Chapter 3
Solid-State Diodes and Diode Circuits

Microelectronic Circuit Design

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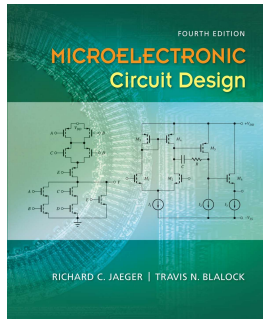
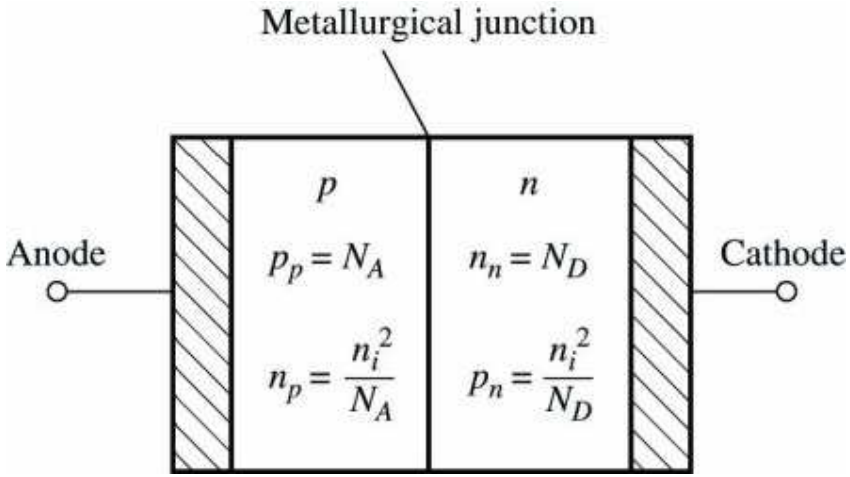
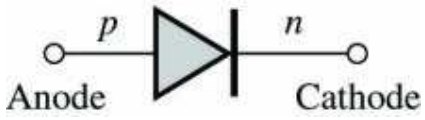


Chapter Goals

- Understand diode structure and basic layout
- Develop electrostatics of the *pn* junction
- Explore various diode models including the mathematical model, the ideal diode model, and the constant voltage drop model
- Understand the SPICE representation and model parameters for the diode
- Define regions of operation of the diode (forward bias, reverse bias, and reverse breakdown)
- Apply the various types of models in circuit analysis
- Explore different types of diodes
- Discuss the dynamic switching behavior of the *pn* junction diode
- Explore diode applications
- Practice simulating diode circuits using SPICE

Diode Introduction

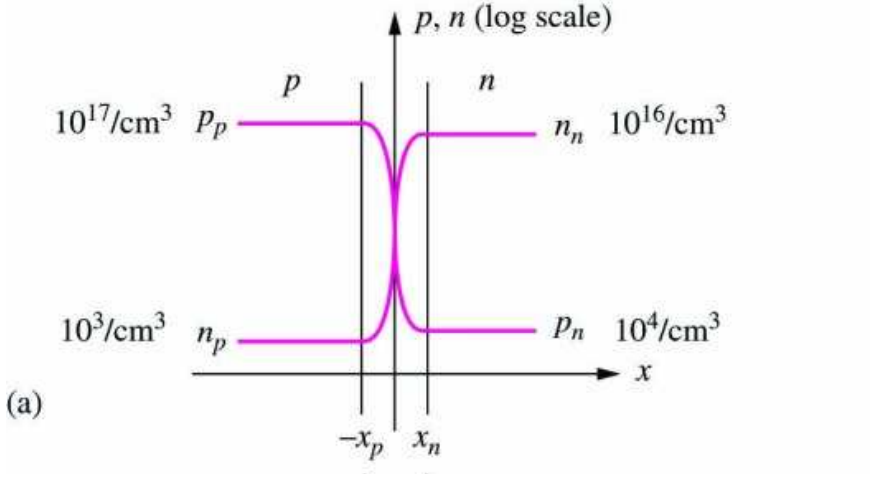


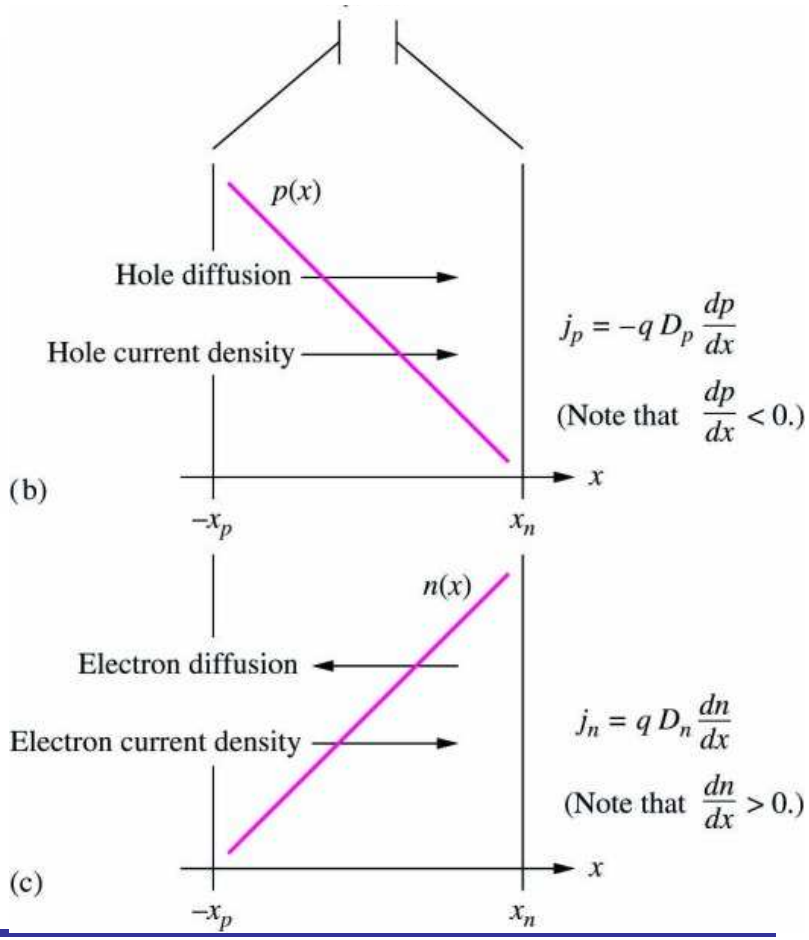


Diode symbol

- A diode is formed by joining an *n*-type semiconductor with a *p*-type semiconductor.
- A ***pn* junction** is the interface between *n* and *p* regions.

pn Junction Electrostatics





Donor and acceptor concentrations on either side of the junction

Concentration gradients give rise to diffusion currents.

Drift Currents

- Diffusion currents lead to localized charge density variations near the *pn* junction.
- Gauss' law predicts an electric field due to the charge distribution:

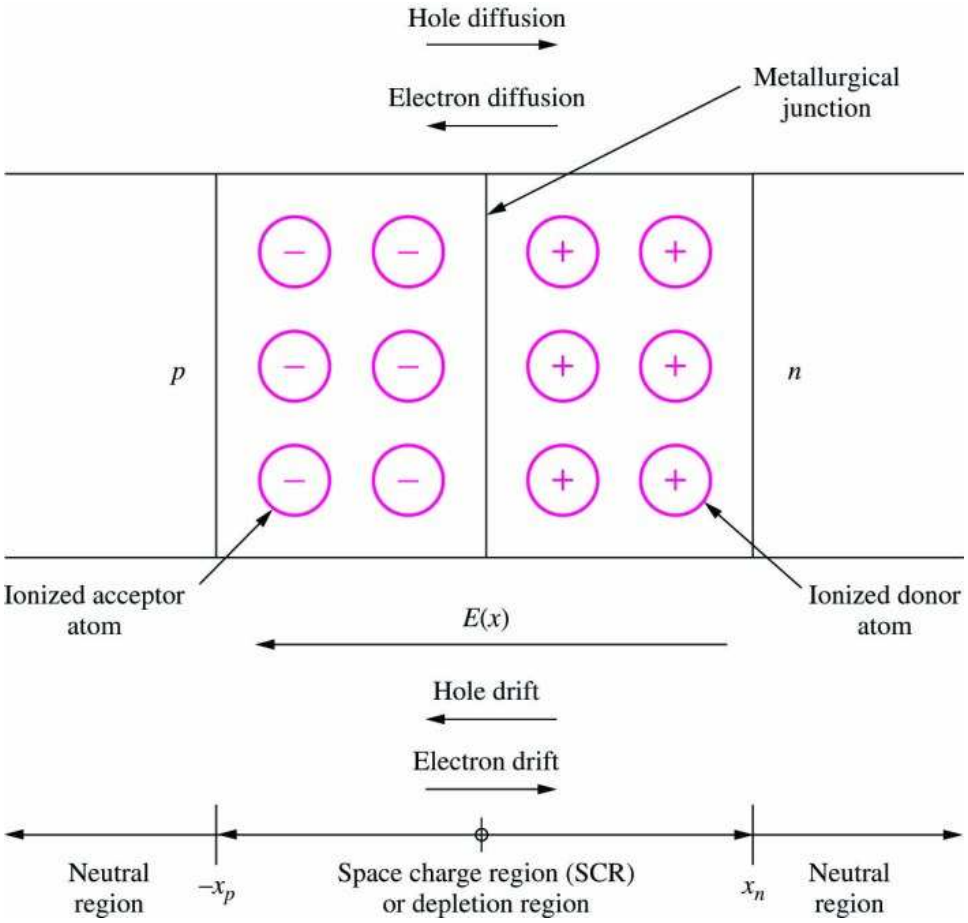
$$\nabla \cdot E = \frac{\rho_c}{\varepsilon_s}$$

- Assuming constant permittivity,

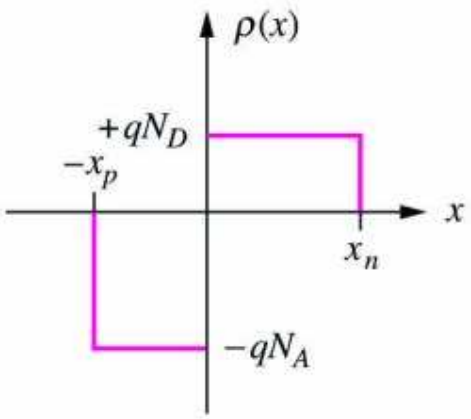
$$E(x) = \frac{1}{\varepsilon_s} \int \rho(x) dx$$

- Resulting electric field gives rise to a drift current. With no external circuit connections, drift and diffusion currents cancel. There is no actual current, since this would imply power dissipation, rather the electric field cancels the diffusion current 'tendency.'

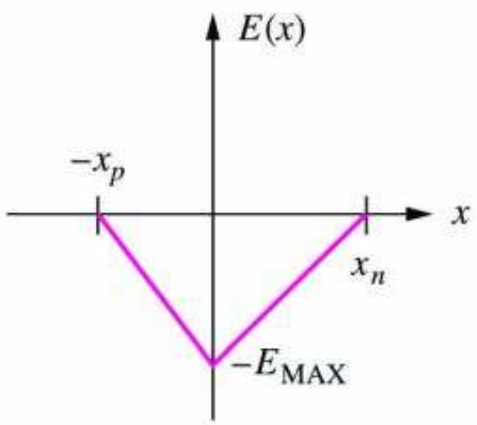
Space-Charge Region Formation at the *pn* Junction



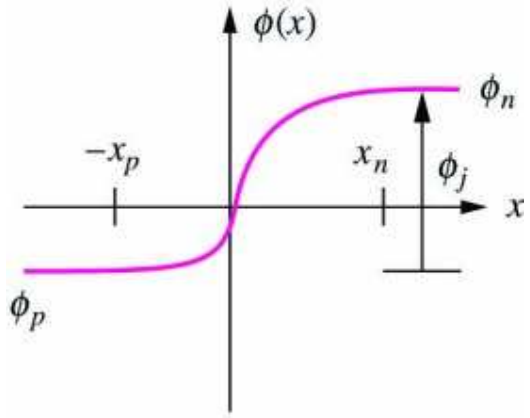
Potential Across the Junction



Charge Density



Electric Field



Potential

$$\phi_j = -\int E(x)dx = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right), \quad V_T = \frac{kT}{q}$$

Width of Depletion Region

Combining the previous expressions, we can form an expression for the width of the space-charge region, or depletion region. The region is referred to as the depletion region since the excess holes and electrons are depleted from the dopant atoms on either side of the junction.

$$w_{d0} = (x_n + x_p) = \sqrt{\frac{2\varepsilon_s}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)\phi_j}$$

EXAMPLE
Width of Depletion Region

Problem: Find built-in potential and depletion-region width for a given diode

Given data: On *p*-type side: $N_A = 10^{17}/\text{cm}^3$; On *n*-type side: $N_D = 10^{20}/\text{cm}^3$

Assumptions: Room-temperature operation with $V_T = 0.025$ V

Analysis:

$$\phi_j = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right) = (0.025\text{ V})\ln\left[\frac{\left(10^{17}/\text{cm}^3\right)\left(10^{20}/\text{cm}^3\right)}{\left(10^{20}/\text{cm}^6\right)}\right] = 0.979\,\text{V}$$

$$w_{d0} = \sqrt{\frac{2\varepsilon_s}{q}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)\phi_j} = 0.113\ \mu\text{m}$$

EXAMPLE
Diode Electric Field

- **Problem:** Find the electric field and size of the individual depletion layers on either side of a *pn* junction for a given diode
- **Given data**: On the *p*-type side: $N_A = 10^{17}/\text{cm}^3$ on the *n*-type side: $N_D = 10^{20}/\text{cm}^3$ from earlier example, $\phi_j = 0.979\text{V}$ $w_{d0} = 0.113\ \mu\text{m}$
- **Assumptions**: Room-temperature operation
- **Analysis:**

$$w_{d0} = x_n + x_p = x_n\left(1+\frac{N_D}{N_A}\right) = x_p\left(1+\frac{N_A}{N_D}\right)$$

$$x_n = \frac{w_{d0}}{\left(1+\frac{N_D}{N_A}\right)} = 1.13\times10^{-4}\ \mu m \qquad x_p = \frac{w_{d0}}{\left(1+\frac{N_A}{N_D}\right)} 0.113\ \mu m$$

$$E_{MAX} = \frac{2\phi_j}{w_{d0}} = \frac{2(0.979V)}{0.113\mu m} = 173\ kV/cm$$

Internal Diode Currents

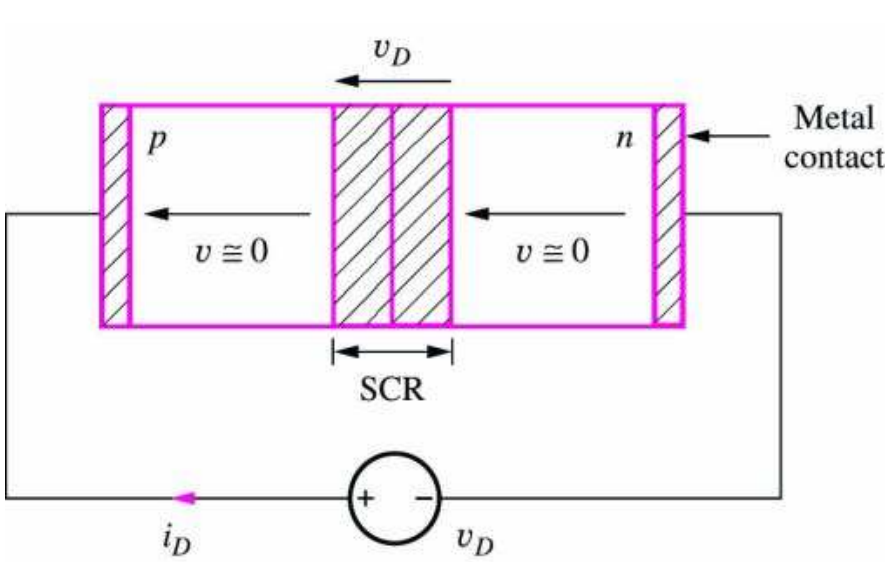
Mathematically, for a diode with no external connections, the total current expressions developed in Chapter 2 are equal to zero. The equations only dictate that the total currents are zero. However, as mentioned earlier, since there is no power dissipation, we must assume that the drift and diffusion current tendencies cancel and the individual hole and electron currents are both zero.

