ECE 202 – Fall 2013

Final Exam

December 12, 2013

Circle your division:

Division 0101: Furgason (8:30 am)

Division 0201: Bermel (9:30 am)

Name (Last, First)______________________________

Purdue ID #_____________________________

There are 18 multiple choice problems (8 points each), and 2 workout problems (28 points each), for a total of 200 points.

Instructions

1. DO NOT START UNTIL TOLD TO DO SO.
2. Write your name, division, professor, and student ID# on your scantron sheet **and** this packet.
3. This is a CLOSED BOOKS and CLOSED NOTES exam.
4. Calculators are not allowed.
5. If extra paper is needed, use back of test pages.
6. Cheating will not be tolerated. Cheating in this exam will result in an F in the course.
7. If you cannot solve a question, be sure to look at the other ones and come back to it if time permits.
8. As described in the course syllabus, we must certify that every student who receives a passing grade in this course has satisfied each of the course outcomes. On this exam, you have the opportunity to satisfy all the course outcomes. (See the course syllabus for a complete description of each outcome.) On the chart below, we list the criteria we use for determining whether you have satisfied these course outcomes.

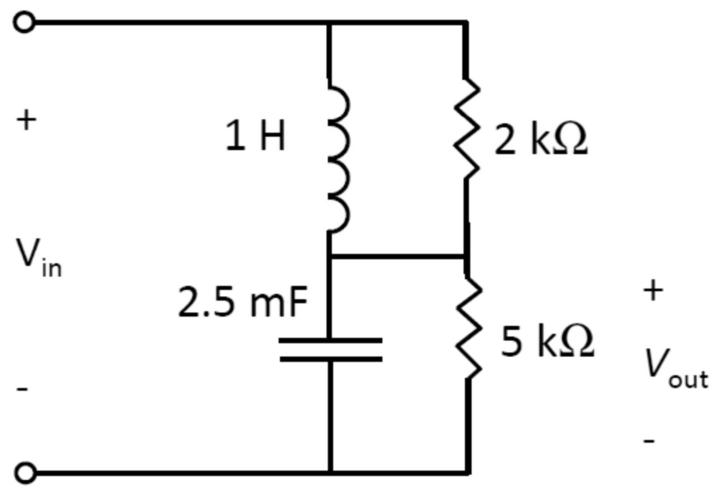
Course Outcome	Exam Questions	Total Questions	Minimum # correct responses required to satisfy course outcome
i	7,10,17	3	2
ii	1,5,11,14	4	2
iii	3,8,9,15	4	2
iv	2,12,16	3	2
v	6,12,19,20	4	2
vi	4,13,18	3	2

If you fail to satisfy any of the course outcomes, don't panic. Your instructor will contact you if there is any problem.

9. **You will find key equations on the final pages of this exam. You can tear them out if needed.**

Workout Problems (28 points each)

1. Consider a low-pass filter depicted below, made from a ‘practical’ inductor and capacitor, with effective internal resistances depicted in parallel for each element:



a. Find the resonant frequency of this circuit, and the resulting quality factor associated with both the practical inductor and capacitor (**hint**: use approximate formulas from the right-hand column of the chart):

$\omega_p =$ ______________________ $Q_L =$ ______________________ $Q_C =$ ________________________

b. Transform the circuit into an equivalent RLC series circuit, and write down the values of each element:

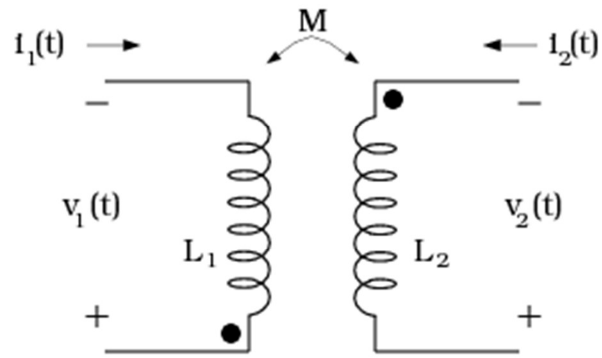
$R =$ _______________________ $L =$ _______________________ $C =$ ________________________

c. Determine the transfer function, $H(s)$, and the quality factor Q_{cir} of the circuit.

$H(s) =$ ____

$Q_{cir} =$ ____

2.



Consider the two-port circuit above. Its z-parameters are defined as follows:

$V_1 = z_{11}I_1 + z_{12}I_2$

$V_2 = z_{21}I_1 + z_{22}I_2$

a. What is the impedance matrix $\boldsymbol{Z} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$?

