Imaging Colorimetry using a Digital Camera with Dental Applications

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5 November 2002
Applications of tooth colorimetry:
assessment of efficacy of whitening agents

Before

After

Image obtained from: http://www.westcentraldental.com/cosmetic.htm
Applications of tooth colorimetry: matching color of restored tooth to neighbors
Colorimetry research at Purdue

• Phase I
  – Development of theoretical framework for model-based colorimetry system and filter design
  – Simulation results only

• Phase II
  – Application to colorimetry with a digital camera
  – Characterization of digital camera
  – Regression-based approach
  – Experimental results with color patches and extracted teeth
Colorimetry research at Purdue (cont.)

• Phase III
  – Refinement of experimental system
  – Development of reflectance estimation based approach
  – Experimental results with extracted and live teeth
Collaborators/Sponsors

• Mark Wolski, General Motors Research Laboratory
• Color Savvy Systems, Inc.
• Indiana University School of Dentistry
  – Dr. Mostafa Analoui, (now with Pfizer, Inc.)
  – Dr. George Stookey
• Indiana 21st Century Fund
Synopsis

• What is color?
• Phase II
  – System architecture
  – Filter design
  – Preliminary results
• Phase III
  – System refinement
  – “Clinical” system
  – Experimental results
• Future research (Phase IV?)
What does color mean?

Camera

PC/Workstation

Printer

Scanner

Copier
Each device has its own language

"I asked you to print sea green."

"That's sea green, isn't it?"

"You said to print RGB = (34, 229, 164)."

"No, it's yellow-green."
Spectral Representation of Color

Illuminant

Reflective Surface

Stimulus

$I(\lambda)$

$S(\lambda)$

$R(\lambda)$
Trichromatic Representation of Color for Human Vision

- 3-D subspace

\[
R_S = \int S(\lambda)Q_R(\lambda)\,d\lambda
\]

\[
G_S = \int S(\lambda)Q_G(\lambda)\,d\lambda
\]

\[
B_S = \int S(\lambda)Q_B(\lambda)\,d\lambda
\]
Finite-Dimensional Approximation

Stimulus

Sensor Response Functions

$Q_R(\lambda)$  $Q_G(\lambda)$  $Q_B(\lambda)$
Finite-Dimensional Approximation (cont.)

\[ Q = \begin{bmatrix} q_R & q_G & q_B \end{bmatrix} = \begin{bmatrix} Q_R(\lambda_1) & Q_G(\lambda_1) & Q_B(\lambda_1) \\ \vdots & \vdots & \vdots \\ Q_R(\lambda_N) & Q_G(\lambda_N) & Q_B(\lambda_N) \end{bmatrix}, \]

\[ c_S = \begin{bmatrix} R_S \\ G_S \\ B_S \end{bmatrix}, \quad s = \begin{bmatrix} S(\lambda_1) \\ \vdots \\ S(\lambda_N) \end{bmatrix}, \]

\[ c_S = Q^T s = \begin{bmatrix} q_R^T s \\ q_G^T s \\ q_B^T s \end{bmatrix} \]
Sensor Subspace

• Example 1: N = 2 wavelengths, one monochromatic sensor

\[ c = s_1^T q = s_2^T q \]

stimuli \( s_1 \) and \( s_2 \) are metamers to sensor \( q \)
Role of Illuminant

\[ S(\lambda) = R(\lambda)I(\lambda) \]

\[ R_S = \int R(\lambda)[I(\lambda)Q_R(\lambda)]d\lambda \]

\[ G_S = \int R(\lambda)[I(\lambda)Q_G(\lambda)]d\lambda \]

\[ B_S = \int R(\lambda)[I(\lambda)Q_B(\lambda)]d\lambda \]

illuminant modifies sensor response
Effect of Illuminant Metamerism

- Example 2: N = 2 wavelengths, two monochromatic sensors

stimuli $s_1$ and $s_2$ are metamers to sensor $q_1$ but not to $q_2$
Challenges of tooth color measurement

- **Smooth surface** gives rise to specular reflection
- **Irregular pattern of scattering of light** gives rise to non-uniform color on the surface
- **Irregular size and shape** complicate measurements
- **Fluorescence** makes color less predictable

A device with high spatial resolution is needed
What is a colorimetry system?

