# 3 A SMALL SIGNAL ATTENUATOR

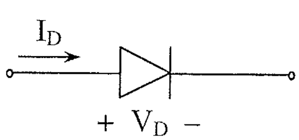
## INSTRUCTIONAL OBJECTIVES

1. Given a diode, you should be able to make DC measurements of the diode current versus voltage over a specified current range.
2. From the collected $I_D$-$V_D$ data, you should be able to use graphical and analytical methods to determine the reverse saturation current ($I_0$) and the ideality factor ($\eta$).
3. Using the ideal diode equation, $I_0$, and $\eta$, you should be able to compute the dynamic resistance ($r_d$) of the given diode for specific values of diode current.
4. You should be able to select the operating points ($V_D$, $I_D$) of a diode in a variable attenuator circuit to obtain various amounts of small signal voltage attenuation and compare theoretical and actual results.
5. You should be able to specify waveform amplitude (small signal) limitations such that the output signal of the attenuator circuit is not observably distorted.
6. You should document your experimental investigation, design and analysis of the results in a clear and concise manner.

## PRELAB

1. The following data has been obtained by measuring diode current vs. voltage using the method given in this experiment.

Table 1. Sample $I_D$ vs. $V_D$ Data




| $V_D$ (volts) | $I_D$ (amps) | rd-theoretical |
|---|---|---|
| 0.312 | $1.30 \times 10^{-6}$ | |
| 0.375 | $7.22 \times 10^{-6}$ | |
| 0.463 | $4.71 \times 10^{-5}$ | |
| 0.515 | $1.89 \times 10^{-4}$ | |
| 0.587 | $1.00 \times 10^{-3}$ | |

(a) Plot the data points ($V_D$, $I_D$) on the included piece of semilog paper. Draw the "best" straight line through the points.

(b) Calculate $\eta V_T$ and $I_0$ using the technique discussed in Section 2.0 of this experiment.

(c) Calculate the values of dynamic resistances, $r_d$, corresponding to the operating points given in Table 1. Enter these values in the table.

2. Using the small signal equivalents of the attenuator circuit as shown in Figure 3(b), derive an expression for $r_d$ in terms of $R_1$, $R_2$, and the attenuation ratio, $v_{out}/v_{in}$.
3. Using the circuit of Figure 3(a) and the diode of Table 1, what value of $V_{cc}$ is required if $R_2 = 3.3K$ and if $V_D = 0.587$ volts?

# 1.0 INTRODUCTION AND DEFINITIONS

In this experiment you will explore the concepts of small signal analysis through the design of a voltage attenuator circuit. Sections 1.1 through 1.3 present the voltage attenuation circuit and describe how a diode may be used to provide variable attenuation. Section 2 provides the theoretical background and equations useful in designing the circuit. Section 3 leads you through the design and testing of your actual attenuator circuit.

### WHAT IS SMALL SIGNAL ANALYSIS?

Small signal analysis is a technique used to analyze non-linear circuits which have reasonably linear operating regions.

Small signal analysis of a device assumes:

- the device is caused to operate at an operating point in a reasonably linear operating region;
- a signal input is applied to the device;
- a signal output results;
- the output signal is linearly related to the input signal.

### HOW LARGE IS A SMALL SIGNAL?

Small signals may be any size -- so long as the output signal is linearly related to the input signal.

### HOW CAN YOU TELL IF THE INPUT AND OUTPUT SIGNALS ARE LINEARLY RELATED?

A good test for linearity is human observation:

- If the input signal closely resembles the output signal, then the signals are proportional and we say that the network is linear for signals of that magnitude. In this case, small signal analysis may be applied effectively.
- If the magnitude of the input signal is increased until the output signal no longer resembles the input signal, we say that the output signal is distorted. In this case, small signal analysis is not effective because a sinewave input of that magnitude does not produce a sinewave output.

## 1.1 THE VOLTAGE DIVIDER AS AN ATTENUATOR

Voltage attenuation is a process whereby the magnitude or amplitude of an electrical signal is decreased. An example of a voltage attenuator is shown in Figure 1 where $R_1$ is fixed and $R_2$ is a variable resistor.

