s) \frac{R_D}{R_3 + (1/sC_3) + R_3} \]

\[ = -g_m (R_3 \parallel R_D) \frac{s}{s + \frac{1}{C_3(R_D + R_3)}} V_{gs}(s) \]

\[ V_g(s) = \frac{s + C_1 R_G}{s + C_1(R_I + R_G) + 1} V_i(s) \]

\[ V_{gs}(s) = V_g - V_s = \frac{s + (1/C_2 R_S)}{s + \frac{1}{C_2[1/g_m] R_S}} V_g(s) \]

\[ A_v(s) = \frac{V_o(s)}{V_i(s)} = A_{mid} F_L(s) \]

\[ A_{mid} = -g_m (R_3 \parallel R_D) \frac{R_G}{R_G + R_I} \]
Direct Determination of Low-Frequency Poles and Zeros: C-S Amplifier (contd.)

The three zero locations are: $s = 0, 0, -1/(R_S C_2)$.

The three pole locations are:

$$s = -\frac{1}{C_1(R_I + R_G)}, -\frac{1}{C_2\left(\frac{1}{g_m}\| R_S\right)}, -\frac{1}{C_2(R_D + R_3)}$$

Each independent capacitor in the circuit contributes one pole and one zero. Series capacitors $C_1$ and $C_3$ contribute the two zeros at $s=0$ (dc), blocking propagation of dc signals through the amplifier. The third zero due to parallel combination of $C_2$ and $R_S$ occurs at frequency where signal current propagation through MOSFET is blocked (output voltage is zero).
Short-Circuit Time Constant Method to Determine $\omega_L$

- Lower cutoff frequency for a network with $n$ coupling and bypass capacitors is given by:

$$\omega_L \approx \frac{1}{\sum_{i=1}^{n} R_i S C_i}$$

where $R_i S$ is resistance at terminals of $i$th capacitor $C_i$ with all other capacitors replaced by short circuits. Product $R_i S C_i$ is short-circuit time constant associated with $C_i$.

Midband gain and upper and lower cutoff frequencies that define bandwidth of amplifier are of more interest than complete transfer function.
Estimate of $\omega_L$ for C-E Amplifier

Using SCTC method, for $C_1$,

$$R_1S = R_I + (R_B || R_{CE}l) = R_2 + (R_B || r_\pi)$$

For $C_2$,

$$R_2S = R_4 || RiE = R_4 \frac{r_\pi + R_{th}}{\beta_o + 1}$$

$$= R_4 \frac{r_\pi + (R_I || R_B)}{\beta_o + 1}$$

For $C_3$,

$$R_3S = R_3 + (R_C || RiC) = R_3 + (R_C || r_o)$$

$$\approx R_3 + R_C$$

$$\omega_L \approx \frac{3}{\sum_{i=1}^{3} \frac{1}{R_iS C_i}}$$

$$f_L = 735 Hz$$
Estimate of $\omega_L$ for C-S Amplifier

Using SCTC method,

For $C_1$,
$$R_{1S} = R_I + (R_G \parallel R_{iG}) = R_I + R_G$$

For $C_2$,
$$R_{2S} = R_S \parallel R_{iS} = R_S \left(\frac{1}{g_{m}}\right)$$

For $C_3$,
$$R_{3S} = R_3 + (R_D \parallel R_{iD}) = R_3 + (R_D \parallel r_o) \\ \cong R_3 + R_D$$
Estimate of $\omega_L$ for C-B Amplifier

Using SCTC method,

For $C_1$,

$$R_{1S} = R_I + (R_E \parallel R_{iE}) = R_I + (R_E \frac{1}{g_m})$$

For $C_2$,

$$R_{2S} = R_3 + (R_C \parallel R_{iC}) \approx R_3 + R_C$$
Estimate of $\omega_L$ for C-G Amplifier

Using SCTC method,

For $C_1$,

$$R_{1S} = R_I + (R_S || R_{iS}) = R_I + (R_S \parallel \frac{1}{g_m})$$

For $C_2$,

$$R_{2S} = R_3 + (R_D || R_{iD}) \approx R_3 + R_D$$
Estimate of $\omega_L$ for C-C Amplifier

Using SCTC method,

For $C_1$,

$$R_{1S} = R_I + (R_B \| R_{iB})$$

$$= R_I + \left( R_B r_\pi + (\beta_0 + 1) \left( R_E \| R_3 \right) \right)$$

For $C_2$,

$$R_{2S} = R_3 + (R_E \| R_{iE}) = R_3 + \left( R_E \left( r_\pi + R_{th} \right) \right)$$
Estimate of $\omega_L$ for C-D Amplifier

Using SCTC method,

For $C_1$,

$$R_{1S} = R_I + (R_G \parallel R_{in}^{CD}) = R_I + R_G$$

For $C_2$,

$$R_{2S} = R_3 + R_S \parallel R_{out}^{CD} = R_3 + R_S \left| \frac{1}{g_m} \right.$$
Frequency-dependent Hybrid-Pi Model for BJT

Capacitance between base and collector terminals is:

\[ C_\pi = g_m \tau_F \]

\( \tau_F \) is forward transit-time of the BJT. \( C_\pi \) appears in parallel with \( r_\pi \). As frequency increases, for a given input signal current, impedance of \( C_\pi \) reduces \( v_{be} \) and thus the current in the controlled source at transistor output.

Capacitance between base and emitter terminals is:

\[ C_\mu = \frac{C_{\mu_0}}{\sqrt{1 + \left( \frac{V_{CB}}{\Phi_{jc}} \right)}} \]

\( C_{\mu_0} \) is total collector-base junction capacitance at zero bias, \( \Phi_{jc} \) is its built-in potential.

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Unity-gain Frequency of BJT

The right-half plane transmission zero $\omega_z = + g_m/C_\mu$ occurring at high frequency can be neglected.

$$\omega_z = \frac{1}{r_\pi (C_\mu + C_\pi)}$$

$\omega_\beta = 1/ r_\pi (C_\mu + C_\pi)$ is the beta-cutoff frequency

$$\beta(s) \equiv \frac{\beta_o \omega_\beta}{s + \omega_\beta} = \frac{\omega_T}{s + \omega_\beta}$$

where

$$\omega_T = \beta_o \omega_\beta = \frac{\beta_o}{s + \omega_\beta}$$

and $f_T = \omega_T / 2\pi$ is the unity gain bandwidth product. Above $f_T$ BJT has no appreciable current gain.
Unity-gain Frequency of BJT (contd.)