- Measurement device
- Calibration mapping

\[
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
\end{bmatrix}
\]
Prior work: colorimetry system design

• Calibration mapping
  – 3 X 6 matrix (Farrell et al, 1994)

• Measurement device
  – Design a device (Wolski et al, 1996)
  – Alter device sensitivity (Chen & Trussell, 1995)
  – Increase the number of device channels (Tominaga 1999, Haneishi et al, 1995)
Prior Work: colorimetry with a digital camera

- Two main approaches have been adopted:
  - Physics-based approach:
    - Model the electro-optics of the camera, [Daligault, 1991], [Quan et al, 2000], no experiments have been reported.
  - Statistically-based approach:
    - Focus on what the camera ‘sees’ instead of how the camera ‘sees’
    - Linear regression is widely used
    - Results have been reported for color patches such as MacBeth Color Checkers ([Lenz, 1996], [Balas, 1997]), Munsell Color Chips ([Finlayson, 1997]) and IT8 target ([Pointer, 2001])
Phase II research

- Digital camera + filters & calibration matrices
- Attack the problem simultaneously on both the design of filters in measurement device and the mapping in color calibration
- Obtain tristimulus values of a sample under pre-selected illumination conditions rather than recover its reflectance
Multi-exposure colorimetry system

illuminant $L$

reflectance $r$

measurement device

Filter 1

Camera

Filter $N_f$

Camera

$R^{(i)}$

$G^{(i)}$

$B^{(i)}$

$R^{(N_f)}$

$G^{(N_f)}$

$B^{(N_f)}$

under illuminant $L'$

$X'$

$Y'$

$Z'$
Multi-exposure colorimetry system

- A calibration problem: if filters are given
  - Model-based or regression-based method
- A system design problem: if filter design is desired
  - Model-based method only
  - Interaction between measurement device and calibration mapping
System design: notation

\[ F = \begin{bmatrix} f_1 & D \\ f_2 & D \\ \vdots \\ f_{N_f} & D \end{bmatrix} \]

\[ R = \begin{bmatrix} r_1^t & r_2^t & \cdots & r_{N_r}^t \end{bmatrix} \]

\[ S = RLF \]

\[ T' = RL'A \]

\[ f_i: \quad \text{31-pt. Filter transmittance of } i\text{th filter} \]

\[ D: \quad \text{camera sensitivities} \]

\[ A: \quad \text{CIE XYZ color matching function} \]
Design goal and approach

• Find the “best” $F-M'$ pair
  – Small perceptual errors
  – Robust system
  – Smooth filter transmittance

• How?
  – RMS errors of linearized $DE$
  – Perturbation of $F$
  – Cost on filter roughness
System design: summary

Minimize:

\[ h(F) = \max_i \{ \varepsilon_t^{(i)} \} + \varepsilon_s \quad i = \text{pre-selected illuminants} \]

Subject to:

\[ 0 \leq \text{all elements of } f_j \leq 1 \quad j = 1, 2, \ldots, N_f \]

- Optimal design through constrained optimization
- Restricted search among candidate solution set
Calibration mappings

- Model-based method

\[
\text{vec}(M''') = \left( LF \otimes I_3 \right)^t B'^t_{eq} B'_{eq} \left( LF \otimes I_3 \right) \cdot \left( LF \otimes I_3 \right)^t B'^t_{eq} B'_{eq} \text{vec}(A^t L') \right]^{1}.
\]

- Regression-based method

\[
M' = (S^t S)^{-1} S^t T'
\]
Evaluation of system performance

\[ \| \varphi(T_1) - \varphi(T_2) \|_{ab} = \frac{1}{N} \sum_i \| \varphi(\text{ith row of } T_1) - \varphi(\text{ith row of } T_2) \|_2 \]

\[
\begin{bmatrix} X & Y & Z \end{bmatrix} \xrightarrow{\varphi} \begin{bmatrix} L^* & a^* & b^* \end{bmatrix}
\]

- \( T_1-T_2 \): error in CIE XYZ space
- \( DE \) in \( L^* a^* b^* \) space
- Mean \( DE \)
Perceptual error - 1

Hence, the estimation error in CIE XYZ space is

\[ \Delta t'_k = t'_k - \hat{t}'_k = r^t_k (L' A - LFM'). \]  \hspace{1cm} (9)