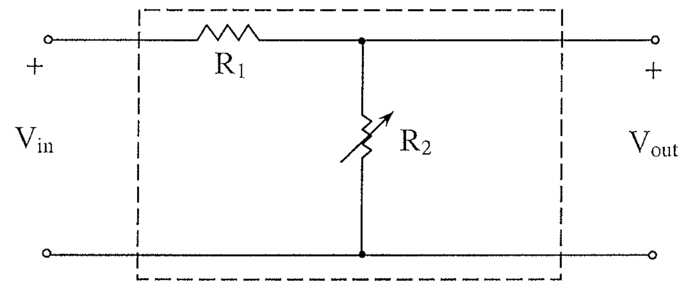



Fig. 1. Simple Voltage Attenuator.

Analyzing the simple voltage divider gives

$$V_{out} = \frac{R_2}{R_1 + R_2} V_{in} \quad (1)$$

Notice that $V_{out}$ is attenuated by different amounts as $R_2$ is varied and that $V_{out}$ can never exceed $V_{in}$.

## 1.2 THE SEMICONDUCTOR DIODE AS A VARIABLE RESISTOR

In many attenuator applications it is desirable to change the amount of attenuation electrically rather than mechanically (i.e. by turning the potentiometer knob). The necessary component to implement this concept is a "resistor" whose value can be changed electrically in a known manner.

One device which can be used as a variable "resistor" is the diode. To see this, consider the diode current vs. voltage (I-V) plots of Figure 2.

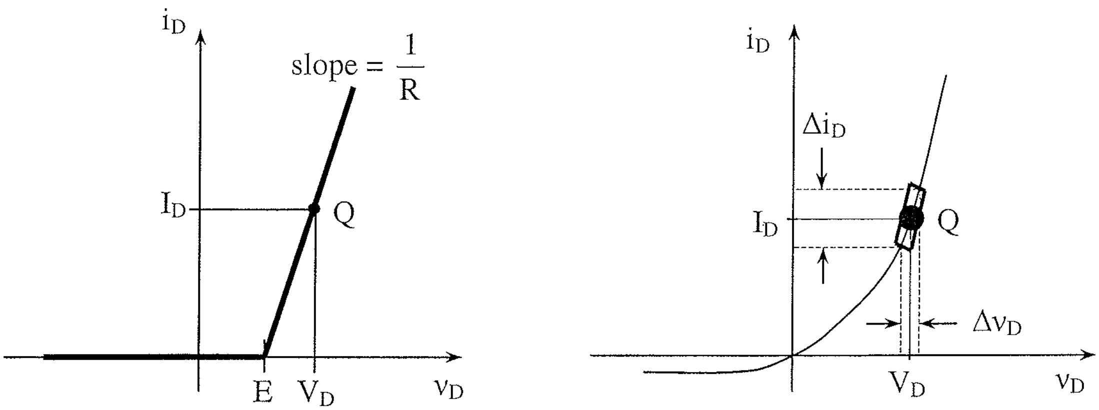



Fig. 2 Diode I-V Characteristics

(a) Linearized Model (resistor and battery)

(b) Diode Equation, $i_D = I_0[\exp(v_D/\eta V_T) - 1]$

Figure 2(a) is a sketch of the linear diode model used in a previous experiment and Figure 2(b) contains a plot of the diode equation, $i_D = I_0[\exp(v_D/\eta V_T) - 1]$. Note that $V_D$ and $I_D$ are DC values whereas $v_D$ and $i_D$ represent the total (DC + AC) diode voltage and current.

Two important concepts of this model are:

(1) The choice of a diode model depends upon the current range. That is, the values of R and E depend on the value of current ($i_D$) flowing through the diode.

(2) The resistor value, R, of the diode model was found by dividing the change in voltage across the diode ($\Delta v_D$) by the corresponding change in current through the diode ($\Delta i_D$). Thus

$$R = \frac{\Delta v_D}{\Delta i_D} \tag{2}$$

Fairly large current swings (eg. 0 to 8 mA) were used when displaying the diode I-V characteristic. A line was then fit to the curve and the resistor value was calculated as the inverse slope of the line.

Now consider the situation depicted in Figure 2(b). A DC voltage, $V_D$, is applied to forward bias the diode. This voltage results in a DC current flow, $I_D$. Then, a small AC voltage signal, $v_D(t)$, is added to this bias. The total voltage, $v_D$, across the diode is

$$v_D = \underset{DC}{V_D} + \underset{AC}{v_D(t)} \tag{3}$$

This small voltage perturbation causes the diode voltage to vary about $V_D$. This in turn causes the current to vary around $I_D$. Refer to $\Delta v_D$ and $\Delta i_D$ of Figure 2(b). Notice that, in a small region about the operating point ($V_D$, $I_D$), the curve approximates a straight line. As a result, a SMALL CHANGE in voltage, causes a PROPORTIONAL SMALL CHANGE in current. This example defines the small signal concept. The small change in stimulus, and the proportional small change in response, are referred to as small signals. The small signal concept is used to analyze non-linear devices which have reasonably linear operating regions.