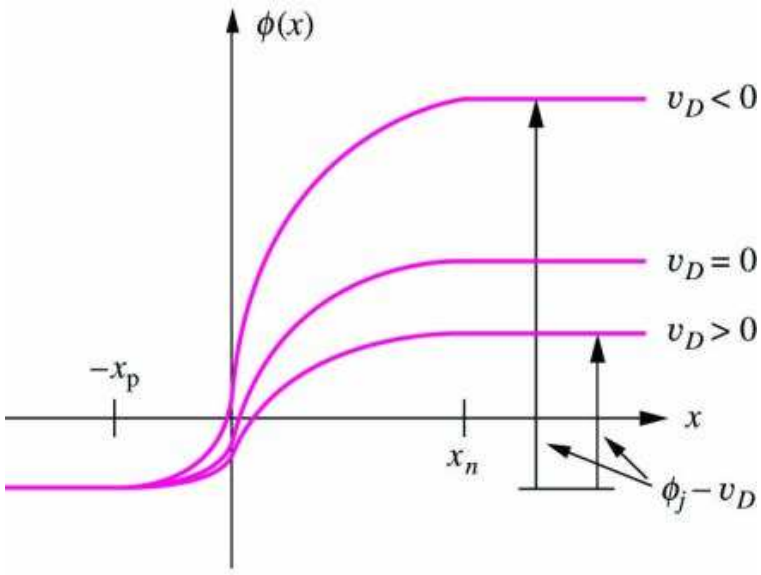
$$j_n^T = q\mu_n nE + qD_n \frac{\partial n}{\partial x} = 0$$

$$j_p^T = q\mu_p pE - qD_p \frac{\partial p}{\partial x} = 0$$

When external bias voltage is applied to the diode, the above equations are no longer equal to zero.

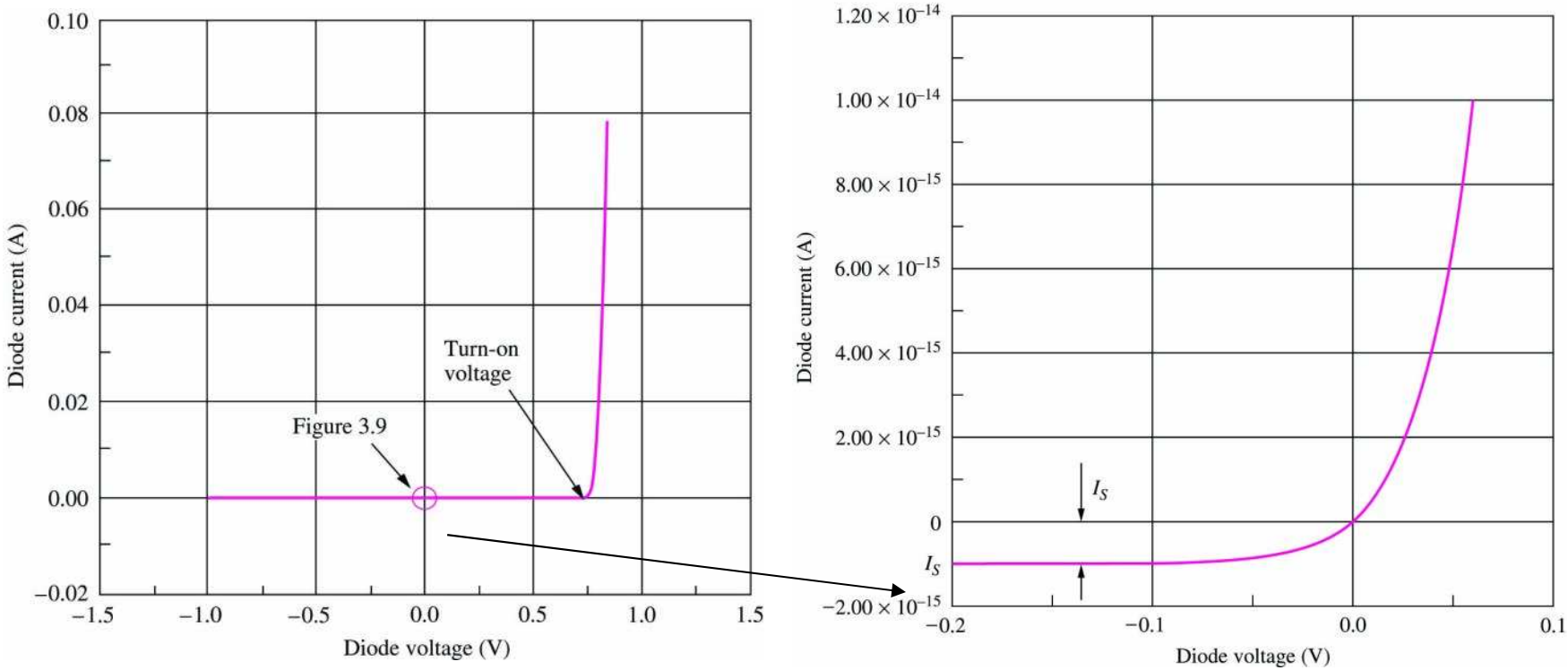
Diode Junction Potential for Different Applied Voltages





Applied voltage v_D appears across the space-charge region
There is negligible voltage drop in the ohmic regions

Diode *i-v* Characteristics

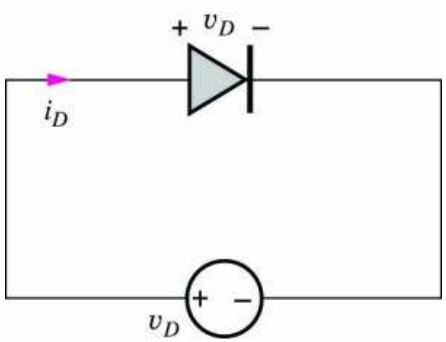


The turn-on voltage marks the point of significant current flow.

I_S is called the reverse saturation current.

Diode Equation

$$i_D = I_S\left[\exp\left(\frac{qv_D}{nkT}\right)-1\right] = I_S\left[\exp\left(\frac{v_D}{nV_T}\right)-1\right]$$



where
- I_S = reverse saturation current (A)
- v_D = voltage applied to diode (V)
- q = electronic charge (1.60 x 10^{-19} C)
- k = Boltzmann's constant (1.38 x 10^{-23} J/K)
- T = absolute temperature (Kelvins)
- n = non-ideality factor (dimensionless)
- V_T = kT/q = thermal voltage (V) (25 mV at room temp.)

I_S is typically between 10^{-18} and 10^{-9} A, and is strongly temperature dependent due to its dependence on n_i^2. The non-ideality factor is typically close to 1, but approaches 2 for devices with high current densities. It is assumed to be 1 in this text.

EXAMPLE
Diode Voltage and Current Calculations

Problem: Find diode voltage for diode with given specifications

Given data: (I_S, I_D) = (0.1 fA, 300 μA), (10 fA, 300 μA), (0.1 fA, 1 mA)

Assumptions: Room-temperature dc operation with $V_T = 0.025$ V

Analysis:

$$\text{With } I_S = 0.1 \text{ fA}: \quad V_D = nV_T \ln\left(1+\frac{I_D}{I_S}\right) = 1(0.025V)\ln\left(1+\frac{3x10^{-4}\,A}{10^{-16}\,A}\right) = 0.718\ V$$

$$\text{With } I_S = 10 \text{ fA}: \quad V_D = (0.025V)\ln\left(1+\frac{3x10^{-4}\,A}{10^{-14}\,A}\right) = 0.603\ V$$

$$\text{With } I_S = 0.1 \text{ fA}, \ I_D = 1 \text{ mA}: \quad V_D = (0.025V)\ln\left(1+\frac{10^{-3}\,A}{10^{-16}\,A}\right) = 0.748\ V$$

Diode Current for Reverse, Zero, and Forward Bias

- Reverse bias:

$$i_D = I_S\left[\exp\left(\frac{v_D}{nV_T}\right)-1\right] \cong I_S\left[0-1\right] \cong -I_S$$

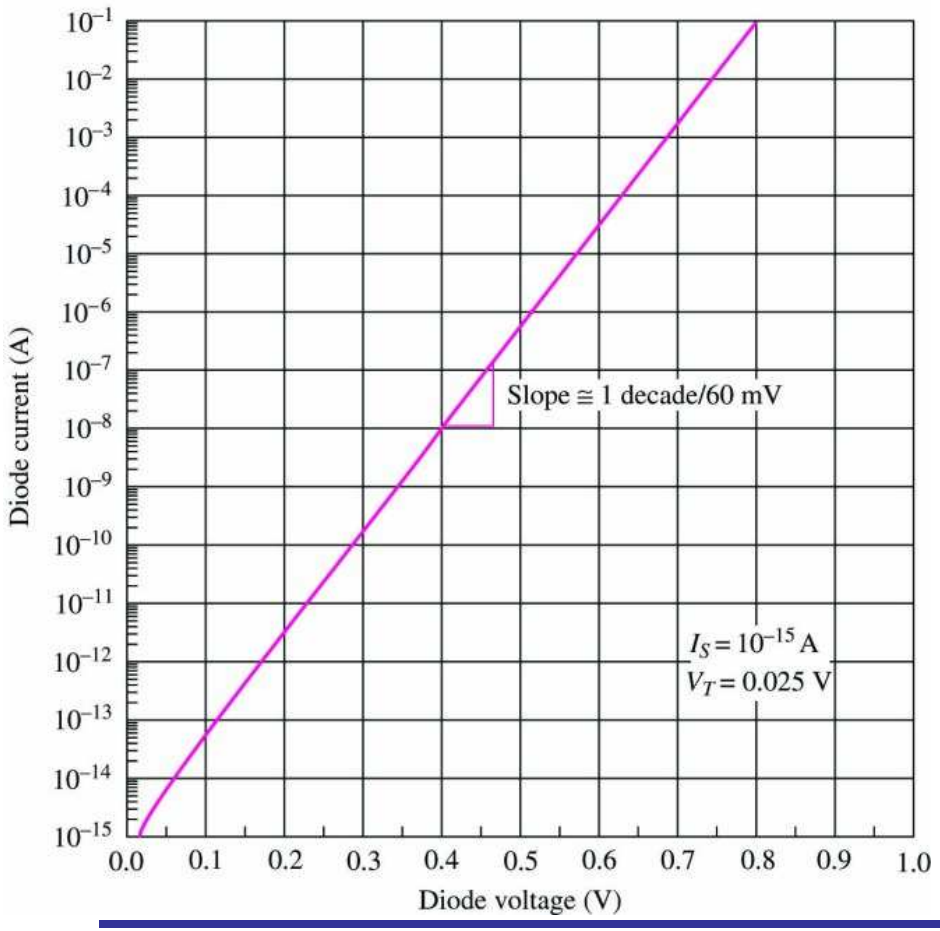
- Zero bias:

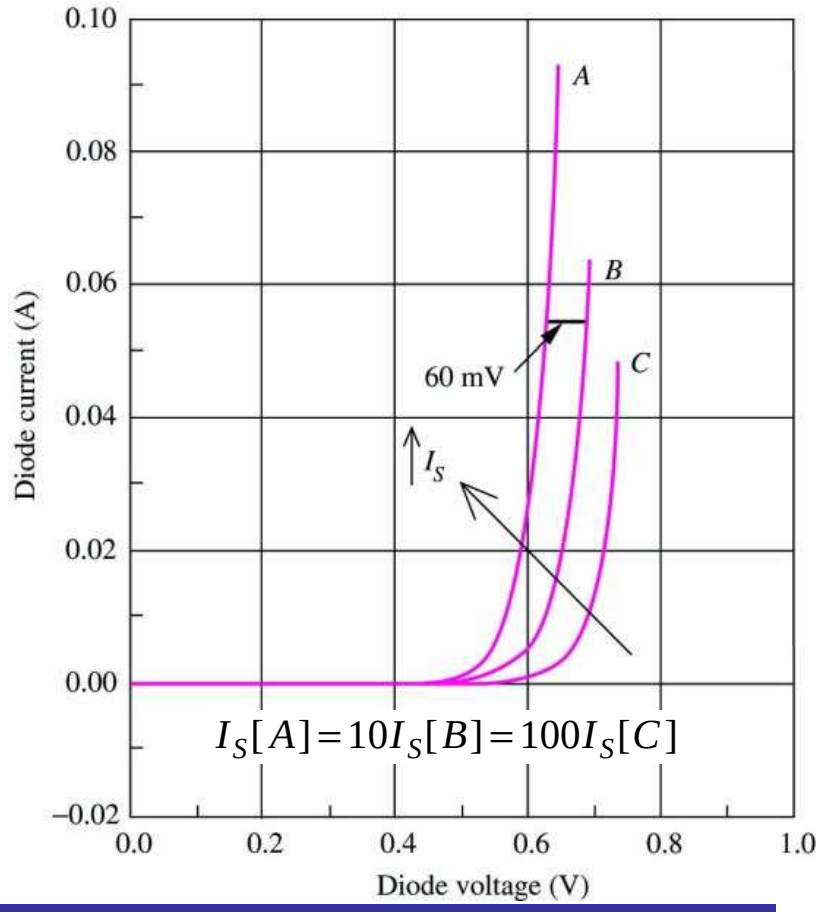
$$i_D = I_S\left[\exp\left(\frac{v_D}{nV_T}\right)-1\right] \cong I_S\left[1-1\right] \cong 0$$