Motivated by the fact that CIE \( L^*a^*b^* \) is approximately a perceptually uniform color space,\(^1,6\) we apply the local linearization technique described in Ref. 14 and approximate the estimation error \( \Delta u'_k \) in CIE \( L^*a^*b^* \) space by weighting \( \Delta t'_k \) with a local Jacobian matrix \( J'_k \). That is

\[
J'_k = \frac{1}{3} \begin{bmatrix}
0 & 116 & 0 \\
500 & -500 & 0 \\
0 & 200 & -200 \\
\end{bmatrix}
\begin{bmatrix}
X_n^{-1/3} X_k^{-2/3} & 0 & 0 \\
0 & Y_n^{-1/3} Y_k^{-2/3} & 0 \\
0 & 0 & Z_n^{-1/3} Z_k^{-2/3} \\
\end{bmatrix}. \hspace{1cm} (10)
\]

\[ \Delta u'^t_k \approx J'_k \Delta t'^t_k = J'_k (A^t L' - M'^t (LF)^t) r_k. \hspace{1cm} (11) \]
Perceptual error - 2

For convenience, we refer to $\Delta E'_k = \|\Delta u'_k\|_2$ as the perceptual error, while recognizing that the CIE $L^*a^*b^*$ space only accounts for a very limited part of how humans perceive color.

Now let us sum the errors for all the color samples in the training set. The square of the total root-mean-squared error can be approximated\(^{14}\) by

$$
(\Delta E_{\text{rms}}')^2 \approx \| B_{eq} \text{vec}(A^t L' - M' t (LF)^t) \|_2^2,
$$

(12)

Here $\text{vec}(\bullet)$ is an operator that creates a vector out of a matrix by stacking its columns; and $B'_{eq}$ is the Cholesky factor\(^{23}\) given by

$$
B'_{eq}^t B'_{eq} = \frac{1}{N_r} \sum_{k=1}^{N_r} B'_k t B_k, \text{ with } B'_k = (r_k \otimes J'_k),
$$

that summarizes the contribution of the training set.
Robustness to filter variation

To investigate the robustness of the device, replacing $F$ in Eq. 12 by $F + \Delta F$, then using linearity of $\text{vec}(\cdot)$ and two 2-norm inequalities, it can be shown that $(\Delta E'_{\text{rms}})^2$ is upper bounded by $2\epsilon'$,

$$(\Delta E'_{\text{rms}})^2 \leq 2\epsilon'(F, M'),$$

where

$$\epsilon'(F, M') = \left\| \begin{bmatrix} B'_e \text{vec}(A^t L') \\ 0 \end{bmatrix} - \begin{bmatrix} B'_e (LF \otimes I_3) \\ \sqrt{K_t} I_{9N_f} \end{bmatrix} \text{vec}(M') \right\|_2^2,$$

$$K_t = \left\| B'_e (L\Delta F \otimes I_3) \right\|_2^2.$$
Smoothness constraint and overall cost function

For filter smoothness, we define a penalty function $\varepsilon_s$ according to

$$\varepsilon_s = K_s \sum_i \| D^2 f_i \|_2^2, \quad (17)$$

using the second order difference operator $D^2$ described in Ref. 14. Consequently, a cost function $h(F)$ involving perceptual errors in CIE $L^*a^*b^*$ space, system robustness, and filter smoothness is obtained; and the calibration problem can thus be formulated as the following constrained optimization problem:

Minimize

$$h(F) = \varepsilon_t + \varepsilon_s, \quad (18)$$

subject to

$$0 \leq \text{all elements of } f_i \leq 1 \quad i = 1,2,\ldots,N_f.$$
Design a colorimetry system

- Camera sensitivity
- Illumination conditions
- Training samples
- Number of exposures
- Robustness and smoothness ($K_t,K_s$)
- Filter design/restricted search
Camera sensitivity

- Request data directly from manufacturer
- Estimate it through calibration
Illumination conditions

- Daylight from viewing booth & camera flash
- E, A, F, D65
Training samples

- Application dependent
  - 120 DuPont paint chips
  - 170 natural objects
  - 1269 Munsell color chips
Robustness and smoothness

\[
\|\phi(RLA) - \phi((RLF + \Delta S)M)\|_{ab}
\]

No noise

Robustness issue: \(K_s = 1\)