$\boldsymbol{Z} =$ ____________________

Its y-parameters are defined as follows:

$I_1 = y_{11}V_1 + y_{12}V_2$

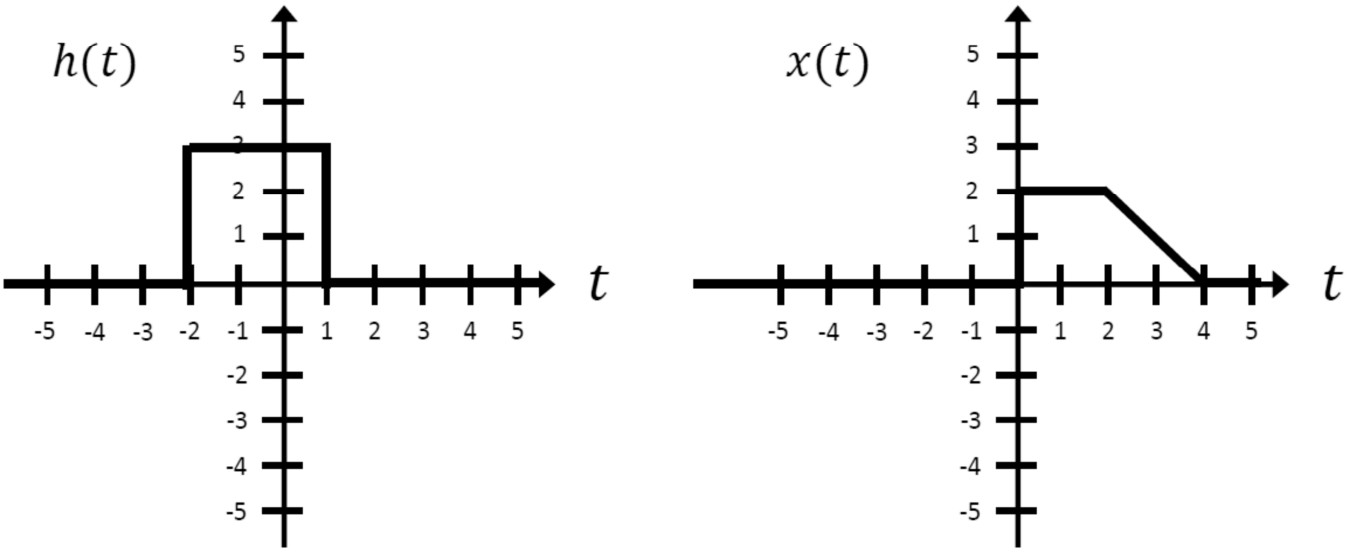
$I_2 = y_{21}V_1 + y_{22}V_2$

b. What is the admittance matrix $\boldsymbol{Y} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$?

$\boldsymbol{Y} =$ ____________________

Multiple Choice Problems (8 points each)

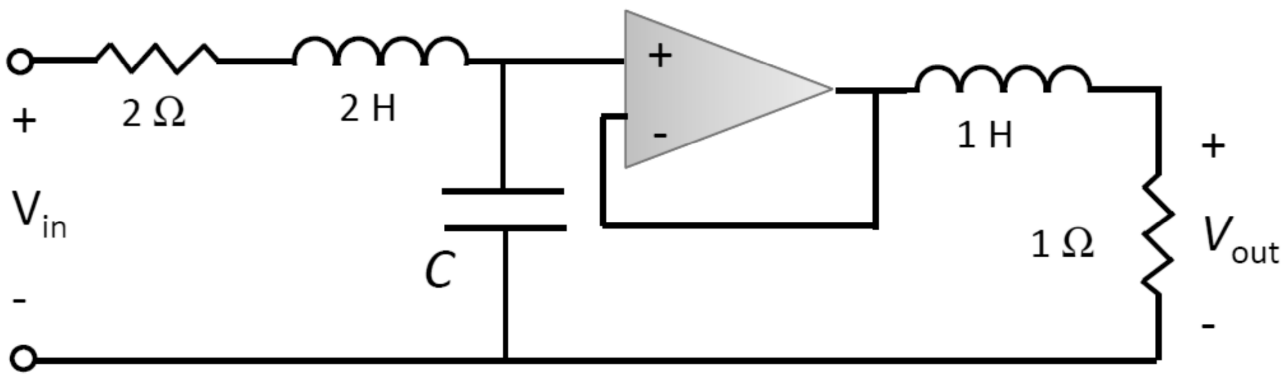
3. What is the numerical value of the convolution function $y(t) = h(t) * x(t)$ when $t = 2$, where both $h(t)$ and $x(t)$ are given in the graphs below?



(1) 0 **(2)** 2 **(3)** 3 **(4)** 4

(5) 6 **(6)** 12 **(7)** 16.5 **(8)** None of these

4. Given the circuit below, what capacitance (in F) is required to realize a third-order Butterworth filter with a filter function $H(s) = \frac{1}{(s+1)(s^2+s+1)}$?