- Forward bias:

$$i_D = I_S\left[\exp\left(\frac{v_D}{nV_T}\right)-1\right] \cong I_S \exp\left(\frac{v_D}{nV_T}\right)$$

Semi-log Plot of Forward Diode Current and Current for Three Different Values of I_S





Diode Temperature Coefficient

Diode voltage under forward bias:

$$v_D = V_T \ln\left(\frac{i_D}{I_S}+1\right) = \frac{kT}{q}\ln\left(\frac{i_D}{I_S}+1\right) \cong \frac{kT}{q}\ln\left(\frac{i_D}{I_S}\right)$$

Taking the derivative with respect to temperature yields

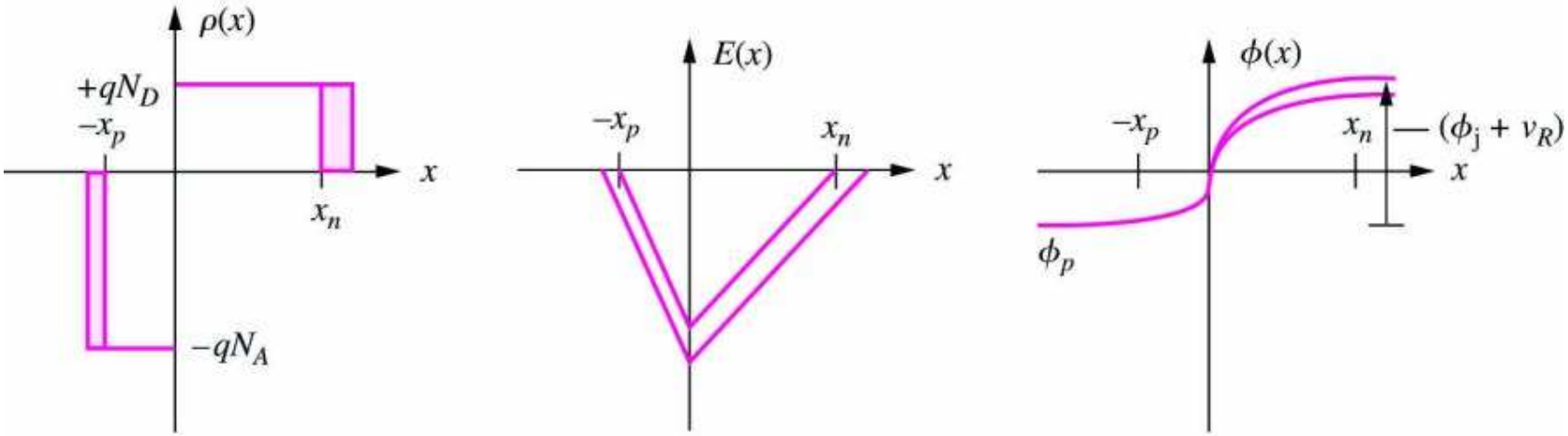
$$\frac{dv_D}{dT} = \frac{k}{q}\ln\left(\frac{i_D}{I_S}\right) - \frac{kT}{q}\frac{1}{I_S}\frac{dI_S}{dT} = \frac{v_D}{T} - V_T\frac{1}{I_S}\frac{dI_S}{dT} = \frac{v_D - V_{GO} - 3V_T}{T} \quad \text{V/K}$$

Assuming $i_D >> I_S$, $I_S \propto n_i^2$, and V_{GO} is the silicon bandgap energy at 0 K. For a typical silicon diode

$$\frac{dv_D}{dT} = \frac{(0.65 - 1.12 - 0.075)V}{300K} = \text{-1.82 mV/K} \approx \textbf{-1.8 mV/°C}$$

Reverse Bias

External reverse bias adds to the built-in potential of the pn junction. The shaded regions below illustrate the increase in the characteristics of the space charge region due to an externally applied reverse bias, v_D.



(a) Space charge density (b) Electric field (c) Electrostatic potential

Reverse Bias (cont.)

External reverse bias also increases the width of the depletion region since the larger electric field must be supported by additional charge.

$$w_d = (x_n + x_p) = \sqrt{\frac{2\varepsilon_s}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)(\phi_j + v_R)}$$

$$w_d = w_{d0}\sqrt{1 + \frac{v_R}{\phi_j}}$$

$$\text{where} \quad w_{d0} = (x_n + x_p) = \sqrt{\frac{2\varepsilon_s}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)\phi_j}$$

Reverse Bias Saturation Current

We earlier assumed that the reverse saturation current was constant. Since it results from thermal generation of electron-hole pairs in the depletion region, it is dependent on the volume of the space charge region. It can be shown that the reverse saturation gradually increases with increased reverse bias.

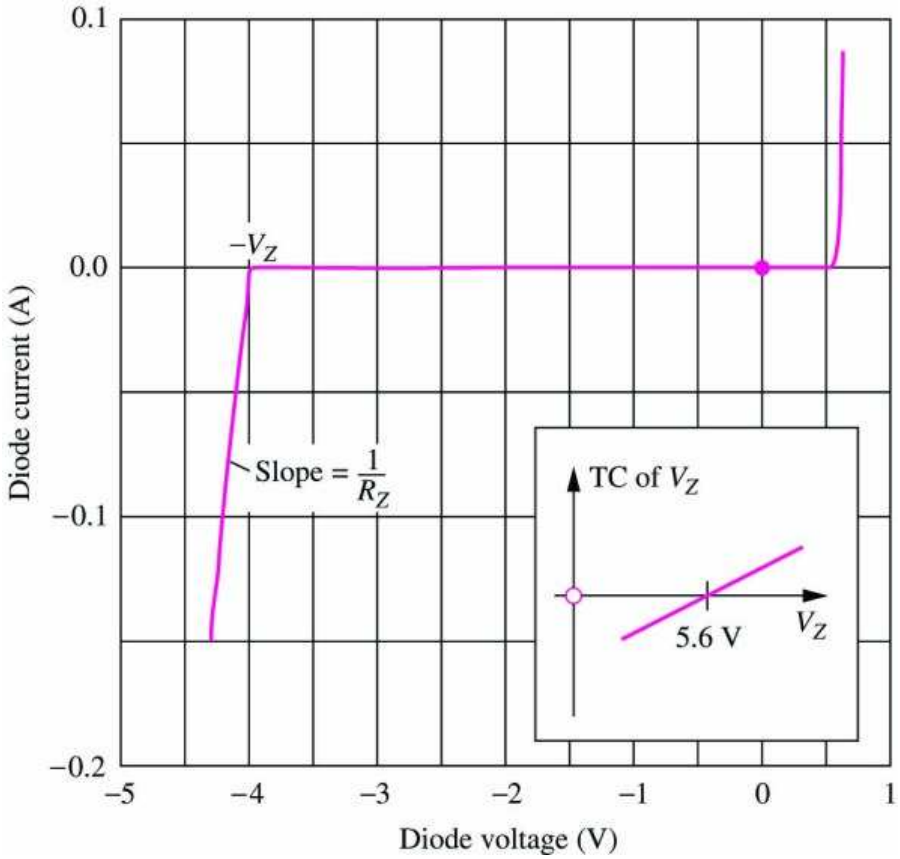
$$I_S = I_{S0}\sqrt{1 + \frac{v_R}{\phi_j}}$$

I_S is approximately constant at I_{S0} under forward bias.

Reverse Breakdown

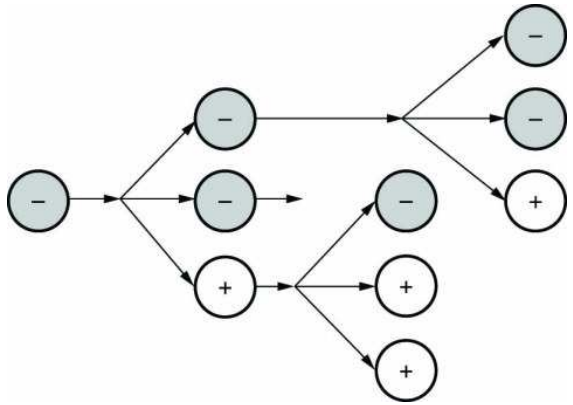
Increased reverse bias eventually results in the diode entering the **breakdown region**, resulting in a sharp increase in the diode current. The voltage at which this occurs is the **breakdown voltage**, V_Z.

$$2\ \text{V} \leq V_Z \leq 2000\ \text{V}$$



Reverse Breakdown Mechanisms

- **Avalanche Breakdown**
  Si diodes with V_Z greater than about 5.6 volts breakdown according to an avalanche mechanism. As the electric field increases, accelerated carriers begin to collide with fixed atoms. As the reverse bias increases, the energy of the accelerated carriers increases, eventually leading to ionization of the impacted ions. The new carriers also accelerate and ionize other atoms. This process feeds on itself and leads rapidly to avalanche breakdown.



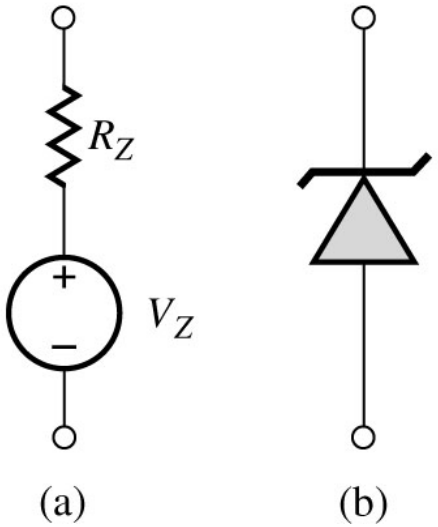
Reverse Breakdown Mechanisms (cont.)

- **Zener Breakdown**
  Zener breakdown occurs in heavily doped diodes. The heavy doping results in a very narrow depletion region at the diode junction. Reverse bias leads to carriers with sufficient energy to tunnel directly between conduction and valence bands moving across the junction. Once the tunneling threshold is reached, additional reverse bias leads to a rapidly increasing reverse current.

- **Breakdown Voltage Temperature Coefficient**
  Temperature coefficient is a quick way to distinguish breakdown mechanisms. Avalanche breakdown voltage increases with temperature, whereas Zener breakdown decreases with temperature.

  For silicon diodes, zero temperature coefficient is achieved at approximately 5.6 V.

Breakdown Region Diode Model



In breakdown, the diode is modeled with a voltage source, V_Z, and a series resistance, R_Z. R_Z models the slope of the *i*-*v* characteristic.

Diodes designed to operate in reverse breakdown are called **Zener diodes** (regardless of the actual mechanism) and use the indicated symbol.

Reverse Bias Capacitance

Changes in voltage lead to changes in depletion width and charge. This leads to a capacitance that we can calculate from the charge-voltage dependence.

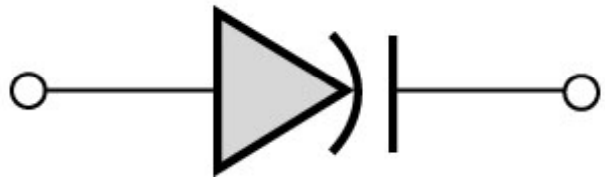
$$Q_n = qN_D x_n A = q\left(\frac{N_A N_D}{N_A + N_D}\right) w_d A \quad \text{Coulombs}$$

$$C_j = \frac{dQ_n}{dv_R} = \frac{C_{j0}A}{\sqrt{1+\frac{v_R}{\phi_j}}} \quad \text{F/cm}^2 \quad \text{where } C_{j0} = \frac{\varepsilon_s}{w_{d0}}$$

C_{j0} is the zero bias junction capacitance per unit area.

Reverse Bias Capacitance (cont.)

Diodes can be designed with hyper-abrupt doping profiles that optimize the reverse-biased diode as a voltage controlled capacitor.



Circuit symbol for the variable capacitance diode

(Varactor diodes)

Forward Bias Capacitance

In forward bias operation, additional charge is stored in the neutral region near the edges of space charge region.

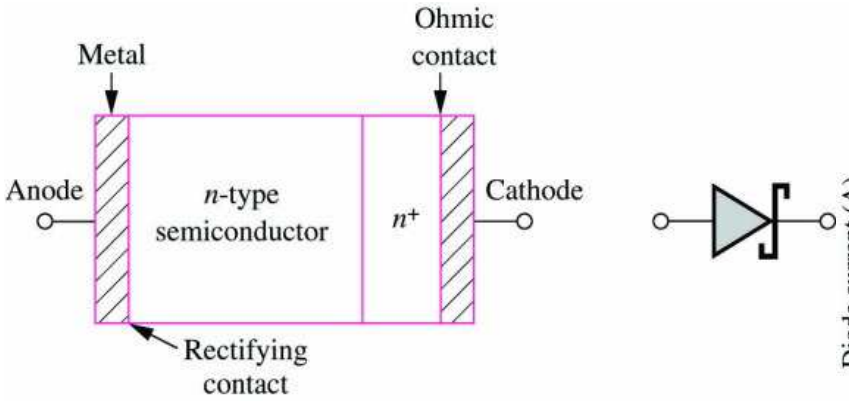
$$Q_D = i_D \tau_T \text{ Coulombs}$$

τ_T is called diode transit time and depends on the size and type of diode.