With noise

Robustness issue: \(K_s = 1\)

Flash-Flash

<table>
<thead>
<tr>
<th>(K_t)</th>
<th>(K_s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(10^{-8})</td>
<td>1</td>
</tr>
<tr>
<td>(10^{-4})</td>
<td>1</td>
</tr>
<tr>
<td>(10^{-3})</td>
<td>1</td>
</tr>
<tr>
<td>(10^{-2})</td>
<td>1</td>
</tr>
<tr>
<td>(10^{-1})</td>
<td>(1)</td>
</tr>
<tr>
<td>(10^{-4})</td>
<td>(10^{-2})</td>
</tr>
<tr>
<td>(10^{-4})</td>
<td>(10^{-1})</td>
</tr>
<tr>
<td>(10^{-4})</td>
<td>(10^1)</td>
</tr>
<tr>
<td>(10^{-4})</td>
<td>(10^2)</td>
</tr>
</tbody>
</table>

Imaging Colorimetry using a Digital Camera with Dental Applications
Procter & Gamble - 5 November 2002
Design results

Optimal filters

Optimal Wratten filters

Optimal filter pairs for 2-exposure Flash-{Flash,E,A,F,D65} colorimetry systems when \((K_t, K_s) = (10^{-4}, 1)\)
Design results
(Channel sensitivities)

Optimal filters

Optimal Wratten filters
Simulation results

\[ \| \varphi(RL'A) - \varphi((RLF)M') \|_{ab} \]

<table>
<thead>
<tr>
<th>Cond.</th>
<th>Flash</th>
<th>E</th>
<th>A</th>
<th>F</th>
<th>D65</th>
<th>Max.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flash</td>
<td>0.2050</td>
<td>0.4571</td>
<td>0.5092</td>
<td>0.2720</td>
<td>0.4504</td>
<td>0.5092</td>
</tr>
<tr>
<td>E</td>
<td>0.1058</td>
<td>0.3600</td>
<td>0.2454</td>
<td>0.3452</td>
<td>0.2662</td>
<td>0.3600</td>
</tr>
<tr>
<td>A</td>
<td>0.4988</td>
<td>0.8363</td>
<td>0.1357</td>
<td>0.6614</td>
<td>0.7469</td>
<td>0.8363</td>
</tr>
<tr>
<td>F</td>
<td>0.3186</td>
<td>0.4321</td>
<td>0.2329</td>
<td>0.1980</td>
<td>0.4038</td>
<td>0.4321</td>
</tr>
<tr>
<td>D65</td>
<td>0.1429</td>
<td>0.4178</td>
<td>0.2466</td>
<td>0.3784</td>
<td>0.3218</td>
<td>0.4178</td>
</tr>
<tr>
<td>All</td>
<td>0.1132</td>
<td>0.3760</td>
<td>0.2433</td>
<td>0.3528</td>
<td>0.2811</td>
<td>0.3760</td>
</tr>
</tbody>
</table>

\[ h(F') = \max_i \{ \varepsilon_t^{(i)} \} + \varepsilon_s \]
Experimental results

\[ \| \varphi(T') - \varphi(SM') \|_{ab} \]

<table>
<thead>
<tr>
<th></th>
<th>Flash</th>
<th>E</th>
<th>A</th>
<th>F</th>
<th>Daylight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model-based</td>
<td>4.4631</td>
<td>4.9830</td>
<td>4.7022</td>
<td>5.8554</td>
<td>4.5970</td>
</tr>
<tr>
<td>Regression-based</td>
<td>1.7792</td>
<td>1.8506</td>
<td>1.8504</td>
<td>1.9743</td>
<td>1.8291</td>
</tr>
</tbody>
</table>

Experimental results on 278 Munsell color chips using Wratten filter pair (WR11+WR38A, WR80B +WR85N6)
Phase II conclusion

- Comparison

<table>
<thead>
<tr>
<th></th>
<th>Regression-based</th>
<th>Model-based</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accuracy</td>
<td>Good</td>
<td>Fair</td>
</tr>
<tr>
<td>Flexibility</td>
<td>Poor</td>
<td>Excellent</td>
</tr>
</tbody>
</table>

- Filters used in regression-based method is the design from model-based technique
Phase III research