(1) 0.1 **(2)** 0.25 **(3)** 0.5 **(4)** 1.0

(5) 2.0 **(6)** 3.0 **(7)** 4.0 **(8)** None of these

5. What is the inverse Laplace transform of $V_{out}(s) = \frac{7s+13}{(s+1)^2(s+3)}$?

(1) $7e^{-t}u(t) + 13\delta(t)$

(2) $[2e^{-t} - 2e^{-3t}]u(t)$

(3) $[2e^{-t} + 3te^{-t} - 2e^{-3t}]u(t)$

(4) $2e^{-t}r(t) + e^{-3t}u(t)$

(5) $[7e^{t} + 13e^{3t}]u(-t)$

(6) $[7\delta(t-1) + 13\delta(t-3)]u(t)$

(7) $7e^{-t} \cdot r(t)$

(8) None of these

6. Given two 2-port networks described by $\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} V_o \\ -I_o \end{bmatrix}$ and $\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ I_1 \end{bmatrix}$, with a cascaded connection, what is the transmission matrix T in $\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = T \begin{bmatrix} V_o \\ -I_o \end{bmatrix}$?

(1) $\begin{bmatrix} 5 & 2 \\ 1 & 0 \end{bmatrix}$

(2) $\begin{bmatrix} 7 & 2 \\ 5 & 2 \end{bmatrix}$

(3) $\begin{bmatrix} 6 & 0 \\ 6 & 1 \end{bmatrix}$

(4) $\begin{bmatrix} 7 & 4 \\ 0 & 1 \end{bmatrix}$

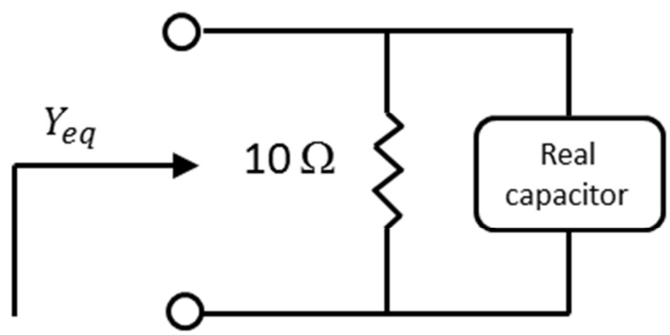
(5) $\begin{bmatrix} 6 & 2 \\ 15 & 5 \end{bmatrix}$

(6) $\begin{bmatrix} 6 & 2 \\ 3 & 1 \end{bmatrix}$

(7) $\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$

(8) None of these

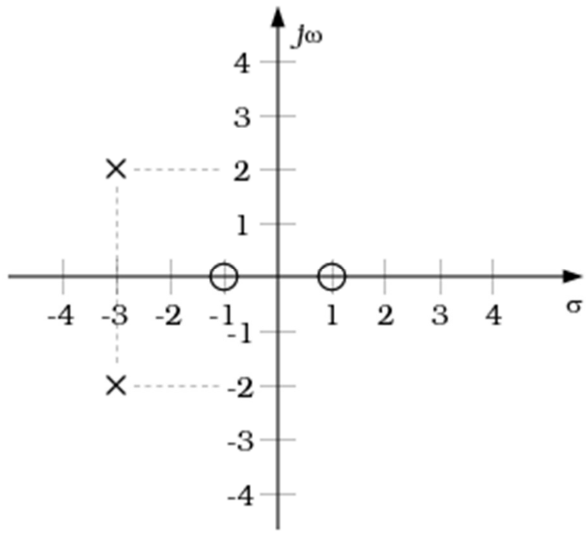
7. The circuit shown below consists of a 10 Ω resistor in parallel with a real 10 μF capacitor having a quality factor Q =10 @ 1000 rad/sec. The input admittance $Y_{eq}(s)$ of this combination (in Ω^{-1}) is:



(1) $0.010 + 10^{-5} \cdot s$ **(2)** $0.011 + 10^{-5} \cdot s$ **(3)** $0.101 + 10^{-5} \cdot s$ **(4)** $0.110 + 10^{-5} \cdot s$

(5) $0.2 + 10^{-5} \cdot s$ **(6)** $1.1 + 10^{-5} \cdot s$ **(7)** $10 + 10^{-5} \cdot s$ **(8)** None of these

The next 2 problems concern the pole-zero plot below, with transfer function $H(s) = K\frac{n(s)}{d(s)}$.