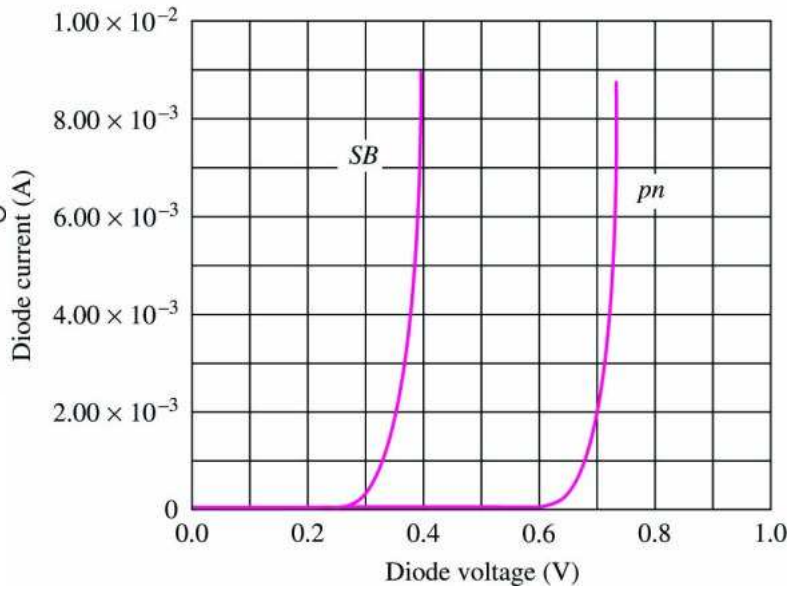
Additional diffusion capacitance, associated with forward region operation is proportional to current and becomes quite large at high currents.

$$C_j = \frac{dQ_D}{dv_D} = \frac{\left(i_D + I_S\right)\tau_T}{V_T} \cong \frac{i_D \tau_T}{V_T} \text{ F}$$

Schottky Barrier Diode

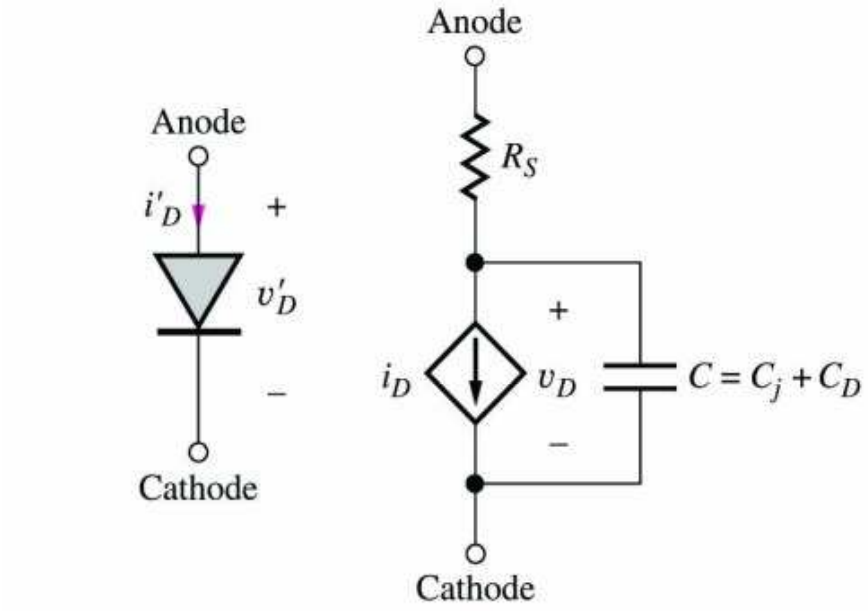


One semiconductor region of the *pn* junction diode can be replaced by a non-ohmic rectifying metal contact. A Schottky contact is easily formed on *n*-type silicon. The metal region becomes the anode. An n^+ region is added to ensure that the cathode contact is ohmic.



Schottky diodes turn on at a lower voltage than *pn* junction diodes and have significantly reduced internal charge storage under forward bias. Thus Schottky diodes switch faster.

Diode Spice Model



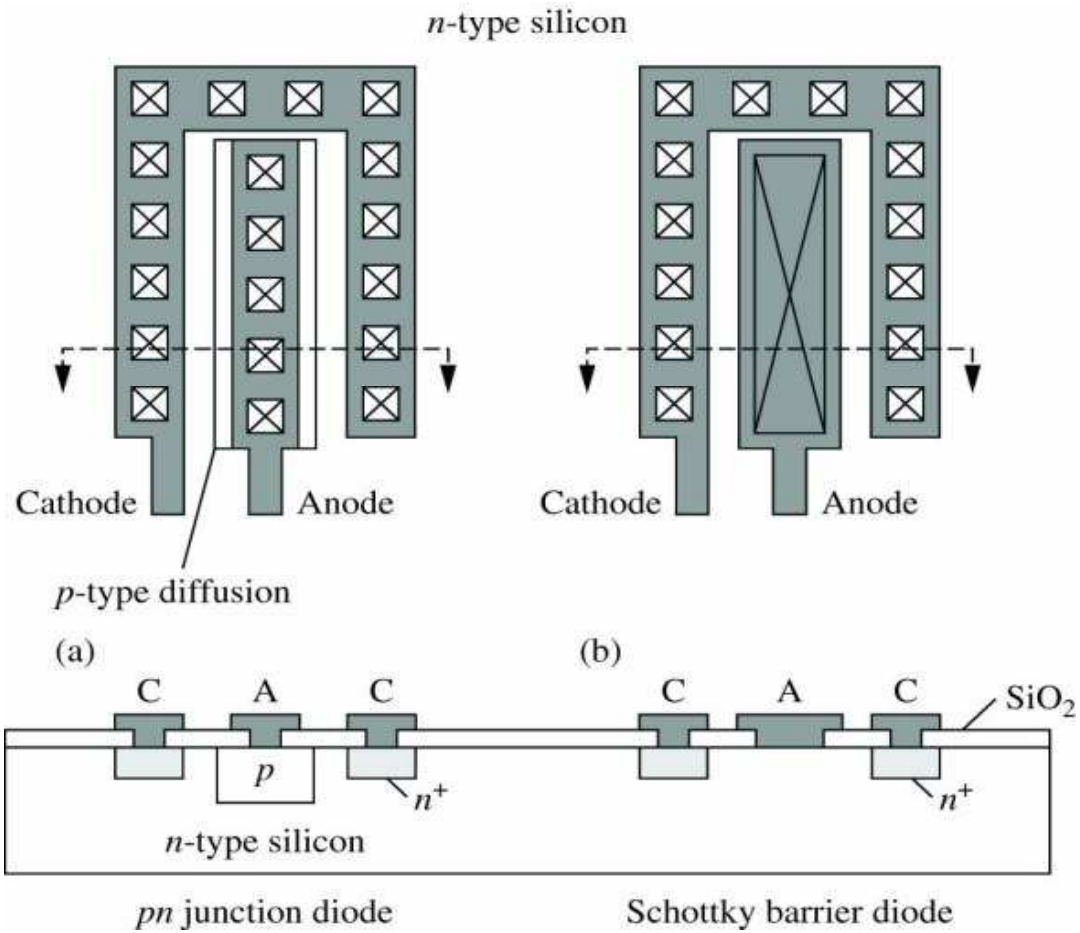
$$i_D = IS\left[\exp\left(\frac{v_D}{NV_T}\right) - 1\right]$$

$$C_D = \mathrm{TT}\frac{i_D}{NV_T} \quad \text{for} \quad v_D \geq 0 \qquad C_j = \frac{CJO}{\left(1 - \frac{v_D}{VJ}\right)^M} RAREA \quad \text{for} \quad v_D \leq 0$$

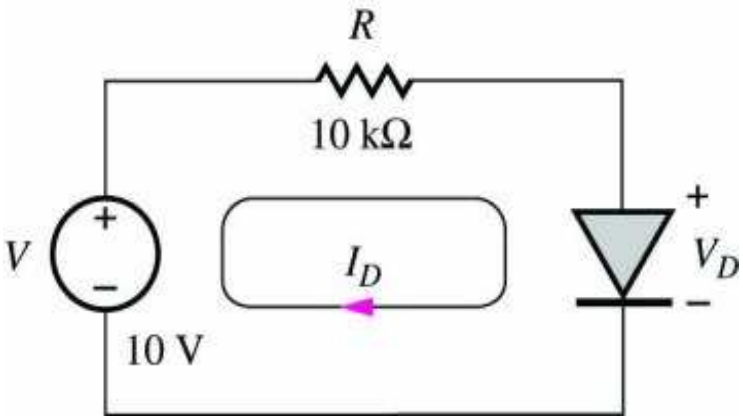
R_s represents the inevitable series resistance of a real device structure. The current controlled current source models the ideal exponential behavior of the diode. Capacitor C includes depletion-layer capacitance for the reverse-bias region as well as diffusion capacitance associated with the junction under forward bias.

Typical default values: Saturation current IS = 10 fA, R_s = 0 Ω, transit time TT = 0 seconds, N = 1

Basic Diode Layout



Diode dc Circuit Analysis: Basics



V and R may represent the Thévenin equivalent of a more complex 2-terminal network. The objective of diode dc circuit analysis is to find the **quiescent operating point** for the diode.

Q-Point = (I_D, V_D)

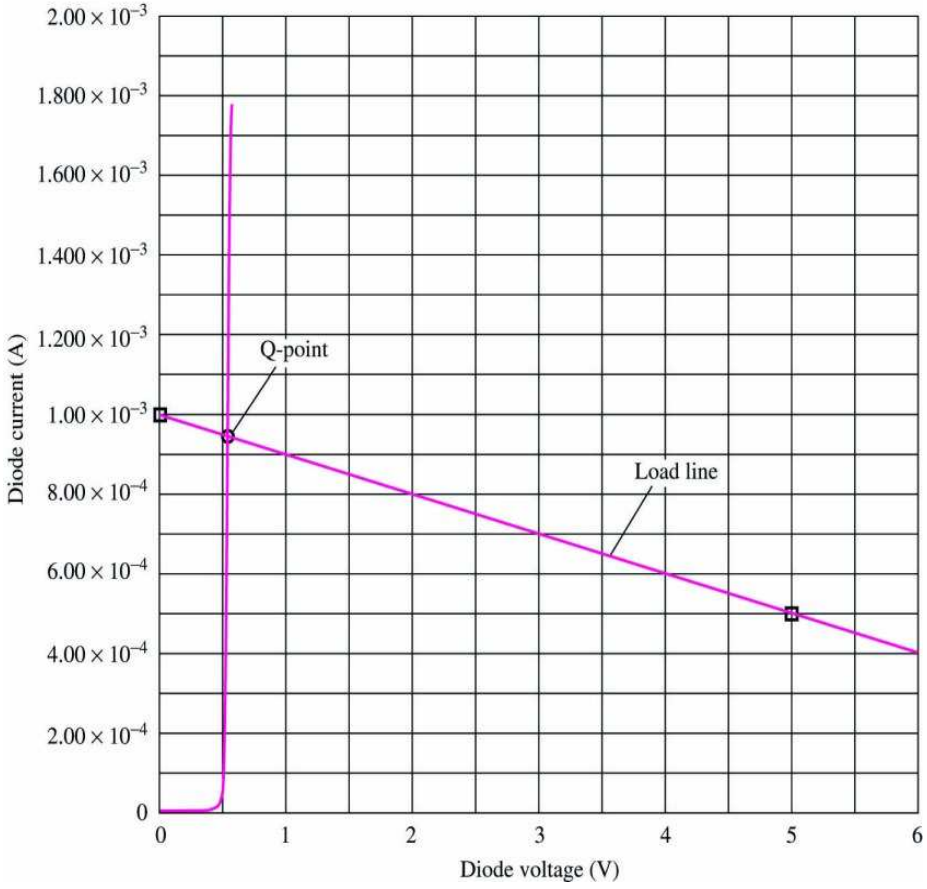
The loop equation for the diode circuit is:

$$V = I_D R + V_D$$

This is also called the **load line** for the diode. The solution to this equation can be found by:

- Graphical analysis using the load-line method.
- Analysis with the diode's mathematical model.
- Simplified analysis with the ideal diode model.
- Simplified analysis using the constant voltage drop (CVD) model.

EXAMPLE
Load-Line Analysis



Problem: Find diode Q-point

Given data: $V = 10$ V, $R = 10$ kΩ.

Analysis:

$$10 = I_D 10^4 + V_D$$

To define the load line we use,

For $V_D = 0$, $I_D = (10V/10k\Omega) = 1\ mA$

For $V_D = 5V$, $I_D = (5V/10k\Omega) = 0.5\ mA$

These points and the resulting load line are plotted. Q-point is given by the intersection of the load line and diode characteristic:

Q-point = (0.95 mA, 0.6 V)

EXAMPLE
Analysis using Diode Mathematical Model

Problem: Find the Q-point for a given diode characteristic.

Given data: I_S = 10^{-13}A, n = 1, V_T = 0.025V

Analysis:

$$I_D = I_S\left[\exp\left(\frac{V_D}{nV_T}\right)-1\right]=10^{-13}\left[\exp(40V_D)-1\right]$$

$$\therefore 10 = 10^{4}10^{-13}\left[\exp(40V_D)-1\right]+V_D$$

The solution is given by a transcendental equation. A numerical answer can be found by using Newton's iterative method.

$$f = 10-10^{4}10^{-13}\left[\exp(40V_D)-1\right]-V_D$$

- Make initial guess $V_D{}^0$.
- Evaluate f and its derivative f' for this value of V_D.
- Calculate new guess for V_D using

$$V_D^1 = V_D^0 - \frac{f\left(V_D{}^0\right)}{f'(V_D{}^0)}$$

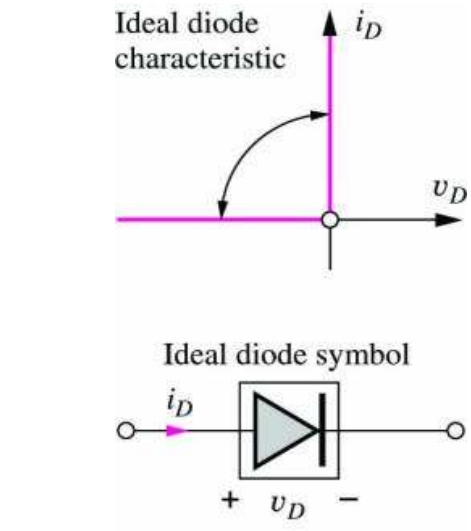
- Repeat steps 2 and 3 till convergence.

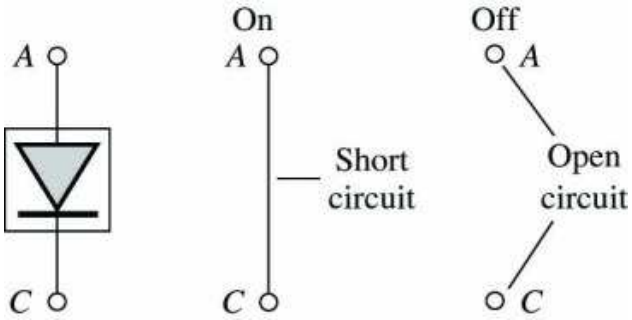
Using a spreadsheet we get :

Q-point = (0.9426 mA, 0.5742 V)

Since, usually we don't have accurate saturation current values and significant tolerances exist for sources and passive components, we need answers precise to only 2 or 3 significant digits.