Let's allow the amplitude of the AC voltage to get smaller and smaller. $\Delta v_D$ and $\Delta i_D$ will both decrease. In the limit, the ratio of the $\Delta v_D$ to $\Delta i_D$ becomes a derivative

$$\lim_{\Delta v_D \to 0} \frac{\Delta v_D}{\Delta i_D} = \frac{dv_D}{di_D}\Big|V_D, I_D = r_d = \text{dynamic resistance} \tag{4}$$

The dynamic resistance, $r_d$, is defined as the inverse slope of the $i_D$ vs. $v_D$ characteristic at a bias point ($V_D$, $I_D$). Notice what happens as Point Q, the *DC operating point*, ($V_D$, $I_D$) is varied. See Figure 2(b). If $V_D$ is decreased, the slope of the curve decreases. Conversely, the dynamic resistance will decrease if $V_D$ increases since the slope will increase. The diode is the variable "resistor" that we were looking for!

## 1.3 A VARIABLE VOLTAGE ATTENUATOR

The variable attenuator circuit that makes use of the dynamic resistance of the diode is shown in Figure 3(a).

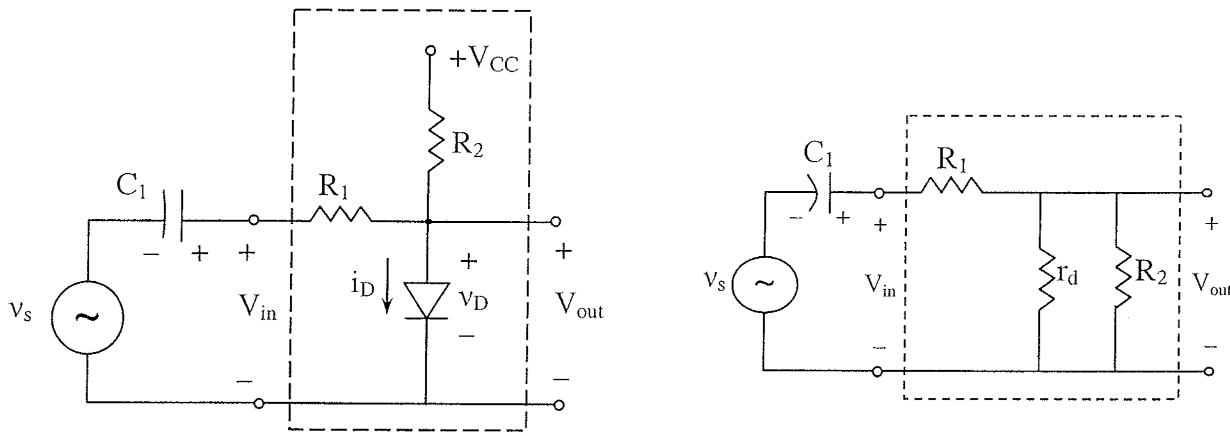



Fig. 3. (a) Diode Attenuator Circuit
(b) Small Signal Equivalent of (a)

In Figure 3(b), it has been assumed that $v_s$ is a small signal. The amount of voltage attenuation (i.e. $v_{out}/v_{in}$) can be controlled by selecting the proper operating point ($V_D$, $I_D$) for the diode.

# 2.0 BACKGROUND: DYNAMIC RESISTANCE AND DIODE PARAMETERS

## 2.1 DERIVATION OF $r_d$ FROM THE IDEAL DIODE EQUATION.

The ideal diode equation relates the diode current, $I_D$, to the voltage across the diode, $V_D$.

$$I_D = I_0[\exp(qV_D/\eta kT) - 1] \qquad (5)$$

where

$I_0$ = reverse saturation current
q = electronic charge = $1.6 \times 10^{-19}$c
$\eta$ = dimensionless ideality factor
k = Boltzmann's constant = $1.38 \times 10^{-23}$ J/K
T = temperature in degrees Kelvin (K)