8. Given that $K = 3$, determine the value of $|H(j2)|$.

(1) 0 **(2)** $\frac{2}{5}$ **(3)** 1 **(4)** $\frac{6}{5}$ **(5)** 2 **(6)** $\sqrt{5}$ **(7)** $2\sqrt{5}$

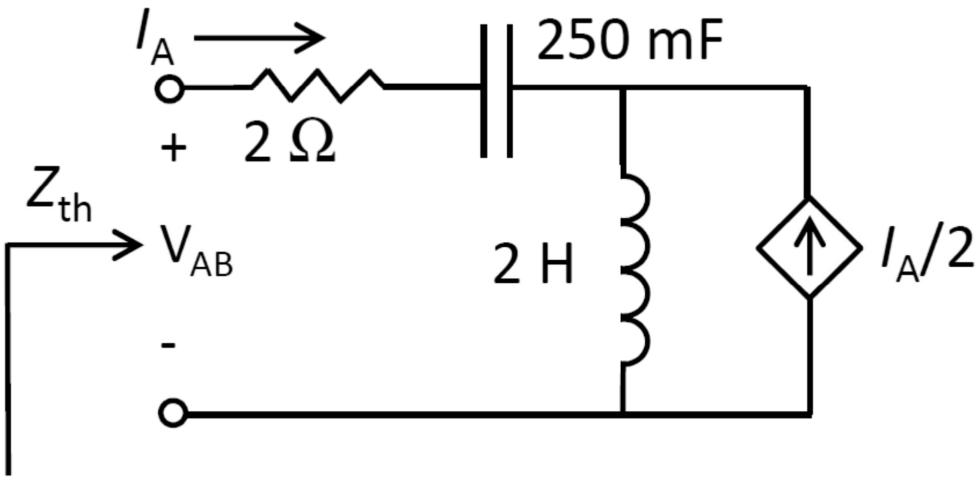
(8) None of these

9. Given that $K = 3$, determine the value of $\angle H(j2)$ (in radians)

(1) 0 **(2)** $\pi/6$ **(3)** $\tan^{-1}(3/4)$ **(4)** $\tan^{-1}(4/3)$

(5) $\frac{\pi}{2} - \tan^{-1}(4/3)$ **(6)** $\frac{\pi}{2} - \tan^{-1}(3/4)$ **(7)** $\pi/2$ **(8)** None of these

10. What is the Thevenin equivalent impedance Z_{th} for this circuit (in Ω)?



(1) 2

(2) $2+\frac{4}{s}+2s$

(3) $2+\frac{s}{4}+\frac{1}{2s}$

(4) $2+\frac{4}{s}+3s$

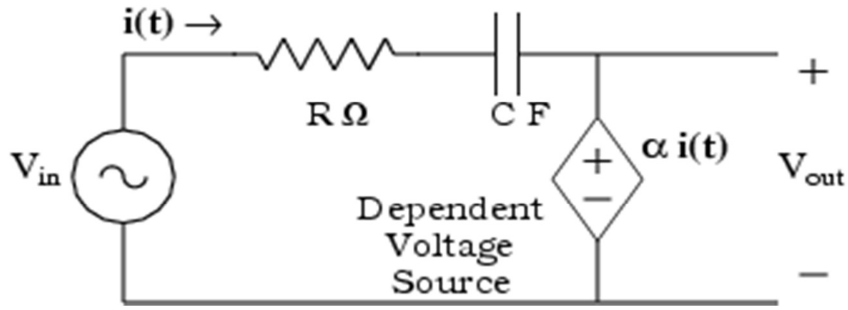
(5) $2s+4/s$

(6) $2+2s+\frac{s^2}{4}$

(7) $\frac{2}{s}+\frac{1}{4s^2}+\frac{2}{s^3}$

(8) None of these

11. The circuit below will be stable provided that:

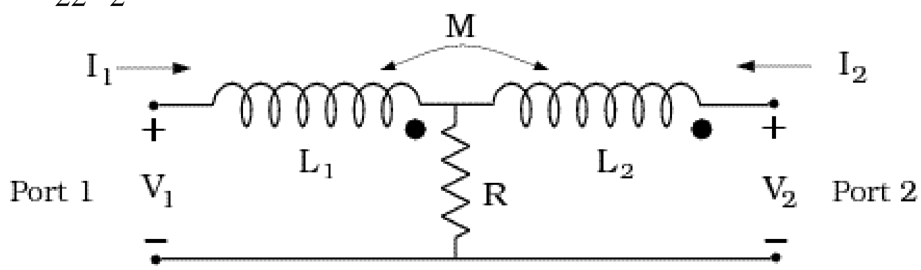


(1) $\alpha > 0$	**(2)** $\alpha < 0$	**(3)** $\alpha > R$	**(4)** $\alpha < R$
(5) $\alpha > -R$	**(6)** $\alpha < -R$	**(7)** $\alpha > RC$	**(8)** None of these

12. Given the following definitions:

$V_1 = z_{11} I_1 + z_{12} I_2$

$V_2 = z_{21} I_1 + z_{22} I_2$



The forward z-parameter, Z_{21}, for the circuit shown above is:

(1) $R - sM$ **(2)** $R * sM$ **(3)** $R + sL_1$ **(4)** $R + s(L_1 - 2M)$

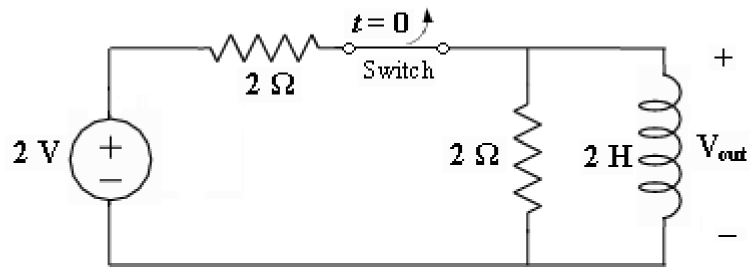
(5) $R + s(L_1 + 2M)$ **(6)** $R + s(L_2 + M)$ **(7)** $R + s(L_1 + L_2 + 2M)$ **(8)** None of these

13. Given a filter function $H(s) = \left(\frac{50}{36-s^2}\right)(6-s) + \left(4 + \frac{24}{s}\right)$, at what frequency ω will the phase $\angle H(j\omega) = -45°$ and the amplitude $|H(j\omega)| = 7\sqrt{2}$?

(1) 0.5 **(2)** 1 **(3)** 2 **(4)** 4

(5) 8 **(6)** 12 **(7)** 16 **(8)** None of these

14. The circuit shown below was built a long time ago, and the switch opens at time, $t = 0$.



After switching, what expression best describes the time dependence of the output voltage $V_{out}(t)$?

(1) $2e^{-2t}u(t)$ **(2)** $-2e^{-2t}u(t)$ **(3)** $2e^{-4t}u(t)$ **(4)** $-2e^{-4t}u(t)$

(5) $2(e^{-t}+1)u(t)$ **(6)** $2(e^{-t}-1)u(t)$ **(7)** $-2e^{-t}u(t)$ **(8)** None of these