Analysis using Ideal Model for Diode





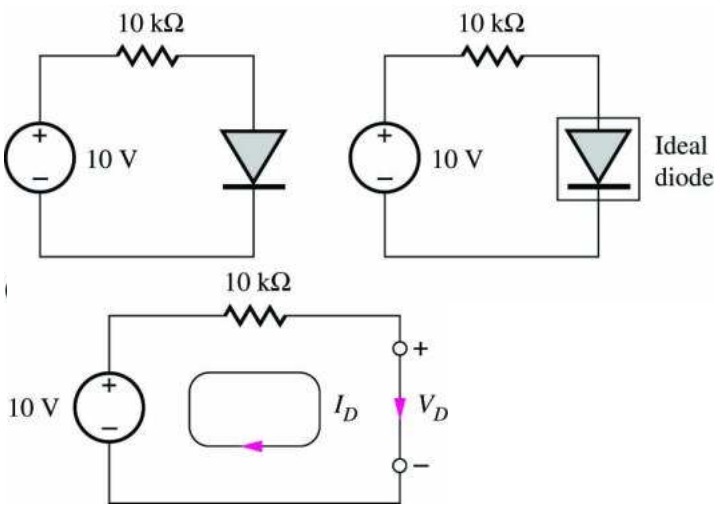
If an ideal diode is forward-biased, the voltage across the diode is zero. If an ideal diode is reverse-biased, the current through the diode is zero.

$$v_D = 0 \text{ for } i_D > 0 \text{ and } i_D = 0 \text{ for } v_D < 0$$

Thus, the diode is assumed to be either on or off. Analysis is conducted in following steps:

- Select a diode model.
- Identify anode and cathode of the diode and label v_D and i_D.
- Guess diode's region of operation from circuit.
- Analyze circuit using diode model appropriate for assumed region of operation.
- Check results to check consistency with assumptions.

EXAMPLE
Analysis using Ideal Model for Diode

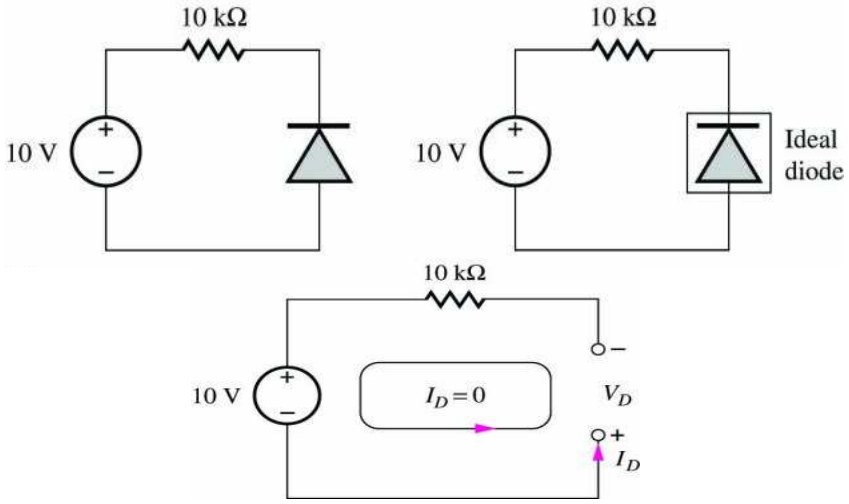


Since source appears to force positive current through diode, assume diode is on.

$$I_D = \frac{(10-0)\text{V}}{10\text{k}\Omega} = 1\ \text{m}A \quad | \quad I_D \geq 0$$

Our assumption is correct, and the

Q-Point = (1 mA, 0V)



Since source is forcing current backward through diode assume, diode is off. Hence $I_D = 0$. Loop equation is:

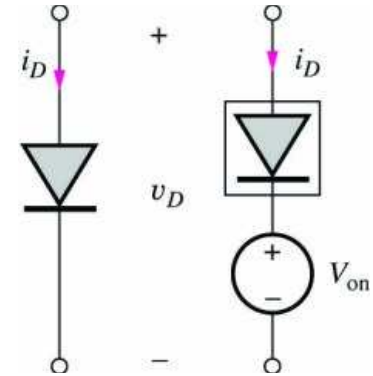
$$10 + V_D + 10^4 I_D = 0$$

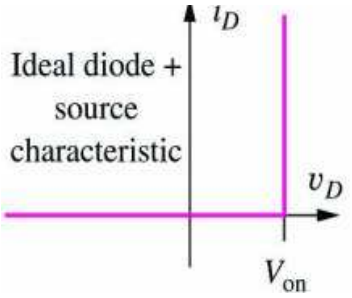
$$V_D = -10V \quad | \quad V_D < 0$$

Our assumption is correct and the

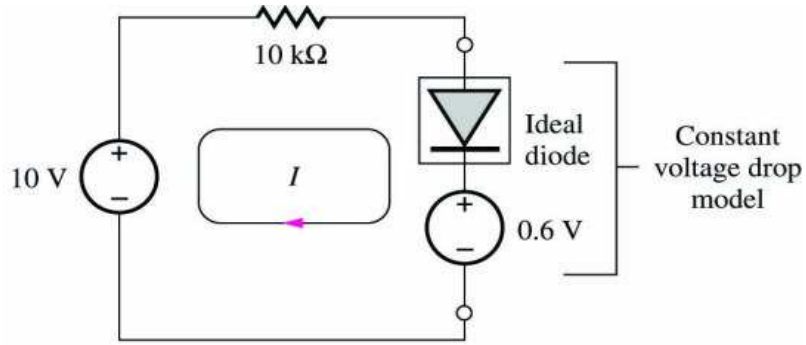
Q-Point = (0, -10 V)

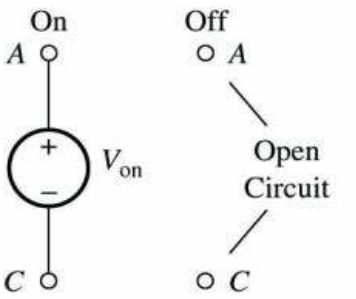
Analysis using Constant Voltage Drop Model for Diode





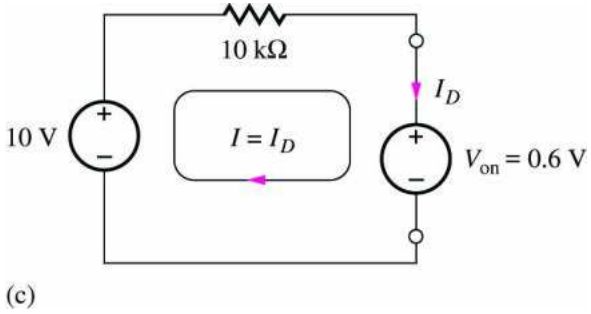
Analysis:





Since the 10-V source appears to force positive current through the diode, assume diode is on.

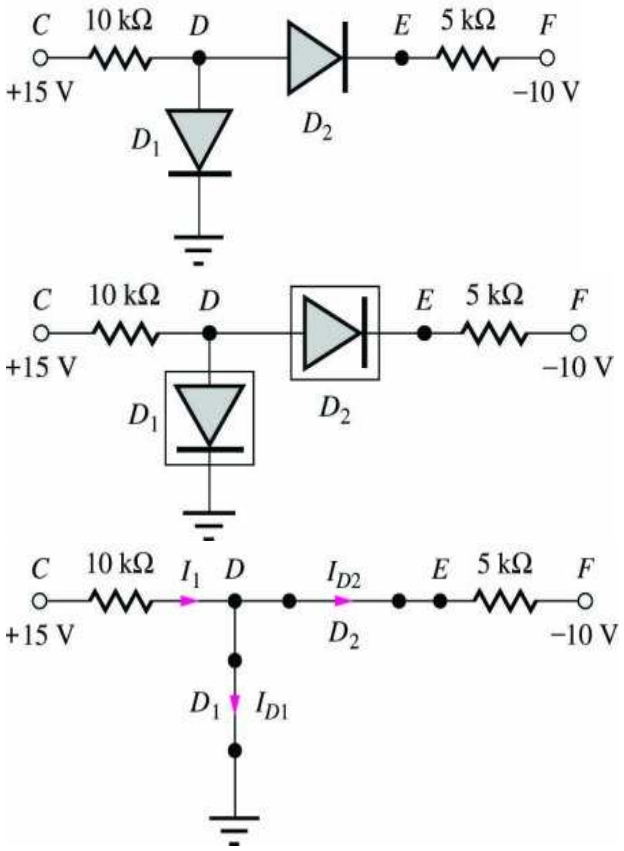
$v_D = V_{on}$ for $i_D > 0$ and
$v_D = 0$ for $v_D < V_{on}$.



$$I_D = \frac{(10 - V_{on})\text{V}}{10\text{k}\Omega}$$

$$= \frac{(10 - 0.6)\text{V}}{10\text{k}\Omega} = 0.940 \text{ mA}$$

EXAMPLE
Two-Diode Circuit Analysis



Analysis: The ideal diode model is chosen. Since the 15-V source appears to force positive current through D_1 and D_2, and the -10-V source is forcing positive current through D_2, assume both diodes are on.

Since the voltage at node D is zero due to the short circuit of ideal diode D_1,

$$I_1 = \frac{(15-0)V}{10k\Omega} = 1.50\ mA$$

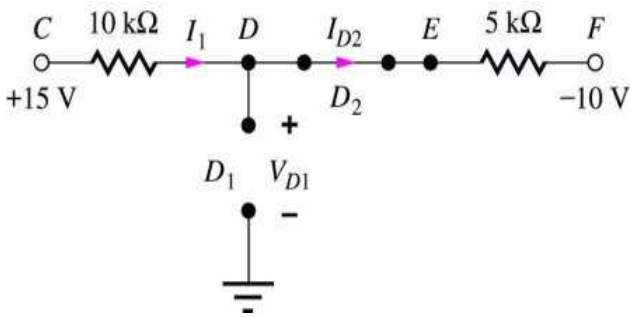
$$I_{D2} = \frac{0-(-10V)}{5k\Omega} = 2.00\ mA$$

$$I_1 = I_{D1} + I_{D2} \quad | \quad I_{D1} = 1.50 - 2.00 = -0.500\ mA$$

The Q-points are (-0.5 mA, 0 V) and (2.0 mA, 0 V)

But, $I_{D1} < 0$ is not allowed by the diode, so try again.

EXAMPLE
Two-Diode Circuit Analysis (cont.)



Analysis: Since current in D_2 is valid, but that in D_1 is not, the second guess is D_1 off and D_2 on.

Since the current in D_1 is zero, $I_{D2} = I_1$,

$$15-10{,}000I_1-5{,}000I_{D2}-(-10)=0$$

$$I_1=\frac{25V}{15{,}000\Omega}=1.67\ mA$$

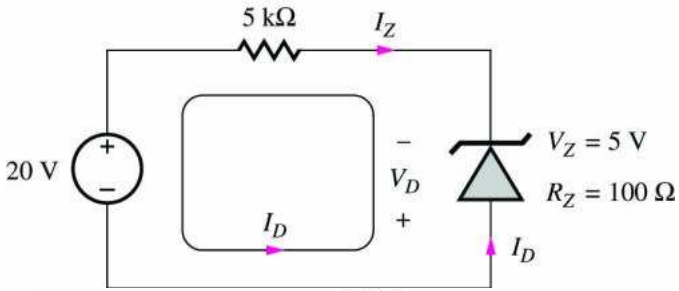
$$V_{D1}=15-10{,}000I_1=15-16.7=-1.67\ V$$

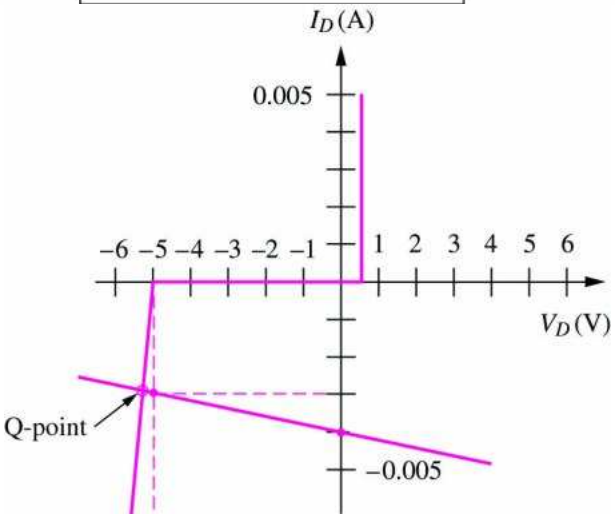
Q-Points are D_1 : (0 mA, -1.67 V): off

D_2 : (1.67 mA, 0 V) : on

Now, the results are consistent with the assumptions.

EXMPLE
Analysis of Diodes in Reverse Breakdown



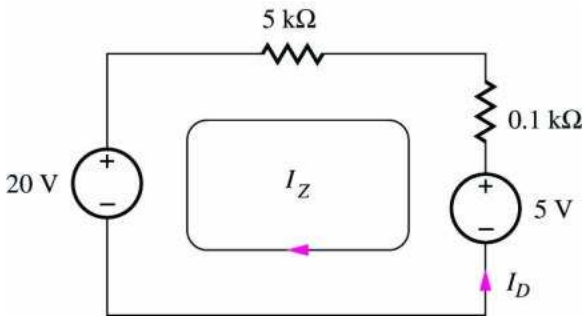


Using load-line analysis:

$$-20 = V_D + 5000 I_D$$

Choose 2 points (0 V, -4 mA) and (-5 V, -3 mA) to draw the load line. It intersects the *i-v* characteristic at the Q-point: (-2.9 mA, -5.2 V).