Current gain is $\beta_o = g_m r_\pi$ at low frequencies and has single pole roll-off at frequencies $> f_\beta$, crossing through unity gain at $\omega_T$. Magnitude of current gain is 3 dB below its low-frequency value at $f_\beta$

$$C_\pi = \frac{g_m}{\omega_T} - C_\mu = \frac{40I_C}{\omega_T} - C_\mu$$

![Graph showing current gain characteristics of BJT with frequency on log scale]
High-frequency Model of MOSFET

\[ I_d(s) = (g_m - sC_{GD})V_{gs}(s) \]

\[ = I_b(s) \frac{(g_m - sC_{GD})}{s(C_{GS} + C_{GD})} \]

\[ \therefore \beta(s) = \frac{I_d(s)}{I_g(s)} = \frac{\omega_T}{s} \left(1 - \frac{s}{\omega_T \left[1 + (C_{GS}/C_{GD})\right]}\right) \]

\[ \omega_T = \frac{g_m}{C_{GS} + C_{GD}} \]

\[ f_T = \frac{\mu_n C_{ox}}{L} \left(V_{GS} - V_{TN}\right) = \frac{3\mu_n}{2} \left(V_{GS} - V_{TN}\right) \]

\[ \frac{\left(V_{GS} - V_{TN}\right)}{L^2} \]
Limitations of High-frequency Models

- Above 0.3 $f_T$, behavior of simple pi-models begins to deviate significantly from the actual device.
- Also, $\omega_T$ depends on operating current as shown and is not constant as assumed.
- For given BJT, a collector current $I_{CM}$ exists that yield $f_{T_{\text{max}}}$.
- For FET in saturation, $C_{GS}$ and $C_{GD}$ are independent of Q-point current, so

$$\omega_T \propto g_m \propto \sqrt{I_D}$$
Effect of Base Resistance on Midband Amplifiers

Base current enters the BJT through external base contact and traverses a high resistance region before entering active area. $r_x$ models voltage drop between base contact and active area of the BJT.

To account for base resistance $r_x$ is absorbed into equivalent pi model and can be used to transform expressions for C-E, C-C and C-B amplifiers.

$$i = g_m v = g_m \frac{r_\pi}{r_\pi + r_x} v_{be} = g_m' v_{be}$$

$$g_m' = g_m \frac{r_\pi}{r_\pi + r_x} = \frac{\beta_o}{r_\pi + r_x}$$

$$r_\pi' = r_\pi + r_x \quad \beta_o' = \beta_o$$
## Summary of BJT Amplifier Equations with Base Resistance

<table>
<thead>
<tr>
<th>Terminal voltage gain</th>
<th>COMMON-EMITTER AMPLIFIER</th>
<th>COMMON-COLLECTOR AMPLIFIER</th>
<th>COMMON-BASE AMPLIFIER</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{ov} = \frac{v_o}{v_1}$</td>
<td>$- \frac{\beta_o R_L}{r''_\pi + (\beta_o + 1)R_E}$</td>
<td>$+ \frac{\beta_o R_L}{r''_\pi + (\beta_o + 1)R_L}$</td>
<td>$+ g'_m R_L$</td>
</tr>
<tr>
<td>$r''<em>\pi = r_x + r</em>\pi$</td>
<td>$\approx - \frac{g'_m R_L}{1 + g'_m R_E}$</td>
<td>$\approx + \frac{g'_m R_L}{1 + g'_m R_L}$</td>
<td>$\approx +1$</td>
</tr>
<tr>
<td>$g'<em>m = \frac{\beta_o}{r''</em>\pi}$</td>
<td></td>
<td></td>
<td></td>
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</table>

<table>
<thead>
<tr>
<th>Signal-source voltage gain</th>
<th>COMMON-EMITTER AMPLIFIER</th>
<th>COMMON-COLLECTOR AMPLIFIER</th>
<th>COMMON-BASE AMPLIFIER</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_v = \frac{v_o}{v_1}$</td>
<td>$- \frac{g'<em>m R_L}{1 + g'<em>m R_E} \left( \frac{R_B | R</em>{iB}}{R_I + R_B | R</em>{iB}} \right)$</td>
<td>$+ \frac{g'<em>m R_L}{1 + g'<em>m R_L} \left( \frac{R_B | R</em>{iE}}{R_I + R_B | R</em>{iE}} \right)$</td>
<td>$\approx +1$</td>
</tr>
</tbody>
</table>

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<thead>
<tr>
<th>Input resistance</th>
<th>COMMON-EMITTER AMPLIFIER</th>
<th>COMMON-COLLECTOR AMPLIFIER</th>
<th>COMMON-BASE AMPLIFIER</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r''_\pi + (\beta_o + 1)R_E$</td>
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</table>

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<tr>
<th>Output resistance</th>
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<th>COMMON-COLLECTOR AMPLIFIER</th>
<th>COMMON-BASE AMPLIFIER</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_o(1 + g'_m R_E)$</td>
<td>$\frac{1}{g'<em>m} + \frac{R</em>{th}}{\beta_o + 1}$</td>
<td>$r_o[1 + g'_m (R_I | R_E)]$</td>
<td>$\approx 0.005[1 + g'_m (R_I | R_E)]$</td>
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</table>

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<tr>
<th>Input signal range</th>
<th>COMMON-EMITTER AMPLIFIER</th>
<th>COMMON-COLLECTOR AMPLIFIER</th>
<th>COMMON-BASE AMPLIFIER</th>
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<tbody>
<tr>
<td>$\approx 0.005(1 + g'_m R_E)$</td>
<td>$\approx 0.005(1 + g'_m R_E)$</td>
<td>$\approx 0.005[1 + g'_m (R_I | R_E)]$</td>
<td>$\alpha_o \approx +1$</td>
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</table>

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<tr>
<th>Current gain</th>
<th>COMMON-EMITTER AMPLIFIER</th>
<th>COMMON-COLLECTOR AMPLIFIER</th>
<th>COMMON-BASE AMPLIFIER</th>
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<tbody>
<tr>
<td>$-\beta_o$</td>
<td></td>
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</table>
Single-Pole High Frequency Response

Let’s first start with a simple two resistor, one capacitor network.

\[
\frac{v_x}{v_i} = \frac{R_2 \left| \frac{1}{sC_1} \right|}{R_1 + R_2 \left| \frac{1}{sC_1} \right|} = \frac{R_2}{R_1 + \frac{R_2}{1 + sR_2C_1}}
\]

\[
= \frac{R_2}{R_1 + R_2 \left( 1 + s \frac{R_1R_2}{R_1 + R_2}C_1 \right)}
\]

\[
= \frac{R_2}{R_1 + R_2 \left( 1 + s \left[ R_1 \parallel R_2 \right] C_1 \right)}
\]
Single-Pole High Frequency Response (cont.)