15. What is the convolution of $f(t) = 6u(t-2) - 6u(t)$ and $g(t) = t^2u(t)$?

(1) $6t^3e^{-2t}u(t)$

(2) $t^2u(t)[6u(2-t) - 6u(-t)]$

(3) $3(t-2)^2u(t-2) - 3t^2u(t)$

(4) $6u(t-2) - 6u(t)$

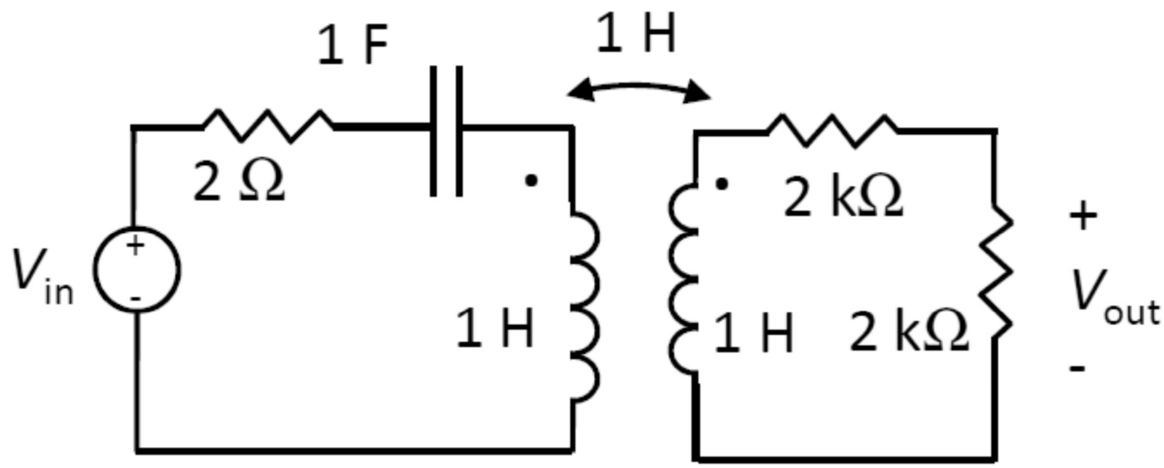
(5) $6r(t-2) - 6r(t)$

(6) $2(t-2)^3u(t-2) - 2t^3u(t)$

(7) $3(t-2)^2u(t-2)$

(8) None of these

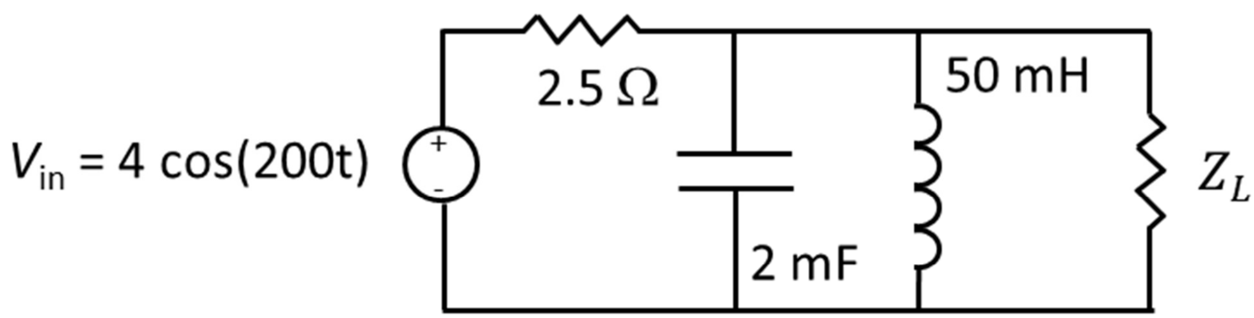
16. Given the mutually inductive system depicted in the circuit below, with $v_{in}(t) = 4\cos(t)$ V, what is the output voltage $v_{out}(t)$ (in V)? **Hint**: the additional voltage in the left-hand inductor from mutual inductance, as well as the self-inductance of the inductor on the right-hand side, can be safely ignored.



(1) $4\cos(t)$ **(2)** $4\sin(t)$ **(3)** $2\cos(t)$ **(4)** $2\sin(t)$

(5) $-\cos(t)$ **(6)** $-\sin(t)$ **(7)** 0 **(8)** None of these

17. Given the circuit below, find the load impedance Z_L that will maximize the power transfer to the load (in Ω).



(1) 1 **(2)** 1+2j **(3)** 1.2+1.6j **(4)** 2

(5) 1.6+1.2j **(6)** 3+4j **(7)** 5 **(8)** None of these

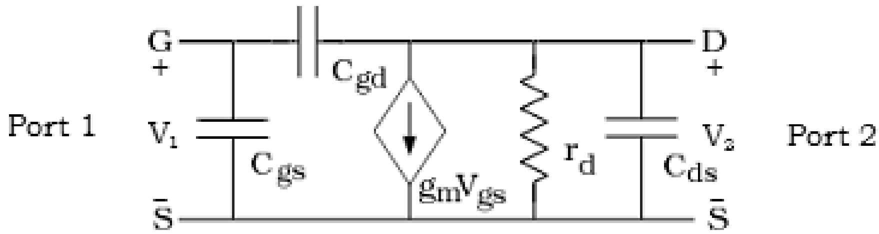
18. If a low-pass filter (not a Butterworth) having a transfer function of $H(s) = \frac{3s+9}{s^2+2s+3}$ is excited by a sinusoidal voltage of $v_S(t) = 3\cos\left(3t - 45^0\right)$ V, the output signal will have the form $v_{out}(t) = \mathrm{A}\cos\left(3t + \theta^0\right)$ V.

The magnitude, A, in volts will be:

(1) 1	**(2)** $\frac{3}{2}$	**(3)** $\frac{3}{\sqrt{2}}$	**(4)** 3
(5) 4.5	**(6)** 6	**(7)** 9	**(8)** None of these

19. Shown below is the small-signal high-frequency model for an FET operating in the linear regime. For this 2-port system, the value of y_{22} is given by:

$$I_1 = y_{11} V_1 + y_{12} V_2$$
$$I_2 = y_{21} V_1 + y_{22} V_2$$

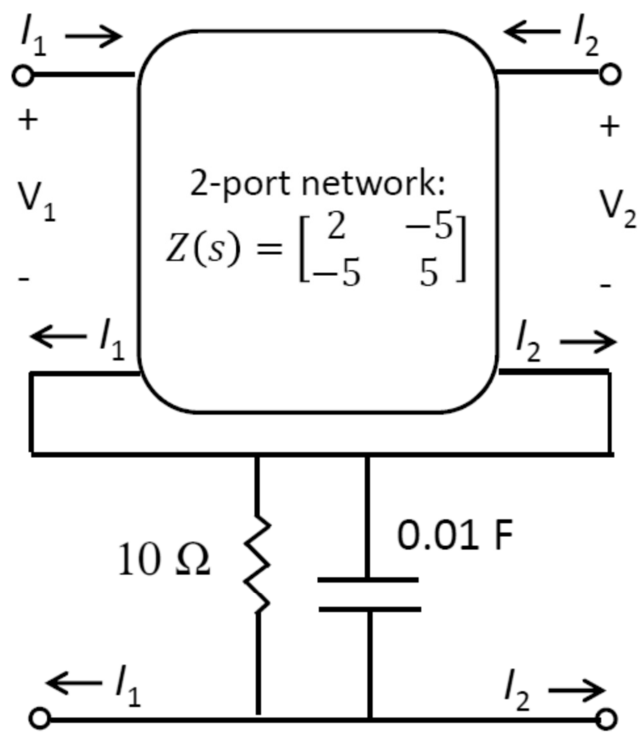


(1) sC_{gs} **(2)** $s(C_{gs} + C_{gd})$ **(3)** $g_m + sC_{gd}$

(4) $g_m + s(C_{gs} + C_{gd})$ **(5)** $g_m + \frac{1}{r_d} + sC_{ds}$ **(6)** $\frac{1}{r_d} + s(C_{gd} + C_{ds})$

(7) $g_m + \frac{1}{r_d} + s(C_{gs} + C_{gd})$ **(8)** None of these

20. Given the 2-port circuit shown below, which consists of a 2-port network with impedance matrix $Z_o = \begin{bmatrix} 2 & -5 \\ -5 & 5 \end{bmatrix}$, connected in series to a resistor of 10 Ω and a capacitor of 0.01 F in parallel, what overall impedance matrix Z_{eq} is obtained at a frequency of 10 rad/s?