Using the piecewise linear model:



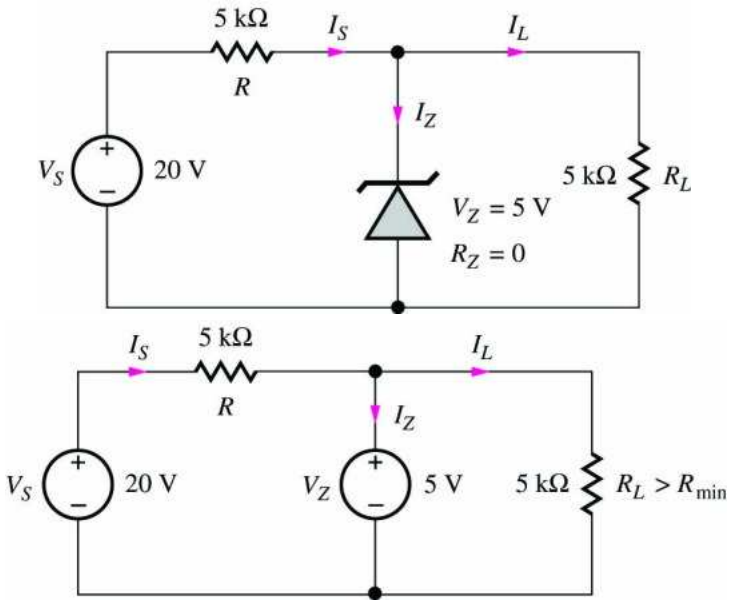
$$I_Z = -I_D > 0$$

$$20 - 5100 I_Z - 5 = 0$$

$$I_Z = \frac{(20-5)\text{V}}{5100\Omega} = 2.94\ \text{mA}$$

Since $I_Z > 0$ ($I_D < 0$), the solution is consistent with Zener breakdown.

EXAMPLE
Voltage Regulator Using the Zener Diode



The Zener diode keeps the voltage across load resistor R_L constant. For Zener breakdown operation, $I_Z > 0$.

$$I_S = \frac{V_S - V_Z}{R} = \frac{(20-5)\text{V}}{5\text{k}\Omega} = 3\text{ mA}$$

$$I_L = \frac{V_Z}{R_L} = \frac{5\text{V}}{5\text{k}\Omega} = 1\ mA \quad | \quad I_Z = I_S - I_L = 2\ mA$$

For proper regulation, Zener current I_Z must be positive. If the Zener current < 0, the Zener diode no longer controls the voltage across the load resistor and the voltage regulator is said to have "dropped out of regulation".

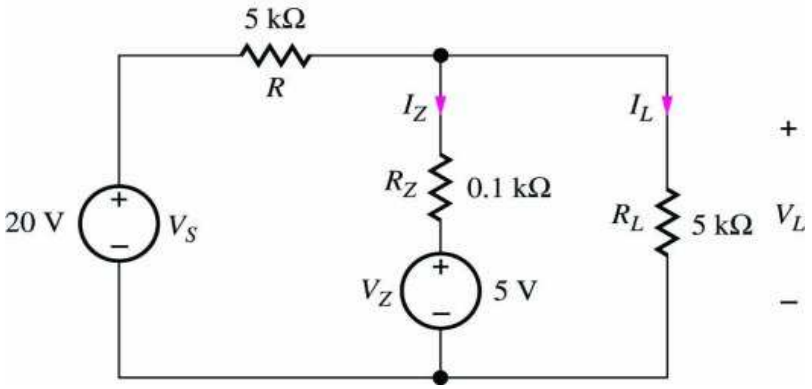
$$I_Z = \frac{V_S}{R} - V_Z\left(\frac{1}{R} + \frac{1}{R_L}\right) > 0 \quad | \quad R_L > \frac{R}{\left(\frac{V_S}{V_Z} - 1\right)} = R_{\text{min}}$$

EXAMPLE
Voltage Regulator Including Zener Resistance

Problem: Find the output voltage and Zener diode current for a Zener diode regulator.

Given data: $V_S = 20$ V, $R = 5$ kΩ, $R_Z = 0.1$ kΩ, $V_Z = 5$ V

Analysis: The output voltage is now a function of the current through the Zener diode.



$$\frac{V_Z - 20\text{V}}{5000\Omega} + \frac{V_L - 5\text{V}}{100\Omega} + \frac{V_L}{5000\Omega} = 0$$

$$V_L = 5.19 \text{ V}$$

$$I_Z = \frac{V_L - 5\text{V}}{100\Omega} = \frac{5.19\text{V} - 5\text{V}}{100\Omega} = 1.9 \text{ mA} > 0$$

Line and Load Regulation

Line regulation characterizes how sensitive the output voltage is to input voltage changes.

$$\text{Line Regulation} = \frac{dV_L}{dV_S} \quad \text{mV/V}$$

$$\text{For a fixed load current, Line Regulation} = \frac{R_Z}{R + R_Z}$$

Load regulation characterizes how sensitive the output voltage is to changes in load current withdrawn from regulator.

$$\text{Load Regulation} = \frac{dV_L}{dI_L} \quad \Omega$$

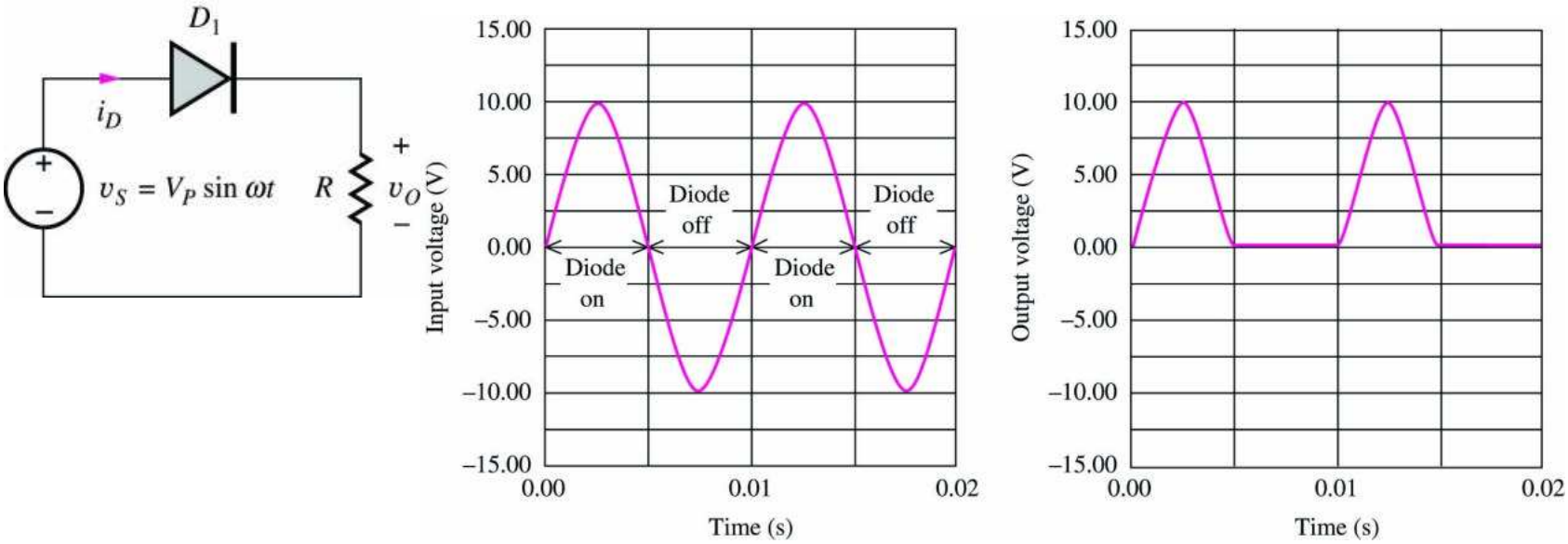
$$\text{For changes in load current, Load Regulation} = -\left(R_Z \| R\right)$$

Load regulation is the Thévenin equivalent resistance looking back into the regulator from the load terminals.

Rectifier Circuits

- A basic rectifier converts an ac voltage to a pulsating dc voltage.
- A filter then eliminates ac components of the waveform to produce a nearly constant dc voltage output.
- Rectifier circuits are used in virtually all electronic devices to convert the 120-V 60-Hz ac power line source to the dc voltages required for operation of electronic devices.
- In rectifier circuits, the diode state changes with time and a given piecewise linear model is valid only for a certain time interval.

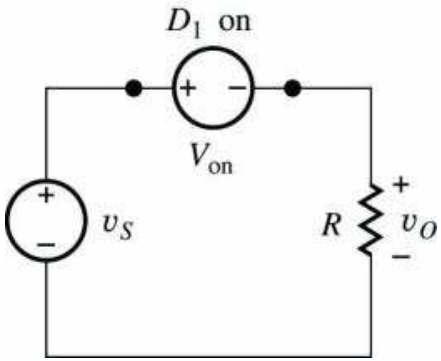
Half-Wave Rectifier Circuit with Resistive Load

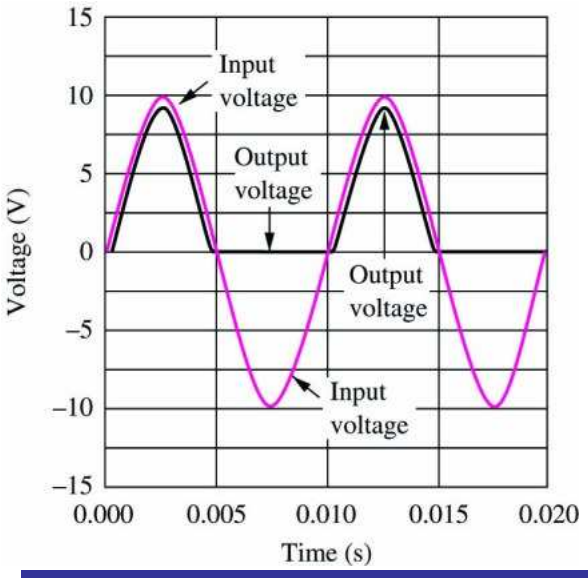


For the positive half-cycle of the input, the source forces positive current through the diode, the diode is on, and $v_O = v_S$ (for an ideal diode).

During the negative half cycle, negative current can't exist in the diode. The diode is off, current in resistor is zero, and $v_O = 0$.

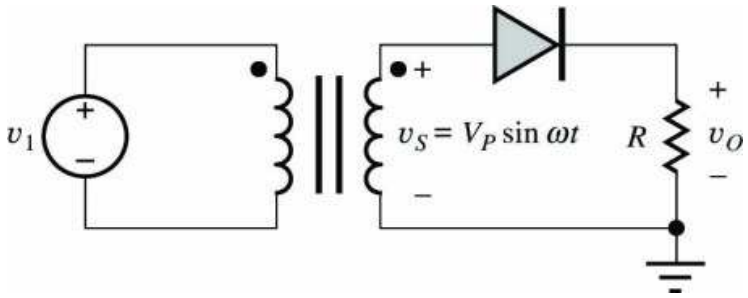
Half-Wave Rectifier Circuit with Resistive Load (cont.)





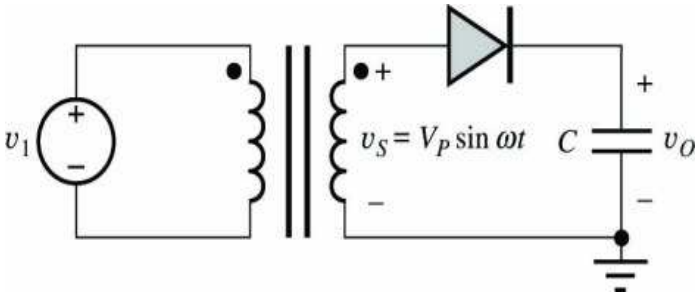
Using the CVD model, during the on-state of the diode $v_O = v_S - V_{on} = (V_P \sin\omega t) - V_{on}$. The output voltage is zero when the diode is off.

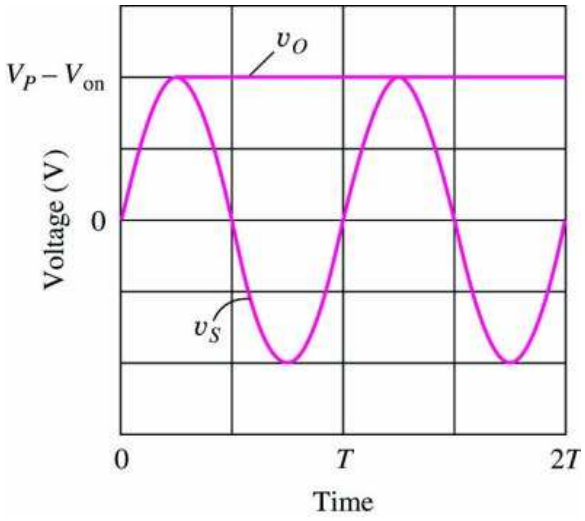
Often a step-up or step-down transformer is used to convert the 120-V, 60-Hz voltage available from the power line to the desired ac voltage level as shown.



Time-varying components in the rectifier output are removed using a filter capacitor.

Peak Detector Circuit



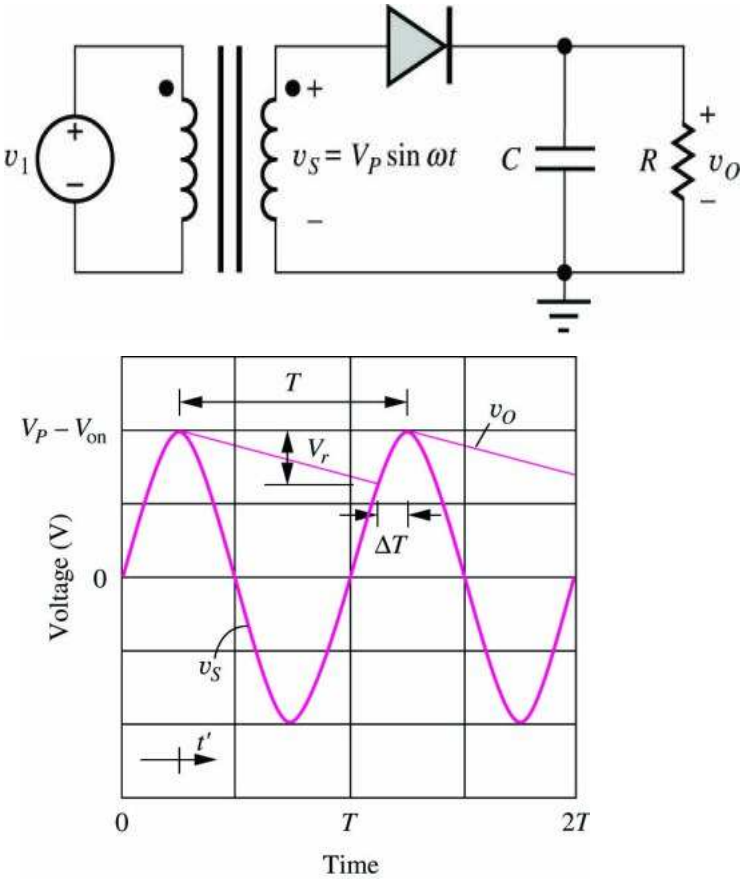


As the input voltage rises, the diode is on, and the capacitor (initially discharged) charges up to the input voltage minus the diode voltage drop.