Substituting $s = j2\pi f$ and using $f_p = 1/(2\pi [R_1 || R_2] C_1)$

\[
\frac{v_x}{v_i} = \left(\frac{1}{R_1 + R_2}\right) \left(1 + j\frac{f}{f_p}\right)
\]

This expression has two parts, the midband gain, $R_2/(R_2+R_1)$, and the high frequency characteristic, $1/(1+jf/f_p)$. 

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We desire to replace $C_{xy}$ with $C_{eq}$ to ground. Starting with the definition of small-signal capacitance:

$$C = \frac{\Delta Q}{\Delta V}$$

Now write an expression for the change in charge for $C_{xy}$:

$$\Delta Q = C_{phy} (\Delta V_x - \Delta V_y) = C_{xy} (\Delta V_x - A_{xy} \Delta V_x)$$

$$= C_{xy} \Delta V_x (1 - A_{xy})$$

We can now find an equivalent capacitance, $C_{eq}$:

$$C_{eq} = \frac{C_{xy} \Delta V_x (1 - A_{xy})}{\Delta V_x} = C_{xy} (1 - A_{xy})$$
C-E Amplifier High Frequency Response using Miller Effect

First, find the simplified small-signal model of the C-A amp.
C-E Amplifier High Frequency
Response using Miller Effect (cont.)

Input gain is found as

\[ A_i = \frac{v_b}{v_i} = \frac{R_{in}}{R_i + R_{in}} \cdot \frac{r_\pi}{r_x + r_\pi} \]

\[ = \frac{R_1 \parallel R_2 \parallel (r_x + r_\pi)}{R_i + R_1 \parallel R_2 \parallel (r_x + r_\pi)} \cdot \frac{r_\pi}{r_x + r_\pi} \]

Terminal gain is

\[ A_{bc} = \frac{v_c}{v_b} = -g_m r_o \parallel R_C \parallel R_3 \equiv -g_m R_C \parallel R_3 \equiv -g_m R_L \]

Using the Miller effect, we find the equivalent capacitance at the base as:

\[ C_{eqB} = C_\mu (1 - A_{bc}) + C_\pi (1 - A_{be}) \]

\[ = C_\mu (1 - [-g_m R_L]) + C_\pi (1 - 0) \]

\[ = C_\mu (1 + g_m R_L) + C_\pi \]
C-E Amplifier High Frequency Response using Miller Effect (cont.)

The total equivalent resistance at the base is

\[ R_{eqB} = (R_{th} + r_x) \parallel R_{inB} \]

\[ = (R_i \parallel R_1 \parallel R_2 + r_x) \parallel r_x = r_{\pi 0} \]

The total capacitance and resistance at the collector is

\[ C_{eqC} = C_\mu + C_L \]

\[ R_{eqC} = r_o \parallel R_C \parallel R_3 \equiv R_C \parallel R_3 \equiv R_L \]

Because of interaction through \( C_\mu \), the two RC time constants interact, giving rise to a dominant pole

\[ \omega_{p1} = \frac{1}{r_{\pi 0}[C_\mu(1 + g_m R_L) + C_\pi] + R_L[C_\mu + C_L]} \]

\[ C_T = [C_\mu(1 + g_m R_L) + C_\pi] + \frac{R_L}{r_{\pi 0}}[C_\mu + C_L] \]

\[ \omega_{p1} = \frac{1}{r_{\pi 0}C_T} = \frac{1}{r_{\pi 0}([C_\mu(1 + g_m R_L) + C_\pi] + \frac{R_L}{r_{\pi 0}}[C_\mu + C_L])} \]
Direct High-Frequency Analysis: C-E Amplifier

The small-signal model can be simplified by using Norton source transformation.

\[ R_L = R_3 \parallel R_C = 100\,\text{k}\Omega \parallel 4.3\,\text{k}\Omega \quad R_B = R_1 \parallel R_2 = 30\,\text{k}\Omega \parallel 10\,\text{k}\Omega \]

\[ v_{th} = i_s \frac{R_B}{R_I + R_B} \]

\[ R_{th} = \frac{R_I R_B}{R_I + R_B} \]

\[ i_s = \frac{v_{th}}{R_{th} + r_x} \]

\[ r_{\pi o} = r_{\pi} \left( R_{th} + r_x \right) \]
Direct High-Frequency Analysis: C-E Amplifier (Pole Determination)

From nodal equations for the circuit in frequency domain,

\[ V_C(s) = I_s(s) \frac{(sC^\mu_m - g_m)}{\Delta} \]

\[ \Delta = s^2 \left( C^\pi C^\mu + C^\mu L + C^\pi C_L \right) \]

\[ + s \left( C^\pi g_L + C^\mu (g_L + g_m + g^\pi) + C_L g^\pi_o \right) + g_L g^\pi_o \]

High-frequency response is given by 2 poles, one finite zero and one zero at infinity. Finite right-half plane zero, \( \omega_Z = + \frac{g_m}{C^\mu} > \omega_T \) can be important in FET amplifiers.

For a polynomial \( s^2 + s A_1 + A_0 \) with roots \( a \) and \( b \), \( a = A_1 \) and \( b = A_0 / A_1 \).

\[ C_T = C^\pi + C^\mu \left( 1 + g_m R_L + \frac{R_L}{r^\pi o} \right) + C_L \frac{R_L}{r^\pi o} \]

\[ \omega_{P1} = \frac{A_0}{A_1} \equiv \frac{1}{r^\pi o C_T} \]

\[ \omega_{P2} \equiv \frac{g_m}{C^\pi \left( 1 + (C_L / C^\mu) \right) + C_L} \equiv \frac{g_m}{C^\pi + C_L} \]

Smallest root that gives first pole limits frequency response and determines \( \omega_H \). Second pole is important in frequency compensation as it can degrade phase margin of feedback amplifiers.
Direct High-Frequency Analysis: C-E Amplifier (Overall Transfer Function)

\[ V_o(s) = \frac{V_{th}(s)}{R_{th} + r_x g_L g_{\pi 0}} \left(1 + \frac{(sC \mu - g_m)}{1 + (s/\omega_{p1})}\right) \left(1 + (s/\omega_{p2})\right) \]

\[ V_o(s) = \frac{V_{th}(s)}{R_{th} + r_x} \left(-g_m R_L r_{\pi 0}\right) g_L g_{\pi 0} \left(1 + \frac{(s/\omega_{p1})}{1 + (s/\omega_{p2})}\right) \]

\[ \therefore V_o(s) = \frac{-V_{th}(s) g_m R_L r_{\pi 0}}{R_{th} + r_x \left(1 + (s/\omega_{p1})\right)} \]

\[ A_{vth}(s) = \frac{V_o(s)}{V_{th}(s)} \equiv \frac{A_{mid}}{1 + (s/\omega_{p1})} \]

\[ A_{mid} = -\frac{\beta_o R_L}{R_{th} + r_x + r_{\pi}} \quad \omega_{p1} = \frac{1}{\pi \omega_{C_T}} \]