(1) $\begin{bmatrix} 12 & -5 \\ -5 & 15 \end{bmatrix}$

(2) $\begin{bmatrix} 2-5j & -5-5j \\ -5-5j & 5-5j \end{bmatrix}$

(3) $\begin{bmatrix} 7 & 0 \\ 0 & 10 \end{bmatrix}$

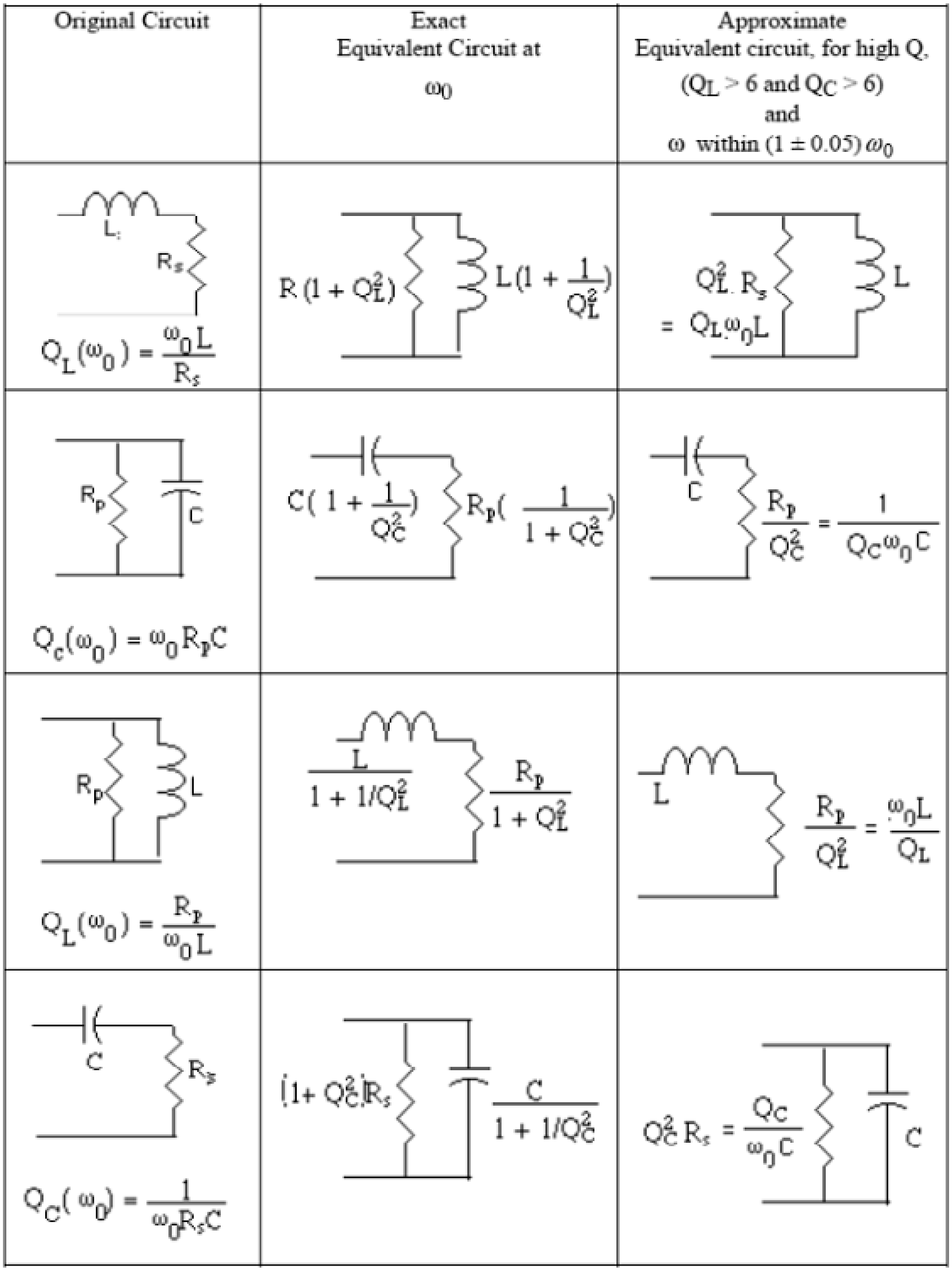
(4) $\begin{bmatrix} 12-10j & 5-10j \\ 5-10j & 15-10j \end{bmatrix}$