At the peak of the input voltage, the diode current tries to reverse, and the diode cuts off. The capacitor has no discharge path and retains a constant voltage providing a constant output voltage:

$$V_{dc} = V_P - V_{on}$$

Half-Wave Rectifier Circuit with *RC* Load



As the input voltage rises during the first quarter cycle, the diode is on and the capacitor (initially discharged) charges up to the peak value of the input voltage.

At the peak of the input, the diode current tries to reverse, the diode cuts off, and the capacitor discharges exponentially through *R*. Discharge continues till the input voltage exceeds the output voltage which occurs near the peak of next cycle. This process then repeats once every cycle.

This circuit can be used to generate negative output voltage if the top plate of capacitor is grounded instead of bottom plate. In this case, $V_{dc} = -(V_P - V_{on})$

Half-Wave Rectifier Circuit with *RC* Load (cont.)

The output voltage is not constant as in an ideal peak detector, but has a **ripple voltage** V_r.

The diode conducts for a short time ΔT called the **conduction interval** during each cycle, and its angular equivalent is called the **conduction angle** θ_c.

$$V_r \cong (V_P - V_{on})\frac{T}{RC}\left(1 - \frac{\Delta T}{T}\right) \cong \frac{(V_P - V_{on})}{R}\frac{T}{C}$$

$$\Delta T \cong \frac{1}{\omega}\sqrt{\frac{2T}{RC}\frac{(V_P - V_{on})}{V_P}} = \frac{1}{\omega}\sqrt{\frac{2V_r}{V_P}}$$

$$\theta_c = \omega \Delta T = \sqrt{\frac{2V_r}{V_P}}$$

EXAMPLE
Half-Wave Rectifier Analysis

Problem: Find the dc output voltage, output current, ripple voltage, conduction interval, and conduction angle for a half-wave rectifier.
Given data: secondary voltage $V_{r\text{ms}}$ = 12.6 (60 Hz), R = 15 Ω, C = 25,000 μF, V_{on} = 1 V
Analysis:

$$V_{dc} = V_P - V_{on} = (12.6\sqrt{2} - 1)V = 16.8\ V$$

$$I_{dc} = \frac{V_P - V_{on}}{R} = \frac{16.8V}{15\Omega} = 1.12\ A$$

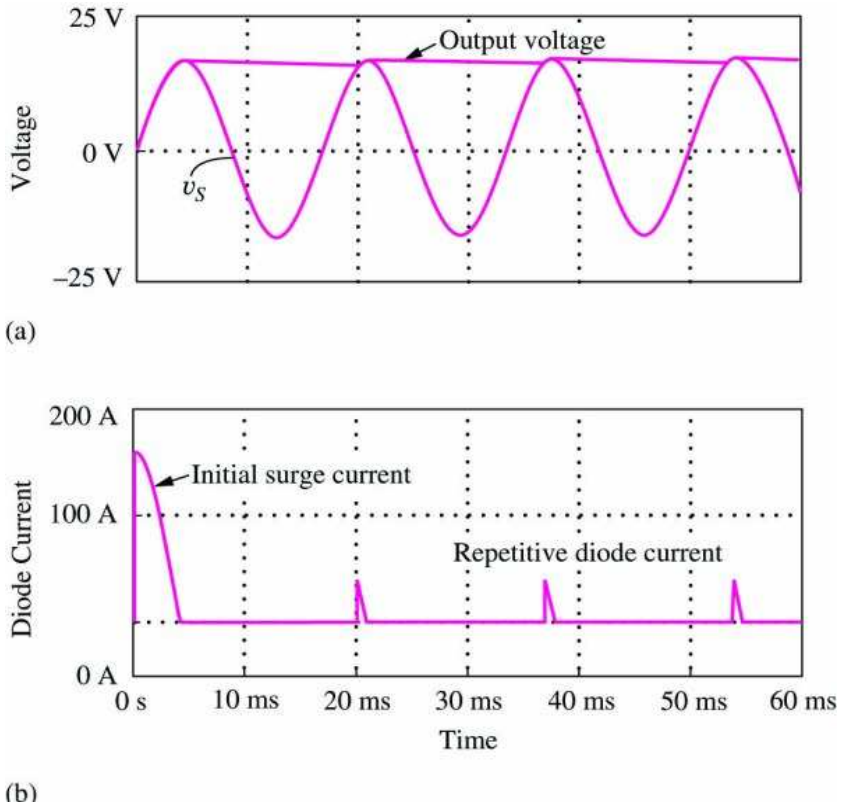
$$V_r \cong \frac{(V_P - V_{on})}{R}\frac{T}{C} = 0.747\ V$$

Using discharge interval T = 1/60 s,

$$\theta_c = \omega\Delta T = \sqrt{\frac{2V_r}{V_P}} = 0.290 \text{ rad} = 16.6^o$$

$$\Delta T = \frac{\theta_c}{\omega} = \frac{\theta_c}{2\pi f} = \frac{0.29}{120\pi} = 0.769 \text{ ms}$$

Peak Diode Current



In rectifiers, nonzero current exists in the diode for only a very small fraction of period T, yet an almost constant dc current flows out of the filter capacitor to load.

The total charge lost from the filter capacitor in each cycle is replenished by the diode during a short conduction interval causing high peak diode currents. If the repetitive current pulse is modeled as a triangle of height I_P and width ΔT,

$$I_P = I_{dc}\frac{2T}{\Delta T} = 48.6 \text{ A}$$

using the values from the previous example.

Surge Current

In addition to the peak diode currents, there is an even larger current through the diode called the **surge current** that occurs when power is first turned on.

During first quarter cycle, current through diode is approximately

$$i_d(t) = i_c(t) \cong C\left(\frac{d}{dt}V_P \sin\omega t\right) = \omega C V_P \cos\omega t$$

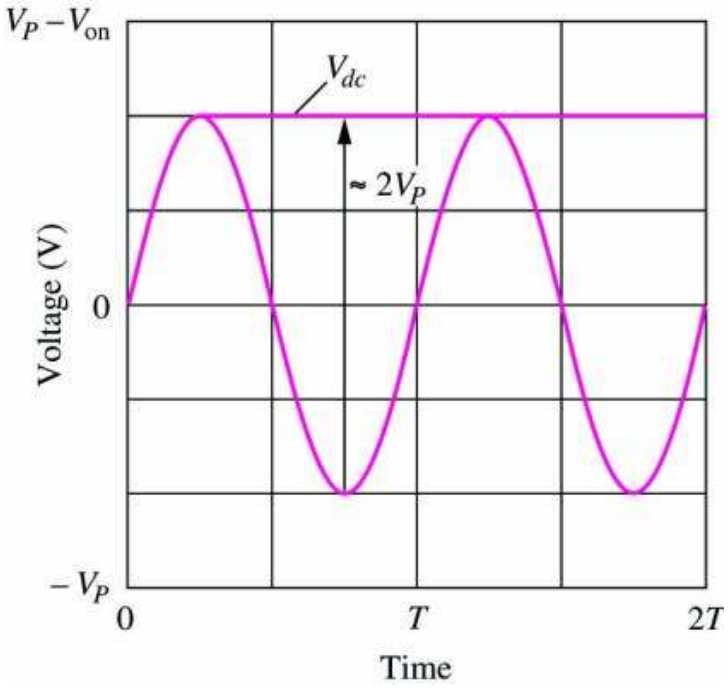
The peak value of this initial surge current occurs at $t = 0^+$:

$$I_{SC} = \omega C V_P = 168 \text{ A}$$

using values from previous example.

Actual values of surge current won't be nearly as large as predicted above because of the neglected series resistances associated with both the rectifier diode and transformer.

Peak Inverse Voltage Rating



The peak inverse voltage (PIV) rating of the rectifier diode is the diode breakdown voltage.

When the diode is off, the reverse-bias across the diode is V_{dc} - v_S. When v_S reaches its negative peak,

$$\text{PIV} \geq V_{dc} - v_s^{\min} = V_P - V_{on} - (-V_P) \cong 2V_P$$

The PIV value corresponds to the minimum value of Zener breakdown voltage required for the rectifier diode.

Diode Power Dissipation

Average power dissipation in a diode is given by

$$P_D = \frac{1}{T}\int_0^T v_D(t) i_D(t) dt = \frac{1}{T}\int_0^T V_{on} i_D(t) dt = V_{on}\frac{I_P}{2}\frac{\Delta T}{T} \cong V_{on} I_{dc}$$

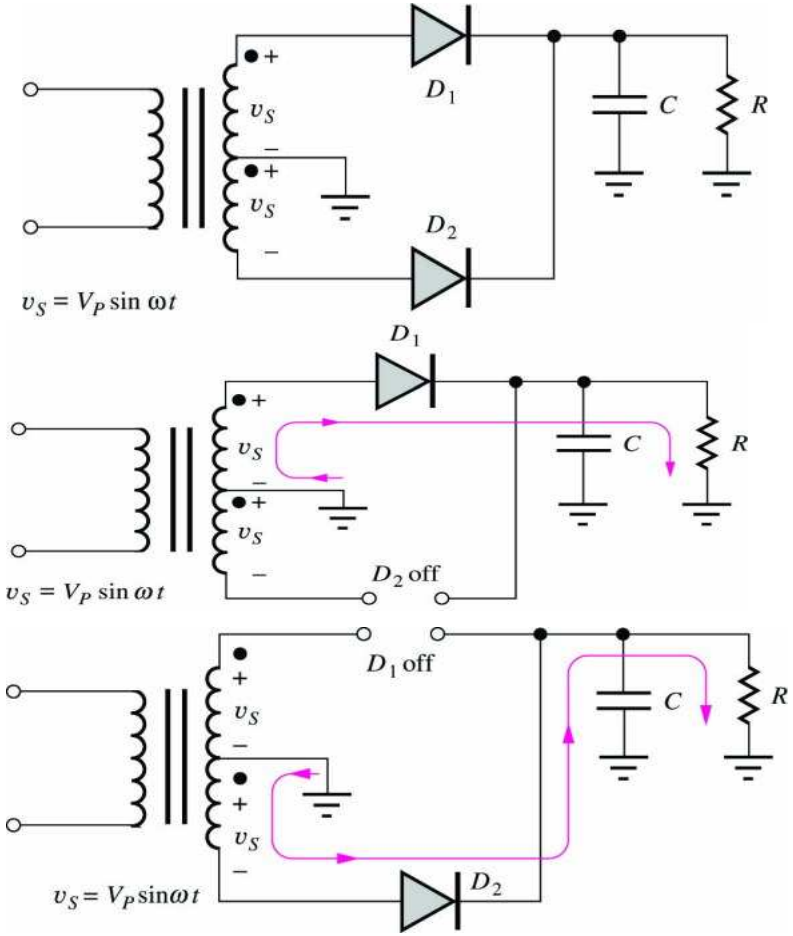
The simplification is done by assuming a triangular approximation for the diode current and the voltage across the diode is constant at V_{dc}.

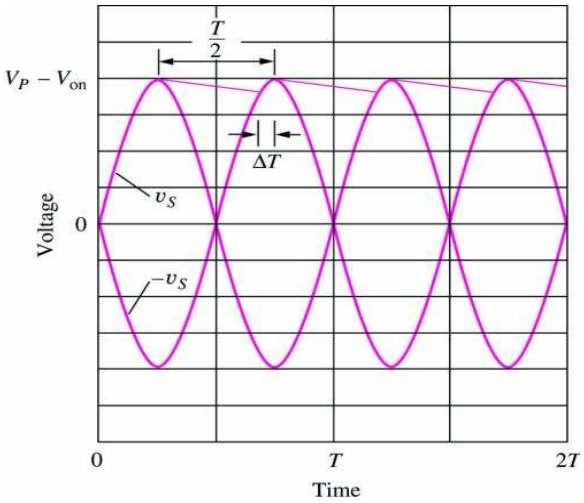
Average power dissipation in the diode series resistance is given by

$$P_D = \frac{1}{T}\int_0^T i_D^2(t) R_S dt = \frac{1}{3} I_D^2 R_S \frac{\Delta T}{T} = \frac{4}{3}\frac{T}{\Delta T} I_{dc}^2 R_S$$

This power dissipation can be reduced by minimizing peak current through the use of a minimum size of filter capacitor or by using full-wave rectifiers.

Full-Wave Rectifiers

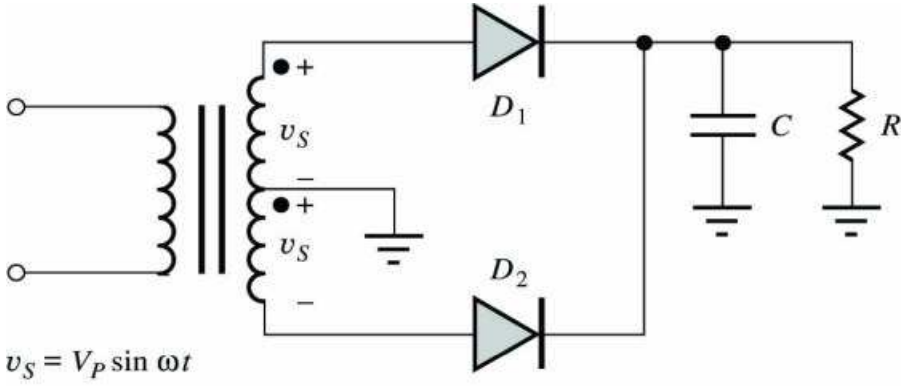


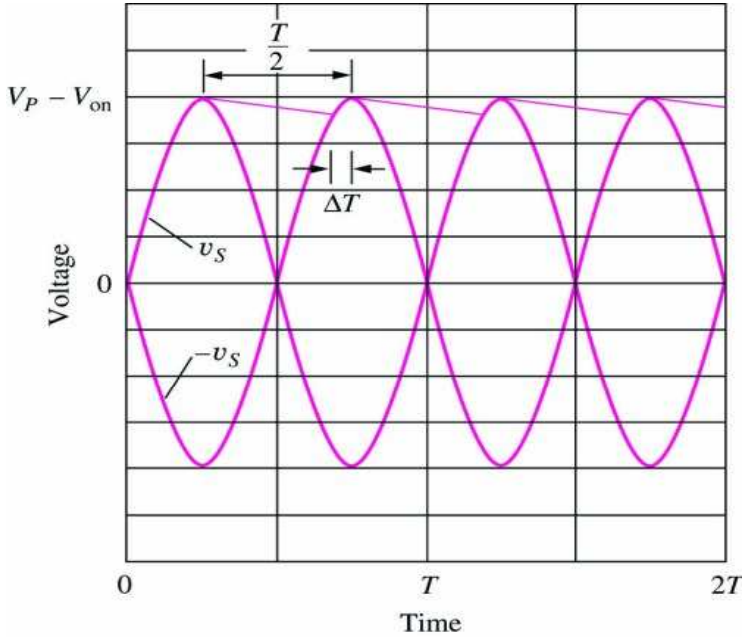


Full-wave rectifiers cut capacitor discharge time in half and require half the filter capacitance to achieve a given ripple voltage. All specifications are the same as for half-wave rectifiers.