Dominant pole model at high frequencies for C-E amplifier is as shown.
Direct High-Frequency Analysis: C-E Amplifier (Example)

- **Problem:** Find midband gain, poles, zeros and $f_L$.
- **Given data:** Q-point = (1.60 mA, 3.00V), $f_T = 500$ MHz, $\beta_o = 100$, $C_\mu = 0.5 \text{ pF}$, $r_x = 250 \Omega$, $C_L = 0$
- **Analysis:**
  
  $$g_m = 40I_C = 40(0.0016) = 64 \text{ mS}, \quad r_\pi = \frac{\beta_o}{g_m} = 1.56 \text{ k}\Omega.$$ 
  
  $$C_\pi = \frac{g_m}{2\pi f_T} - C_\mu = 19.9 \text{ pF}$$
  
  $$R_L = R_3 || R_C = 100 \text{k}\Omega || 4.3 \text{k}\Omega = 4.12 \text{k}\Omega$$
  
  $$R_{th} = R_B || R_I = 7.5 \text{k}\Omega || 1 \text{k}\Omega = 882 \Omega$$
  
  $$r_{\pi o} = r_\pi (R_{th} + r_x) = 656 \Omega$$
  
  $$C_T = C_\pi + C_\mu \left( 1 + g_m R_L \right) + \frac{R_L}{r_{\pi o}} (C_\mu + C_L) = 156 \text{ pF}$$
  
  $$f_{P1} = \frac{1}{2\pi r_\pi C_T} = 1.56 \text{ MHz}$$
  
  $$f_{P2} = \frac{g_m}{2\pi (C_\pi + C_L)} = 512 \text{ MHz}$$
  
  $$f_Z = \frac{g_m}{2\pi C_\mu} = 20.4 \text{ GHz}$$
  
  $$A_{vth} = A_i A_{bc} = 0.512(-264) = -135$$
Spice Simulation of Example C-E Amplifier
Estimation of $\omega_H$ using the Open-Circuit Time Constant Method

At high frequencies, impedances of coupling and bypass capacitors are small enough to be considered short circuits. Open-circuit time constants associated with impedances of device capacitances are considered instead.

$$\omega_H \cong \frac{1}{m \sum_{i=1}^{m} R_{io} C_i}$$

where $R_{io}$ is resistance at terminals of $i$th capacitor $C_i$ with all other capacitors open-circuited.

For a C-E amplifier, assuming $C_L = 0$

$$R_{\pi 0} = r_{\pi 0}$$

$$R_{\mu 0} = \frac{V_x}{I_x} = r_{\pi 0} + (1 + g_m R_L + \frac{R_L}{r_{\pi 0}})$$

$$\omega_H \cong \frac{1}{R_{\pi 0} C_{\pi} + R_{\mu 0} C_{\mu}} \frac{1}{r_{\pi 0} C_T}$$
High-Frequency Analysis: C-S Amplifier

Analysis similar to the C-E case yields the following equations:

\[ R_{th} = R_l \parallel R_G \]

\[ R_L = R_D \parallel R_3 \]

\[ v_{th} = v_i R_I + R_G \]

\[ C_T = C_{GS} + C_{GD}(1 + g_m R_L) + \frac{R_L}{R_{th}}(C_{GD} + C_L) \]

\[ \omega_{P1} = \frac{1}{R_{th} C_T} \]

\[ \omega_{P2} = \frac{g_m}{C_{GS} + C_L} \]

\[ \omega_Z = \frac{+g_m}{C_{GD}} \]
C-S Amplifier High Frequency Response with Source Degeneration Resistance

First, find the simplified small-signal model of the C-A amp.

Recall that we can define an effective $g_m$ to account for the unbypassed source resistance.

$$g_m' = \frac{g_m}{1 + g_m R_S}$$
C-S Amplifier High Frequency Response with Source Degeneration Resistance (cont.)

Input gain is found as

\[ A_i = \frac{v_g}{v_i} = \frac{R_G}{R_i + R_G} \]

\[ = \frac{R_1 \parallel R_2}{R_i + R_1 \parallel R_2} \]

Terminal gain is

\[ A_{gd} = \frac{v_d}{v_g} = -g_m' (R_{iD} \parallel R_D \parallel R_3) \equiv \frac{-g_m R_D \parallel R_3}{1 + g_m R_S} \]

Again, we use the Miller effect to find the equivalent capacitance at the gate as:

\[ C_{eqG} = C_{GD} (1 - A_{gd}) + C_{GS} (1 - A_{gs}) \]

\[ = C_{GD} (1 - \left[ -\frac{g_m R_L}{1 + g_m R_S} \right]) + C_{GS} (1 - \frac{g_m R_S}{1 + g_m R_S}) \]

\[ = C_{GD} (1 + \frac{g_m R_D \parallel R_L}{1 + g_m R_S}) + \frac{C_{GS}}{1 + g_m R_3} \]
C-S Amplifier High Frequency Response with Source Degeneration Resistance (cont.)

The total equivalent resistance at the gate is

\[ R_{eqG} = R_G \parallel R_I = R_{th} \]

The total capacitance and resistance at the collector is

\[ C_{eqD} = C_{GD} + C_L \]

\[ R_{eqD} = R_{iD} \parallel R_D \parallel R_3 \equiv R_D \parallel R_3 = R_L \]

Because of interaction through \( C_{GD} \), the two RC time constants interact, giving rise to the dominant pole:

\[ \omega_{p1} = \frac{1}{R_{th} \left[ C_{GD} \left( 1 + \frac{g_m R_L}{1 + g_m R_S} \right) + \frac{C_{GS}}{1 + g_m R_S} + \frac{R_L}{R_{th}} \left( C_{GD} + C_L \right) \right]} \]

And from previous analysis:

\[ \omega_{p2} = \frac{g_m'}{(C_{GS} + C_L)} = \frac{g_m}{(1 + g_m R_S)(C_{GS} + C_L)} \]

\[ \omega_z = \frac{+g_m'}{C_{GD}} = \frac{+g_m}{(1 + g_m R_S)(C_{GD})} \]
C-E Amplifier with Emitter Degeneration Resistance