The expression is often written as

$$\boxed{I_D = I_0[\exp(V_D/\eta V_T) - 1]} \qquad (6)$$

where

$$V_T = \text{thermal voltage} = kT/q \qquad (7)$$

The values $V_D$ and $I_D$ above are to be interpreted as DC quantities. They specify the operating point of the diode, $(V_D, I_D)$. If a small signal voltage, $v_d$, is added to $V_D$, the total voltage applied to the diode is

$$v_D = V_D + v_d \tag{8}$$

The varying voltage results in a varying diode current, $i_d$, and the total instantaneous current is

$$i_D = I_D + i_d \tag{9}$$

Including these small signal variations into the ideal diode equation gives

$$i_D = I_0[\exp(v_D/\eta V_T) - 1] \tag{10}$$

The dynamic conductance, $g_d$ is a small signal parameter and is defined as the ratio of the differential current to the differential voltage at a specific operating point $(V_D, I_D)$. Thus,

$$g_d = \left.\frac{di_D}{dv_D}\right|_{(V_D,I_D)} = \left.\frac{1}{\eta V_T} I_0 \exp(v_D/\eta V_T)\right|_{(V_D,I_D)}$$

$$= \frac{1}{\eta V_T} I_0 \exp(V_D/\eta V_T) \tag{11}$$

rearranging Equation 10 we find that

$$I_0 \exp(v_D/\eta V_T) = i_D + I_0 \tag{12}$$

Then

$$g_d = \frac{(I_D + I_0)1}{(\eta V_T)} \tag{13}$$

The dynamic resistance, $r_d$, is defined as $1/g_d$ or

$$r_d = \left.\frac{dV_D}{di_D}\right|_{(V_D,I_D)} \tag{14}$$

$$\boxed{r_d = \frac{\eta V_T}{I_D + I_0}} \tag{15}$$

By definition, $r_d$ is the inverse slope of the $I_D - V_D$ diode characteristic at the operating point $(V_D, I_D)$. It is the small signal resistance of the diode at a given DC bias.

## 2.2 CALCULATION OF $\eta V_T$ AND $I_0$ FROM $I_D$ vs. $V_D$ DATA

The diode parameters $\eta V_T$ and $I_0$ can be determined by using static DC measurements of diode current versus diode voltage. Note that $\eta$ and the calculation of $I_0$ are dependent on the current range used.

Recall the ideal diode equation in terms of DC values

$$I_D = I_0[\exp(V_D/\eta V_T)-1] \tag{16}$$

For $V_D >> \eta V_T$, $\exp(V_D/\eta V_T) >> 1$ and the equation can be reduced to

$$I_D = I_0 \exp(V_D/\eta V_T) \tag{17}$$

Taking the natural log of both sides

$$\ln(I_D) = \frac{1}{\eta V_T} V_D + \ln(I_0) \tag{18}$$

This equation can be thought of as an equation of a line $y = ax + b$, where $y = \ln(I_D)$, $a = \frac{1}{\eta V_T}$, $x = V_D$, and $b = \ln(I_0)$. Since any line can be defined by two points, consider the DC points $(V_{D1}, I_{D1})$ and $(V_{D2}, I_{D2})$

$$\ln(I_{D1}) = \frac{1}{\eta V_T} V_{D1} + \ln(I_0) \qquad \ln(I_{D2}) = \frac{1}{\eta V_T} V_{D2} + \ln(I_0) \tag{19}$$

subtracting and rearranging gives

$$\eta V_T = \frac{V_{D1} - V_{D2}}{\ln I_{D1} - \ln I_{D2}} \tag{20}$$

Thus, the $\eta V_T$ product can be obtained using two $(V_D, I_D)$ data points. Once $\eta V_T$ has been determined, the ideal diode equation and a $(V_D, I_D)$ data point can be used to solve for $I_0$:

$$\boxed{I_0 = \frac{I_D}{\exp(V_D/\eta V_T) - 1}} \tag{21}$$

## 2.3 SEMI-LOG GRAPHICAL CONSIDERATIONS

In the previous section, $\eta V_T$ was determined using just two ($V_D$, $I_D$) data points. The problem with "this technique is that some error is present in any measurement and if one or both of the ($V_D$, $I_D$) points is incorrect," the $\eta V_T$ calculation could be way off. A better method would be to obtain a number of data points and use some sort of average to minimize any possible error. This "averaging" can be accomplished by plotting the points on semi-log paper and getting two ($V_D$, $I_D$) points graphically.

Semi-log graph paper has one axis which is scaled linearly and one which has a logarithmic (base 10) scale. Since the magnitude of $I_D$ varies much more than $V_D$, it is appropriate to plot $I_D$ on the logarithmic axis. Recalling Equation (18)

$$\ln(I_D) = \frac{1}{\eta V_T} V_D + \ln(I_0) \qquad (22)$$

it can be seen that ($V_D$, $I_D$) data points should plot as a straight line, the slope of which is 1/ $\eta V_T$. Due to measurement error the points won't be exactly linear and a best "fit" line will have to be drawn.