Reversing polarity of the diodes gives a full-wave rectifier with negative output voltage.

Full-Wave Rectifier Equations





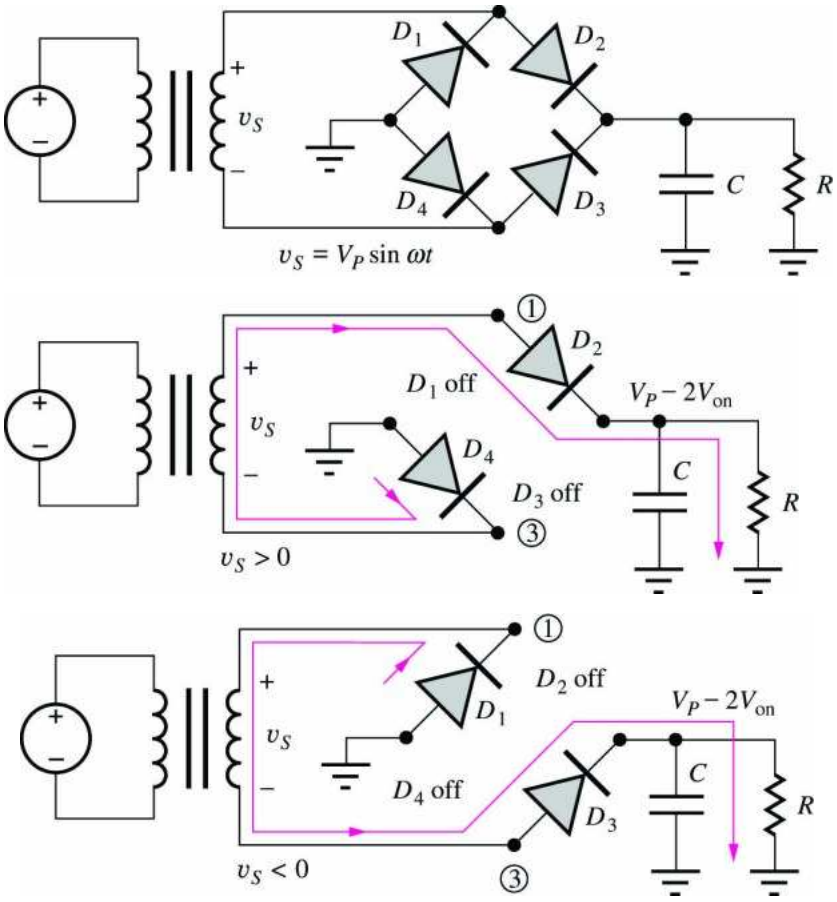
$$V_{dc} \cong V_P - V_{on} \qquad V_r \cong \frac{(V_P - V_{on})}{R}\frac{T}{2C}$$

$$\Delta T \cong \frac{1}{\omega}\sqrt{\frac{T}{RC}\frac{(V_P - V_{on})}{V_P}} = \frac{1}{\omega}\sqrt{\frac{2V_r}{V_P}}$$

$$\theta_c = \omega \Delta T = \sqrt{\frac{2V_r}{V_P}} \qquad I_P = I_{dc}\frac{T}{\Delta T}$$

$$PIV = 2V_P$$

Full-Wave Bridge Rectification



The requirement for a center-tapped transformer in the full-wave rectifier is eliminated through use of 2 extra diodes. All other specifications are the same as for a half-wave rectifier except PIV = V_P.

Rectifier Topology Comparison

- Filter capacitors are a major factor in determining cost, size and weight in design of rectifiers.
- For a given ripple voltage, a full-wave rectifier requires half the filter capacitance as that in a half-wave rectifier. Reduced peak current can reduce heat dissipation in diodes. Benefits of full-wave rectification outweigh increased expenses and circuit complexity (an extra diode and center-tapped transformer).
- The bridge rectifier eliminates the center-tapped transformer, and the PIV rating of the diodes is reduced. Cost of extra diodes is negligible.

Rectifier Topology Comparison and Design Tradeoffs

Rectifier Parameter	Half-Wave Rectifier	Full-Wave Rectifier	Full-Wave Bridge Rectifier
Filter Capacitor	$C = \frac{V_P - V_{on}}{V_r} \frac{T}{R}$	$C = \frac{V_P - V_{on}}{V_r} \frac{T}{2R}$	$C = \frac{V_P - V_{on}}{V_r} \frac{T}{2R}$
PIV Rating	$2V_P$	$2V_P$	V_P
Peak Diode Current (Constant V_r)	Highest I_P	Reduced $I_P/2$	Reduced $I_P/2$
Surge Current	Highest	Reduced (αC)	Reduced (αC)
Comments	Least Complexity	Smaller Capacitor Center-taped Transformer Two Diodes	Smaller Capacitor No Center-taped Transformer Four Diodes

DESIGN EXAMPLE
Rectifier Design Analysis

Problem: Design a rectifier with given specifications.

Given data: $V_{\text{dc}} = 15$ V, $V_{\text{r}} < 0.15$ V, $I_{\text{dc}} = 2$ A **Assume:** $V_{on} = 1$ V.

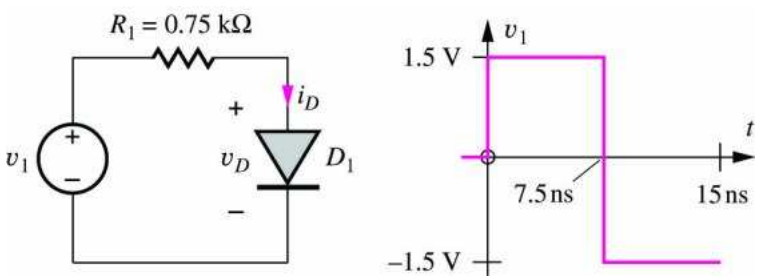
Analysis: Use a full-wave bridge rectifier that needs a smaller value of filter capacitance, smaller diode PIV rating, and no center-tapped transformer.

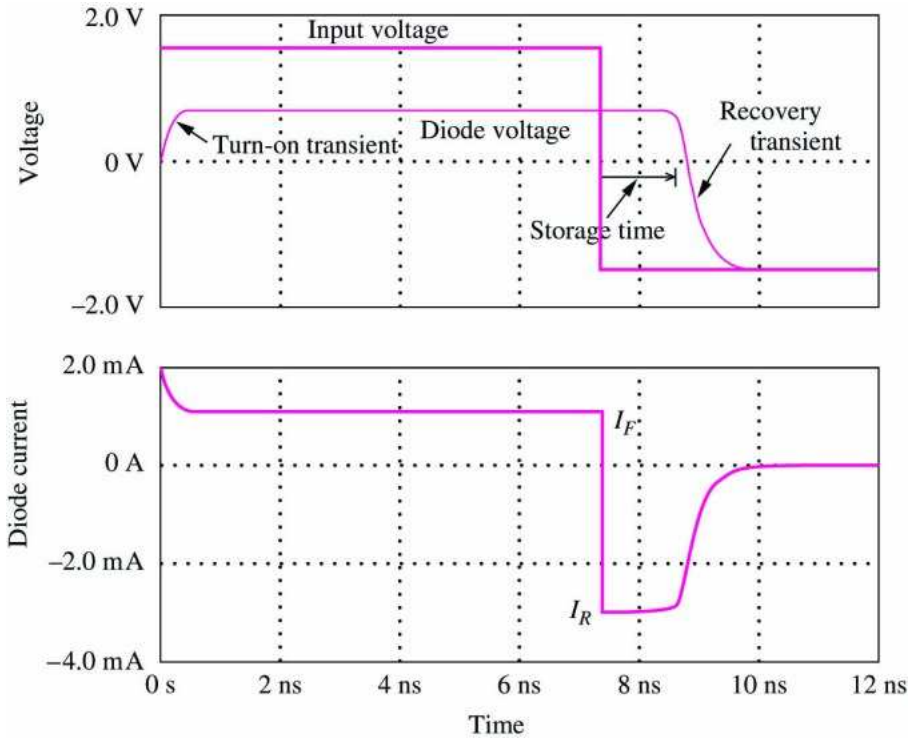
$$V = \frac{V_P}{\sqrt{2}} = \frac{V_{dc} + 2V_{on}}{\sqrt{2}} = \frac{15+2}{\sqrt{2}} V = 12.0\ V_{rms} \quad | \quad C = I_{dc}\left(\frac{T/2}{V_r}\right) = 2A\left(\frac{1}{120}s\right)\left(\frac{1}{0.15V}\right) = 0.111\,F$$

$$\Delta T = \frac{1}{\omega}\sqrt{\frac{2V_r}{V_P}} = \frac{1}{120\pi}\sqrt{\frac{2(0.15V)}{17V}} = 0.352\ ms \quad | \quad I_P = I_{dc}\left(\frac{2}{\Delta T}\right)\left(\frac{T}{2}\right) = 2A\frac{(1/60)s}{0.352\,ms} = 94.7\ A$$

$$I_{surge} = \omega C V_P = 120\pi(0.111)(17) = 711\ A \quad | \quad PIV = V_P = 17\ V$$

Dynamic Switching Behavior of Diodes

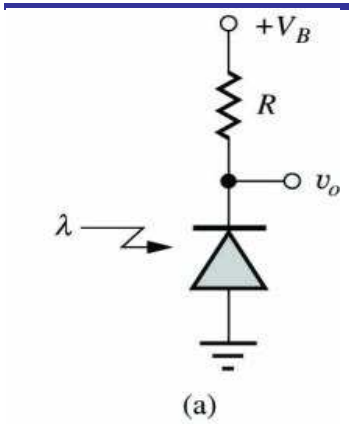




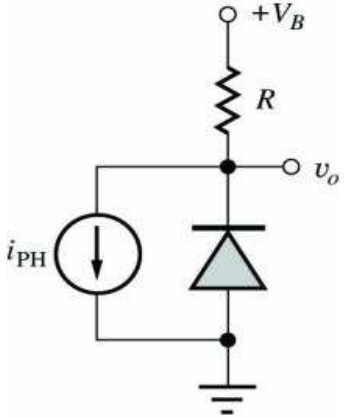
The non-linear depletion-layer capacitance of the diode prevents the diode voltage from changing instantaneously and determines turn-on and recovery times. Both forward and reverse current overshoot the final values when the diode switches on and off as shown. Storage time is given by:

$$\tau_S = \tau_T \ln\left(1 - \frac{I_F}{I_R}\right)$$

Photo Diodes and Photodetectors



(a)



(b)

If the depletion region of a *pn* junction diode is illuminated with light with sufficiently high frequency, photons can provide enough energy to cause electrons to jump the semiconductor bandgap to generate electron-hole pairs:

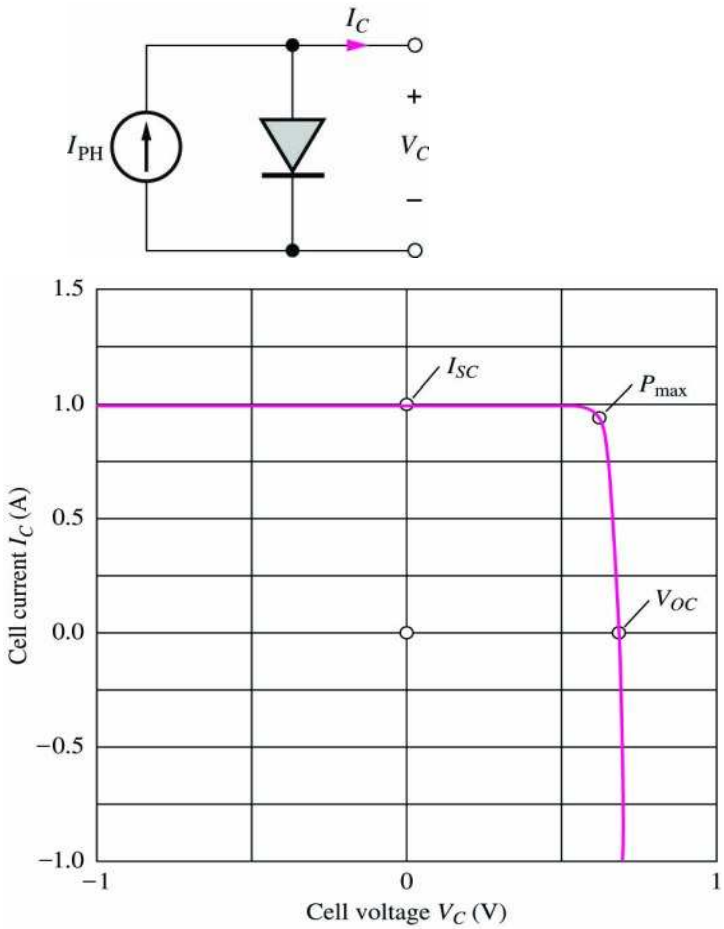
$$E_P = h\upsilon = \frac{hc}{\lambda} \geq E_G$$

h =Planck's constant = 6.626 x 10^{-34} J-s
υ = frequency of optical illumination
λ = wavelength of optical illumination
c = velocity of light = 3 x 10^8 m/s
Photon-generated current can be used in photodetector circuits to generate an output voltage

$$v_O = i_{PH} R$$

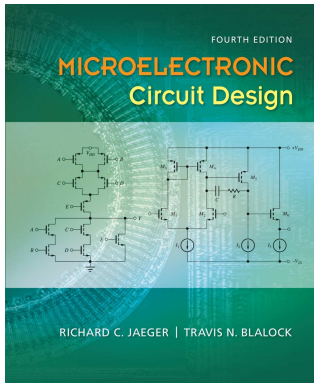
The diode is reverse-biased to enhance depletion-region width and electric field.

Solar Cells and Light-Emitting Diodes



In solar cell applications, optical illumination is steady, and dc current I_{PH} is generated. The goal is to extract power from the cell, and the *i-v* characteristics are plotted in terms of cell current and cell voltage. For a solar cell to supply power to an external circuit, the $I_C V_C$ product must be positive, and the cell should be operated near the point of maximum output power P_{max}.

Light-Emitting Diodes (LEDs) use recombination processes in the forward-biased *pn* junction diode to produce light. When a hole and electron recombine, an energy equal to the bandgap of the semiconductor is released as a photon.



End of Chapter 3