Analysis similar to the C-S case yields the following equations:

\[ r_{\pi 0} = R_{eqB} = (R_{th} + r_x) \parallel [r_\pi + (\beta + 1)R_E]\]

\[ R_L = R_C \parallel R_3 \]

\[ \omega_{p1} = \frac{1}{r_{\pi 0}C_T} \]

\[ = \frac{1}{r_{\pi 0}([C_\mu(1 + \frac{g_m R_L}{1 + g_m R_E}) + \frac{C_\pi}{1 + g_m R_E}] + \frac{R_L}{r_{\pi 0}([C_\mu + C_L])})} \]

\[ \omega_{p2} \approx \frac{g_m}{2\pi (1 + g_m R_E)(C_\pi + C_L)} \]

\[ \omega_z = \frac{+g_m}{2\pi [1 + g_m R_E][C_\mu]} \]
Gain-Bandwidth Trade-offs Using Source/Emitter Degeneration Resistor

Adding source resistance to the CS amp caused gain to decrease and dominant pole frequency to increase.

\[ A_{gd} = \frac{v_d}{v_g} = \frac{-g_m R_D \parallel R_3}{1 + g_m R_S} \]

\[ \omega_{p1} = \frac{1}{R_{th}[C_{GD}(1 + \frac{g_m R_L}{1 + g_m R_S}) + \frac{C_{GS}}{1 + g_m R_S} + \frac{R_L}{R_{th}}(C_{GD} + C_L)]} \]

However, decreasing the gain also decreased the frequency of the second pole.

\[ \omega_{p2} = \frac{g_m}{(1 + g_m R_S)(C_{GS} + C_L)} \]

Increasing the gain of the C-E/C-S stage causes pole-splitting, or increase of the difference in frequency between the first and second poles.
High Frequency Poles for the C-B Amplifier

\[ A_i \cong \frac{1}{1 + g_m R_i} \]
\[ A_{ec} = \frac{v_c}{v_e} = g_m R_i \parallel R_L \cong g_m R_L \]
\[ R_{iC} = r_o (1 + g_m r_\pi \parallel R_i) \]

Since \( C_\mu \) does not couple input and output, input and output poles can be found directly.

\[ C_{eqE} = C_\pi \]
\[ R_{eqE} = \frac{1}{g_m} \parallel R_E \parallel R_i \]
\[ \omega_{p1} = \frac{1}{\left( \frac{1}{g_m} \parallel R_E \parallel R_i \right) C_\pi} \cong \frac{g_m}{C_\pi} \]

\[ C_{eqC} = C_\mu + C_L \]
\[ R_{eqC} = R_{iC} \parallel R_L \cong R_L \]
\[ \omega_{p2} = \frac{1}{(R_{iC} \parallel R_L)(C_\mu + C_L)} \cong \frac{1}{R_L (C_\mu + C_L)} \]
High Frequency Poles for the C-G Amplifier

Similar to the C-B, since $C_{GD}$ does not couple the input and output, input and output poles can be found directly.

$$C_{eqS} = C_{GS}$$

$$R_{eqS} = \frac{1}{g_m} \parallel \frac{1}{R_4} \parallel \frac{1}{R_I}$$

$$\omega_{p1} = \frac{1}{(\frac{1}{g_m} \parallel \frac{1}{R_4} \parallel \frac{1}{R_I})C_{GD}} \approx \frac{g_m}{C_{GD}}$$

$$C_{eqD} = C_{GD} + C_L$$

$$R_{eqD} = \frac{R_{id}}{R_L} \parallel R_L \approx R_L$$

$$\omega_{p2} = \frac{1}{(\frac{R_{id}}{R_L})(C_{GD} + C_L)} \approx \frac{1}{R_L(C_{GD} + C_L)}$$
High Frequency Poles for the C-C Amplifier

\[ A_i = \frac{v_b}{v_i} = \frac{R_{in}}{R_i + R_{in}} \]

\[ A_{be} = \frac{v_e}{v_b} = \frac{g_m R_L}{1 + g_m R_L} \]

\[ C_{eqB} = C_\mu (1 - A_{bc}) + C_\pi (1 - A_{be}) = C_\mu (1 - 0) + C_\pi \left(1 - \frac{g_m R_L}{1 + g_m R_L}\right) \]

\[ = C_\mu + \frac{C_\pi}{1 + g_m R_L} \]

\[ R_{eqB} = R_i \parallel R_{in} = \left[(R_i \parallel R_B) + r_x\right] \parallel \left[r_\pi + (\beta + 1)R_L\right] = (R_{th} + r_x) \parallel \left[r_\pi + (\beta + 1)R_L\right] \]

\[ C_{eqE} = C_\pi + C_L \]

\[ R_{eqE} = R_{iE} \parallel R_L \equiv \left[1/g_m + \frac{(R_{th} + r_x)}{\beta + 1}\right] \parallel R_L \]
High Frequency Poles for the C-C Amplifier (cont.)

The low impedance at the output makes the input and output time constants relatively well decoupled, leading to two poles.

\[
\omega_{p1} = \frac{1}{([R_{th} + r_x] \parallel [r_\pi + (\beta + 1)R_L])(C_\mu + \frac{C_\pi}{1 + g_m R_L})}
\]
\[
\omega_{p2} = \frac{1}{[R_{iE} \parallel R_L][C_\pi + C_L]} \approx \frac{1}{[(1/g_m + \frac{R_{th} + r_x}{\beta + 1}) \parallel R_L][C_\pi + C_L]}
\]

The feed-forward high-frequency path through \( C_p \) leads to a zero in the C-C response. Both the zero and the second pole are quite high frequency and are often neglected, although their effect can be significant with large load capacitances.

\[
\omega_z \approx \frac{g_m}{C_\pi}
\]
High Frequency Poles for the C-D Amplifier

\[ \omega_{p1} = \frac{1}{R_{th} \left( C_{GD} + \frac{C_{GS}}{1 + g_m R_L} \right)} \]

\[ \omega_{p2} = \frac{1}{\left[ R_{is} \parallel R_L \right] \left[ C_{GS} + C_L \right]} \approx \frac{1}{\left[ 1/g_m \parallel R_L \right] \left[ C_{GS} + C_L \right]} \]

\[ \omega_z \approx \frac{g_m}{C_{GS}} \]

Similar the the C-C amplifier, the high frequency response is dominated by the first pole due to the low impedance at the output of the C-C amplifier.
### Summary of the Upper-Cutoff Frequencies of the Single-Stage Amplifiers (pg.1037)