After the ($V_D$, $I_D$) points are plotted and a best "fit" line is obtained, any two points on the line can be used find $\eta V_T$. Equation 22 can not be used in its present form since it contains natural logarithm terms (base e) while the log axis of the semi-log paper is base 10. The following conversion is required.

$$\ln(x) = \frac{\log_{10}(x)}{\log_{10}(e)} \qquad (23)$$

Then Equation 20 becomes

$$\boxed{\eta V_T = \log_{10}(e)\left[\frac{V_{D_1} - V_{D_2}}{\log_{10}(I_{D_1}) - \log_{10}(I_{D_2})}\right]} \qquad (24)$$

Note that this equation is simplified if the ($V_D$, $I_D$) points are chosen such that the $I_D$ values are a decade (ie. a factor of 10) apart.

$I_0$ can be determined using $\eta V_T$ calculated above and any ($V_D$, $I_D$) point on the line. See Equation 21.

## 3.0 EXERCISES

### 3.1 Determine Diode Parameters

A. Acquire Diode Current vs. Diode Voltage Data.

Using the circuit shown in Figure 4, adjust $V_{CC}$ to obtain diode currents of from $10^{-6}$ to $10^{-3}$ amperes. Choose 10 points over the range with one point near each decade (for example: $1 \times 10^{-5}$). Record $V_D$ for each $I_D$. $I_D$ should be determined by measuring the voltage across R with a digital multimeter. Don't forget to measure the exact value of R with the digital multimeter.

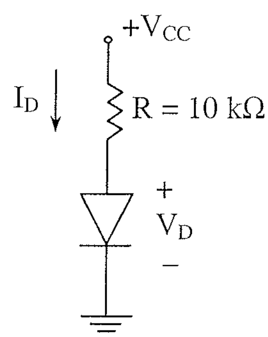



Fig. 4. Circuit used to measure $I_D - V_D$ data.

B. Plot data and calculate parameters.

Plot the $I_D - V_D$ data collected in the previous section on the sheet of semi-log paper included at the end of this experiment. Draw the "best" straight line through the data points that you can. Then, calculate values of $\eta V_T$ and $I_0$ using the method presented in the "Diode Parameters" sections (2.1 & 2.2) of this experiment.

C. Theoretical $r_d$ values.

Using the values of $\eta V_T$ and $I_0$ determined above along with the theoretical dynamic resistance expression (Equation 15), calculate $r_d$ values corresponding to each of the measured $I_D - V_D$ data points. What are the maximum and minimum values of $r_d$ for $I_D$ in the range of 1 μA to 1 mA?

### 3.2 Design the Attenuator

The schematic of the variable attenuator is given in Figure 3(a). Let the frequency of the source be 5 kHz and $R_1 = 10\ K\Omega$. Choose a value for $C_1$ such that its impedance is negligible at this frequency. Using the maximum magnitude of $r_d$ obtained in 3.1.C, determine $R_2$ such that the attenuation ratio ($V_{out}/V_{in}$) is 1/2.

Then, calculate values of $r_d$ to obtain attenuation ratios of 1/5 and 1/10 keeping $R_2$ fixed. Compute the operating points and $V_{CC}$ required to obtain these three values of attenuation.

### 3.3 Determine a Maximum "Small Signal" Input, $v_{in}$.

Use a triangle wave to establish the maximum input signal, $v_{in}$, which can be considered a "small signal" over the entire attenuation range (1/2 to 1/10).

See section 1.0 for definition of "small signal".

Without changing the amplitude, switch the input waveform to a sinewave. Does the output signal appear to be undistorted by the attenuator over the attenuation range: 1/2 to 1/10?

Measure the rms value of this input sinewave using the DMM. This will be the rms value of the small signal used to test the attenuator in Section 3.4.

### 3.4 Test the Attenuator for Small Signals

Using the small signal sinusoid determined in 3.3, set the diode current to the currents designed in 3.2 and compare the actual attenuations to the predicted attenuations (1/2, 1/5, and 1/10).

### 3.5 Investigate distortion of large signals

Adjust the source voltage, $v_s$, so that $v_{in}$ is a 10 volt peak-to-peak triangle waveform (5 KHz). Make a sketch of $v_{out}$ vs. time for an attenuation ratio of 1/10.

Discuss the distortion and its cause.