(5) $\begin{bmatrix} 7-5j & -5j \\ -5 & 5 \end{bmatrix}$

(6) $\begin{bmatrix} 2 & -5 \\ -5j & 10-5j \end{bmatrix}$

(7) $\begin{bmatrix} 7-5j & -5j \\ -5j & 10-5j \end{bmatrix}$

(8) None of these



Butterworth Transfer Functions

First order	Second order	Third order
$1/(s+1)$	$1/(s^2 + \sqrt{2}s + 1)$	$1/[(s+1)(s^2+s+1)]$

Table 12.1 LAPLACE TRANSFORM PAIRS

Item Number	$f(t)$	$\mathcal{L}[f(t)] = F(s)$
1	$K\delta(t)$	K
2	$Ku(t)$ or K	K/s
3	$r(t)$	$1/s^2$
4	$t^n u(t)$	$\dfrac{n!}{s^{n+1}}$
5	$e^{-at}u(t)$	$1/(s+a)$
6	$te^{-at}u(t)$	$1/(s+a)^2$
7	$t^n e^{-at}u(t)$	$\dfrac{n!}{(s+a)^{n+1}}$
8	$\sin(\omega t)u(t)$	$\dfrac{\omega}{s^2+\omega^2}$
9	$\cos(\omega t)u(t)$	$\dfrac{s}{s^2+\omega^2}$
10	$e^{-at}\sin(\omega t)u(t)$	$\dfrac{\omega}{(s+a)^2+\omega^2}$
11	$e^{-at}\cos(\omega t)u(t)$	$\dfrac{(s+a)}{(s+a)^2+\omega^2}$
12	$t\sin(\omega t)u(t)$	$\dfrac{2\omega s}{(s^2+\omega^2)^2}$
13	$t\cos(\omega t)u(t)$	$\dfrac{s^2-\omega^2}{(s^2+\omega^2)^2}$
14	$\sin(\omega t+\phi)u(t)$	$\dfrac{s\sin(\phi)+\omega\cos(\phi)}{s^2+\omega^2}$
15	$\cos(\omega t+\phi)u(t)$	$\dfrac{s\cos(\phi)-\omega\sin(\phi)}{s^2+\omega^2}$
16	$e^{-at}[\sin(\omega t)-\omega t\cos(\omega t)]u(t)$	$\dfrac{2\omega^3}{[(s+a)^2+\omega^2]^2}$

17	$te^{-at}\sin(\omega t)u(t)$	$2\omega\dfrac{s+a}{[(s+a)^2+\omega^2]^2}$
18	$e^{-at}\left[C_1\cos(\omega t)+\left(\dfrac{C_2-C_1a}{\omega}\right)\sin(\omega t)\right]u(t)$	$\dfrac{C_1s+C_2}{(s+a)^2+\omega^2}$

Table 12.2 LAPLACE TRANSFORM PROPERTIES

Property	Transform Pair
Linearity	$\mathcal{L}[a_1f_1(t)+a_2f_2(t)] = a_1F_1(s)+a_2F_2(s)$
Time Shift	$\mathcal{L}[f(t-T)u(t-T)] = e^{-sT}F(s),\ T>0$
Multiplication by t	$\mathcal{L}[tf(t)u(t)] = -\dfrac{d}{ds}F(s)$
Multiplication by t^n	$\mathrm{L}[t^nf(t)]=(-1)^n\dfrac{d^nF(s)}{ds^n}$
Frequency Shift	$\mathcal{L}[e^{-at}f(t)] = F(s+a)$
Time Differentiation	$\mathcal{L}\left[\dfrac{d}{dt}f(t)\right] = sF(s)-f(0^-)$
Second-Order Differentiation	$\mathrm{L}\left[\dfrac{d^2f(t)}{dt^2}\right]=s^2F(s)-sf(0^-)-f^{(1)}(0^-)$
nth-Order Differentiation	$\mathrm{L}\left[\dfrac{d^nf(t)}{dt^n}\right]=s^nF(s)-s^{n-1}f(0^-)-s^{n-2}f^{(1)}(0^-)$ $-\ldots-f^{(n-1)}(0^-)$
Time Integration	(i) $\mathrm{L}\left[\int_{-\infty}^{t}f(q)dq\right]=\dfrac{F(s)}{s}+\dfrac{\int_{-\infty}^{0^-}f(q)dq}{s}$ (ii) $\mathrm{L}\left[\int_{0^-}^{t}f(q)dq\right]=\dfrac{F(s)}{s}$
Time/Frequency Scaling	$\mathcal{L}[f(at)] = \dfrac{1}{a}F\left(\dfrac{s}{a}\right)$