<table>
<thead>
<tr>
<th>Type</th>
<th>Formula</th>
<th>Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Common-emitter</td>
<td>$\frac{1}{\tau_0 C_T} = \frac{1}{r_\pi 0 \left[ C_\pi + C_\mu (1 + g_m R_L) + \left( C_a + C_L \right) \frac{R_L}{r_\pi 0} \right]}$</td>
<td>$r_\pi 0 = r_\pi</td>
</tr>
<tr>
<td>Common-source</td>
<td>$\frac{1}{R_{lb} C_T} = \frac{1}{R_{th} \left[ C_{GS} + C_{GD} \left( 1 + g_m R_L \right) + \left( C_{GD} + C_L \right) \frac{R_L}{R_{lb}} \right]}$</td>
<td>$R_{th} = R_I</td>
</tr>
<tr>
<td>Common-emitter with emitter resistor $R_E$</td>
<td>$\frac{1}{r_\pi 0 \left[ C_\pi + C_\mu \left( 1 + \frac{g_m R_L}{1 + g_m R_E} \right) + \left( C_a + C_L \right) \frac{R_L}{r_\pi 0} \right]}$</td>
<td>$r_\pi 0 = r_\pi</td>
</tr>
<tr>
<td>Common-source with source resistor $R_S$</td>
<td>$\frac{1}{R_{th} \left[ C_{GS} + C_{GD} \left( 1 + \frac{g_m R_L}{1 + g_m R_S} \right) + \left( C_{GD} + C_L \right) \frac{R_L}{R_{lb}} \right]}$</td>
<td>$R_{th} = R_I</td>
</tr>
<tr>
<td>Common-base</td>
<td>$\frac{1}{R_L (C_\mu + C_L)}$</td>
<td></td>
</tr>
<tr>
<td>Common-gate</td>
<td>$\frac{1}{R_L (C_{GD} + C_L)}$</td>
<td></td>
</tr>
<tr>
<td>Common-collector</td>
<td>$\frac{1}{[(R_I</td>
<td></td>
</tr>
<tr>
<td>Common-drain</td>
<td>$\frac{1}{(R_I</td>
<td></td>
</tr>
</tbody>
</table>
Frequency Response: Differential Amplifier

$C_{EE}$ is total capacitance at emitter node of the differential pair.

Differential mode half-circuit is similar to a C-E stage. Bandwidth is determined by the $r\pi C T$ product. As emitter is a virtual ground, $C_{EE}$ has no effect on differential-mode signals.

For common-mode signals, at very low frequencies,

$|A_{cc}(0)| \equiv \frac{R C}{2 R_{EE}} << 1$

Transmission zero due to $C_{EE}$ is

$s = -\omega_{Z} = -\frac{1}{C_{EE} R_{EE}}$
Frequency Response: Differential Amplifier (contd.)

Common-mode half-circuit is similar to a C-E stage with emitter resistor $2R_{EE}$. OCTC for $C_\pi$ and $C_\mu$ is similar to the C-E stage. OCTC for $C_{EE}/2$ is:

$$R_{EEO} = 2R_{EE} \left[ \frac{r_\pi + r_x}{\beta_o + 1} \frac{1}{g_m} \right]$$

$$\omega_p \approx \frac{C_\pi}{1 + 2g_m R_{EE} \left( 1 + \frac{2R_{EE}}{r_x} \right) + C_\mu \left( 1 + \frac{g_m R_C}{1 + 2g_m R_{EE} \frac{R_C}{r_x}} \right)} + \frac{C_{EE}}{2g_m}$$

As $R_{EE}$ is usually designed to be large,

$$\omega_p \approx \frac{C_\pi + C}{2g_m} + C_\mu \left( R_C + r_x \right)$$

$$|A_{cc}|$$

Jaeger/Blalock 8/10/10
Frequency Response: Common-Collector/ Common-Base Cascade

$R_{EE}$ is assumed to be large and neglected.

\[
R_{CC1}^{out} = \frac{r_x1}{\beta_{o1}} + 1 \approx \frac{1}{g_m1} \\
R_{CB2}^{in} = \frac{r_x2}{\beta_{o2}} + 1 \approx \frac{1}{g_m2}
\]

The intermediate node pole is neglected since the impedance is quite low. We are left with the input pole for a C-D and the output pole of a C-B stage.

\[
\omega_{pB1} = \frac{1}{([R_{th} + r_x1] || [r_{x1} + (\beta + 1)R_L])(C_{\mu1} + \frac{C_{\pi1}}{1 + g_mR_L})} = \frac{1}{([R_{th} + r_x1] || [2r_{x1}])(C_{\mu1} + \frac{C_{\pi2}}{2})}
\]

\[
\omega_{pC2} \equiv \frac{1}{R_C(C_{\mu} + C_L)}
\]
Frequency Response: Cascode Amplifier

There are two important poles, the input pole for the C-E and the output pole for the C-B stage. The intermediate node pole can usually be neglected because of the low impedance at the input of the C-B stage. \( R_{L1} \) is small, so the second term in the first pole can be neglected. Also note the \( R_{L1} \) is equal to \( 1/g_{m2} \).

\[
\omega_{pB1} = \frac{1}{r_{\pi0}C_T} = \frac{1}{r_{\pi01}([C_{\mu1}(1 + \frac{g_{m1}R_{L1}}{1 + g_{m1}R_E}) + C_{\pi1}] + \frac{R_{L1}}{r_{\pi0}}[C_{\mu1} + C_{L1}])} = \frac{1}{r_{\pi01}(2C_{\mu1} + C_{\pi1})}
\]

\[
\omega_{pC2} \equiv \frac{1}{R_L(C_{\mu2} + C_L)}
\]
Frequency Response: MOS Current Mirror

This is very similar to the C-S stage simplified model, so we will apply the C-S equations with relevant changes.

$$\omega_{P1} = \frac{1}{(1/g_{m1})C_T}$$

$$= \frac{1}{g_{m1}(C_{GS1} + C_{GS2} + C_{GD2}(1 + g_{m2}r_{o2}) + \frac{r_{o2}}{1/g_{m1}}C_{GD2})}$$

$$= \frac{1}{2C_{GS1}g_{m1} + 2C_{GD2}r_{o2}}$$

Assumes matched transistors.
**Frequency Response: Multistage Amplifier**

- **Problem:** Use open-circuit and short-circuit time constant methods to estimate upper and lower cutoff frequencies and bandwidth.
- **Approach:** Coupling and bypass capacitors determine low-frequency response, device capacitances affect high-frequency response.

At high frequencies, ac model for multi-stage amplifier is as shown.
Frequency Response: Multistage Amplifier Parameters

Parameters and operation point information for the example multistage amplifier.

<table>
<thead>
<tr>
<th>Transistor Parameters</th>
<th>$g_m$</th>
<th>$r_\pi$</th>
<th>$r_o$</th>
<th>$\beta_o$</th>
<th>$C_{GS}/C_\pi$</th>
<th>$C_{GD}/C_\mu$</th>
<th>$r_x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_1$</td>
<td>10 mS</td>
<td>$\infty$</td>
<td>12.2 kΩ</td>
<td>$\infty$</td>
<td>5 pF</td>
<td>1 pF</td>
<td>0 Ω</td>
</tr>
<tr>
<td>$Q_2$</td>
<td>67.8 mS</td>
<td>2.39 kΩ</td>
<td>54.2 kΩ</td>
<td>150</td>
<td>39 pF</td>
<td>1 pF</td>
<td>250 Ω</td>
</tr>
<tr>
<td>$Q_3$</td>
<td>79.6 mS</td>
<td>1.00 kΩ</td>
<td>34.4 kΩ</td>
<td>80</td>
<td>50 pF</td>
<td>1 pF</td>
<td>250 Ω</td>
</tr>
</tbody>
</table>
Frequency Response: Multistage Amplifier (SCTC Estimate of $\omega_L$)

SCTC for each of the six independent coupling and bypass capacitors are calculated as follows:

\[
R_{1S} = R_I + \left( R_G \parallel R_{in1} \right) = 10k\Omega + 1M\Omega = 1.01M\Omega
\]

\[
R_{2S} = \frac{1}{R_{S1} g_{m1} 0.01S} = 200\Omega \parallel \frac{1}{66.7\Omega} = 66.7\Omega
\]

\[
R_{3S} = \left( R_{D1} \parallel R_{O1} \right) + \left( R_{B2} \parallel R_{in2} \right) = \left( R_{D1} \parallel r_{o1} \right) + \left( R_{B2} \parallel r_{\pi2} \right) = 2.69k\Omega
\]

\[
R_{th2} = R_{B2} \parallel R_{D1} \parallel r_{o1} = 571\Omega
\]

\[
R_{4S} = R_{E2} \parallel \frac{R_{th2} + r_{\pi2}}{\beta_{o2} + 1} = 19.4\Omega
\]

\[
R_{5S} = \left( R_{C2} \parallel R_{O2} \right) + \left( R_{B3} \parallel R_{in3} \right) = \left( R_{C2} \parallel r_{o2} \right) + \left( R_{B3} \parallel r_{\pi3} + (\beta_{o3} + 1)(R_{E3} \parallel R_{L}) \right) = 18.4k\Omega
\]

\[
R_{th3} = R_{B3} \parallel R_{C2} \parallel r_{o2} = 3.99k\Omega
\]

\[
R_{6S} = R_L + R_{E3} \parallel \frac{R_{th3} + r_{\pi3}}{\beta_{o3} + 1} = 311\Omega
\]

\[
\omega_L \equiv \sum_{i=1}^{n} \frac{1}{R_{iS} C_i} = 3330\text{rad/s}
\]

\[
f_L = \frac{\omega_L}{2\pi} = 530\text{Hz}
\]
Frequency Response: Multistage Amplifier (High-Frequency Poles)

High-frequency pole at the gate of M1: Using our equation for the C-S input pole:

\[
f_{p1} = \frac{1}{2\pi R_{th1}[C_{GD1}(1 + g_{m1}R_L) + C_{GS1} + \frac{R_{L1}}{R_{th1}}(C_{GD1} + C_{L1})]}
\]

\[
R_{L1} = R_{I12} \parallel r_{\pi2} \parallel r_{o1} = 598\,\Omega \parallel (2.39k\Omega + 250\Omega) \parallel 12.2k\Omega = 469\,\Omega
\]

\[
C_{L1} = C_{\pi2} + C_{\mu2}(1 + g_{m2}R_L)
\]

\[
R_{L2} = R_{I23} \parallel R_{in3} \parallel r_{o2} = R_{I23} \parallel [r_{x3} + r_{\pi3} + (\beta_{03} + 1)(R_{E3} \parallel R_L)] \parallel r_{o2} = 3.33k\Omega
\]

\[
C_{L1} = 39pF + 1pF[1 + 67.8mS(3.33k\Omega)] = 266pF
\]

\[
f_{p1} = \frac{1}{2\pi 9.9k\Omega[1pF(1 + 0.01S(469\Omega)] + 5pF + \frac{469\Omega}{9.9k\Omega}(1pF + 266pF)]} = 689 \, \text{KHz}
\]
Frequency Response: Multistage Amplifier (High-Frequency Poles cont.)

High-frequency pole at the base of Q2: From the detailed analysis of the C-S amp, we find the following expression for the pole at the output of the M1 C-S stage:

\[ f_p^2 = \frac{C_{GS1}g_{L1} + C_{GD1}(g_m + g_{th1} + g_{L1}) + C_{L1}g_{th1}}{2\pi\left[C_{GS1}(C_{GD1} + C_{L1}) + C_{GD1}C_{L1}\right]} \]

For this particular case, \( C_{L1} \) (Q2 input capacitance) is much larger than the other capacitances, so \( f_p^2 \) simplifies to:

\[ f_p^2 \approx \frac{C_{L1}g_{th1}}{2\pi\left[C_{GS1}C_{L1} + C_{GD1}C_{L1}\right]} \approx \frac{1}{2\pi R_{th1}(C_{GS1} + C_{GD1})} \]

\[ f_p^2 = \frac{1}{2\pi 9.9k\Omega(5pF+1pF)} = 2.68 \text{ MHz} \]
High-frequency pole at the base of Q3: Again, due to the pole-splitting behavior of the C-E second stage, we expect that the pole at the base of Q3 will be set by equation 16.95:

\[ f_{p3} \cong \frac{g_{m2}}{2\pi[C_{\pi2}(1+\frac{C_{L2}}{C_{\mu2}})+C_{L2}]} \]

The load capacitance of Q2 is the input capacitance of the C-C stage.

\[ C_{L2} = C_{\mu3} + \frac{C_{\pi3}}{1+g_{m3}R_{E3} \parallel R_L} = 1\text{pF} + \frac{50\text{pF}}{1 + 79.6\text{mS}(3.3\text{k}\Omega \parallel 250\Omega)} = 3.55\text{ pF} \]

\[ f_{p3} \cong \frac{67.8\text{mS}[1\text{k}\Omega/(1\text{k}\Omega+250\Omega)]}{2\pi[39\text{pF}(1+\frac{3.55\text{pF}}{1\text{pF}})+3.55\text{pF}]} = 47.7\text{ MHz} \]
Frequency Response: Multistage Amplifier (f_H estimate)

There is an additional pole at the output of Q3, but it is expected to be at a very high frequency due to the low output impedance of the C-C stage. We can estimate f_H from eq. 16.23 using the calculated pole frequencies.

\[
f_H = \frac{1}{\sqrt{\frac{1}{f_{p1}^2} + \frac{1}{f_{p2}^2} + \frac{1}{f_{p3}^2}}} = 667 \text{ kHz}
\]

The SPICE simulation of the circuit on the next slide shows an f_H of 667 KHz and an f_L of 530 Hz. The phase and gain characteristics of our calculated high frequency response is quite close to that of the SPICE simulation. It was quite important to take into account the pole-splitting behavior of the C-S and C-E stages. Not doing so would have resulted in a calculated f_H of less than 550 KHz.
Frequency Response: Multistage Amplifier (SPICE Simulation)
Intro to RF Amplifiers

• Amplifiers with narrow bandwidth are often required in radio frequency (RF) applications to be able to select one signal from a large number of signals.
• Frequencies of interest > unity gain frequency of op amps, so active RC filters can’t be used.
• These amplifiers have high Q ($f_H$ and $f_L$ close together relative to center frequency)
• These applications use resonant RLC circuits to form frequency selective tuned amplifiers.
The Shunt-Peaked Amplifier

- As the frequency goes up, the gain is enhanced by the increasing impedance of the inductor.

\[
A_v(s) = \frac{(-gmR)(1+sL/R)}{1+sRC+s^2LC} \quad \text{where} \quad C = C_L + C_{GD}
\]

- The gain improvement can be plotted as a function of parameter, \(m\), defined below:

\[
A_{vn}(s) = \frac{1+ms}{1+s+ms^2}
\]

where \(L = mR^2C\)
The Shunt-Peaked Amplifier

![Graph showing normalized voltage gain vs. normalized frequency](image)

- No peaking ($m = 0$)
- Maximally flat ($m = 0.41$)
- Maximum bandwidth ($m = 0.71$)
- BW reference ($A_{vn} = 0.707$)

Graph parameters:
- $m = 0.71$
- $m = 0.41$
- $m = 0$
- $|A_{vn}| = 0.707$
Single-Tuned Amplifiers

- RLC network selects the frequency, parallel combination of \( R_D, R_3 \) and \( r_o \) set the Q and bandwidth.

\[
A_v(s) = \frac{V_o(s)}{V_i(s)} = \frac{sC_{GD} - g_m}{G_P + s(C + C_{GD}) + \left( \frac{1}{sL} \right)}
\]

\( G_P = g_o + G_D + G_3 \)

- Neglecting right-half plane zero,

\[
A_v(s) = A_{mid} \frac{s \omega_o}{Q} \frac{Q}{s^2 + \frac{\omega_o}{Q} + \omega_o^2}
\]

\[
\omega_o = \frac{1}{\sqrt{L(C + C_{GD})}} \quad Q = \omega_o R_P (C + C_{GD}) = \frac{R_P}{\omega_o L}
\]
Single-Tuned Amplifiers (contd.)

- At center frequency, \( s = j\omega_o \),
  \[ A_v = A_{mid} \]
  \[ A_{mid} = -g_m R_P = -g_m (r_o \frac{R_D}{R_3}) \]

\[
BW = \frac{\omega_o}{Q} = \frac{1}{R_P (C+C_{GD})} = \frac{\omega_o^2 L}{R_P}
\]
Use of tapped Inductor- Auto Transformer

$C_{GD}$ and $r_o$ can often be small enough to degrade characteristics of the tuned amplifier. Inductor can be made to work as an auto transformer to solve this problem.

$$\frac{V_0(s)}{I_2(s)} = \frac{nV_1(s)}{I_s(s)/n} = n^2 \frac{V_1(s)}{I_s(s)} \quad Z_s(s) = n^2 Z_p(s)$$

These results can be used to transform the resonant circuit and higher $Q$ can be obtained and center frequency doesn’t shift significantly due to changes in $C_{GD}$.

Similar solution can be used if tuned circuit is placed at amplifier input instead of output.
Multiple Tuned Circuits

- Tuned circuits can be placed at both input and output to tailor frequency response.
- Radio-frequency choke (an open circuit at operating frequency) is used for biasing.
- Synchronous tuning uses two circuits tuned to same center frequency for high Q.
  \[ \text{BW}_n = \text{BW}_1 \sqrt{2^{1/n} - 1} \]
- Stagger tuning uses two circuits tuned to slightly different center frequencies to realize broader band amplifiers.

Cascode stage is used to provide isolation between the two tuned circuits and eliminate feedback path between them due to Miller multiplication.
CS Amp with Inductive Degeneration

- Typically need to match input resistance to antenna impedance at center frequency, usually 50 ohms.
- Using our follower analyses, the input impedance is found as:

\[
Z_{\text{in}}(s) = Z_{gs} + Z_s + (g_m Z_{gs}) Z_s
\]

\[
Z_{\text{in}}(s) = \frac{1}{sC_{GS}} + sL_s + R_{eq}
\]

where \( R_{eq} = +g_m L_s / C_{GS} \)
- The following slide shows a complete low noise CS amp where a series inductor resonates with the input capacitance to leave only the resistance at the center frequency.
Complete Cascode LNA
Mixer Introduction

- A mixer is a circuit that multiplies two signals to produce sum and difference frequencies:

\[ S_0 = S_2 \cdot S_1 = \sin \omega_2 t \cdot \sin \omega_1 t \]

\[ = \frac{\cos(\omega_2 - \omega_1) t - \cos(\omega_2 + \omega_1) t}{2} \]

- A filter is usually used to reject either the sum or difference frequency to implement up-conversion or down-conversion.
Single-Balanced Mixer

- This basic mixer form is essentially a switched circuit that 'chops' the sine wave input with a square wave function

\[ v_1(t) = A \sin \omega_1 t \]

\[ s_s(t) = \frac{1}{2} + \sum_{n \text{ odd}} \frac{1}{\pi n} \sin n \omega_2 t \]
Single-Balanced Mixer Output Spectra

\[ v_o(t) = \frac{A}{2} \sin \omega_1 t + \sum_{n \text{ odd}}^{\infty} \frac{A}{\pi n} \cos(n \omega_2 - \omega_1) t - \cos(n \omega_2 + \omega_1) t \]

\[ v_o(\omega) \]
Differential Pair as Single-Balanced Mixer

\[ v_o(t) = \sum_{n \text{ odd}} \frac{4}{n \pi} \left[ I_{EE} R C \sin n \omega_2 t + I_1 R C \frac{\cos(n \omega_2 - \omega_1) t - \cos(n \omega_2 + \omega_1) t}{2} \right] \]
Gilbert Multiplier as a Double-Balanced Mixer

- The Gilbert Multiplier is an extension of the differential single-balanced mixer.
- The input polarity is reversed on the second diff pair and the signal v1 selects between the two diff pairs.
- The currents are summed in the load resistors and the DC component is zero.
- Only sum and difference frequencies are present at the output.
$$v_O(t) = V_m \frac{R}{R_1} \sum_{n \text{ odd}} \frac{2}{n \pi} \left[ \cos(n \omega_c - \omega_m) \cdot t - \cos(n \omega_c + \omega_m) \cdot t \right]$$
End of Chapter 17