• Continuation of Wu’s work.
• Modify the measurement setup to achieve higher accuracy. Improve prediction of the system.
• Make the prediction more robust to different illumination conditions.
• Adapt the system for clinical use.
• Adopt the statistical approach.
Regression Models

- **Illuminant-Dependent Colorimeter (IDC):**
  - Slight modification of Wu’s Regression Model, use an affine instead of homogeneous calibration matrix
  - Only output CIE XYZ values
  - In this model, we calibrate, measure and **match the output under the same illuminant**
  - Prediction is only meaningful under the same lighting condition under which the calibration matrix was derived

- **Illuminant-Independent Colorimeter (IIC):**
  - Output spectral reflectance
  - In this model, we calibrate and measure under the same illuminant but can **match the output under any illuminant**
IIC and IDC for 2-exposure system

\[ c_j : \text{coordinates of } r_j \text{ in the eigenspace of the training samples} \]
\[ \lambda_i : \text{ith component of the 31-point wavelength vector} \]
\[ r_j : 31 \times 1 \text{ estimated spectral reflectance} \]
\[ D : 31 \times 3 \text{ camera sensitivity matrix} \]
\[ M : \text{calibration matrix, } 7 \times 3 \text{ for IDC model and } 7 \times 6 \text{ for IIC model} \]
\[ A : 31 \times 3 \text{ CIE XYZ color matching functions} \]
\[ w_j : \text{light reflected from the tooth measured by the spectroradiometer} \]
\[ w^* : \text{light reflected from a standard white measured by the spectroradiometer} \]
\[ T^* : \text{predicted output matrix} \]
\[ T : \text{true output matrix} \]
\[ \text{IDC} : \text{Illuminant Dependent Colorimeter} \]
\[ \text{IIC} : \text{Illuminant Independent Colorimeter} \]
\[ S : \text{as defined in Sec. 2.1} \]
\[ V_{tr} : \text{as defined in Sec. 2.3} \]
\[ (\cdot)^T : \text{matrix transposition} \]
Experiment

• Apparatus
  – Kodak DCS460 Digital Camera
  – Kodak wratten filters (WR11+WR85N6 and WR38A+WR80B)
  – Photo Research PR705 spectroradiometer

• Experiment 1
  – 85 extracted teeth were measured under 3 different light sources:
    » MacBeth Daylight simulator
    » 2 AC-powered Vivitar 285 flash units
    » PL900 DC regulated Halogen light with bifurcated fiber optic light guide

• Experiment 2
  – 21 human subjects were recruited. The top two incisors of each subject were measured with Halogen Light
Setup For Experiment 1

- For Daylight simulator, the teeth were placed in the viewing booth. The incident angle of the light is approximately 45 degree.
Picture for Experiment 1 with flash units
Picture for Experiment 2 with a subject
Performance Evaluation

• Experiment 1
  – In each random selection, 70 of 85 extracted teeth were randomly picked as training set, the remaining 15 comprised the test set.

• Experiment 2
  – In each random selection, 34 of 42 live human teeth were randomly picked as training set, the remaining 8 comprised the test set.

• In both experiments, 10,000 selections were performed.

• The average $\Delta E$ of each selection was recorded.
Results

• Experiment 1
  – IDC, measured with 3 different light sources:

<table>
<thead>
<tr>
<th>Illuminant</th>
<th>Mean</th>
<th>StdDev</th>
<th>Max</th>
<th>% &lt; 1.5*</th>
</tr>
</thead>
<tbody>
<tr>
<td>D65</td>
<td>1.101</td>
<td>0.169</td>
<td>1.885</td>
<td>98.5</td>
</tr>
<tr>
<td>Flash</td>
<td>0.965</td>
<td>0.198</td>
<td>1.775</td>
<td>99.4</td>
</tr>
<tr>
<td>Halogen</td>
<td>0.843</td>
<td>0.115</td>
<td>1.362</td>
<td>100</td>
</tr>
</tbody>
</table>

– IIC, measured with Halogen light

<table>
<thead>
<tr>
<th>Illuminant</th>
<th>Mean</th>
<th>StdDev</th>
<th>Max</th>
<th>% &lt; 1.5*</th>
</tr>
</thead>
<tbody>
<tr>
<td>D65 (IIC)</td>
<td>0.833</td>
<td>0.109</td>
<td>1.354</td>
<td>100</td>
</tr>
<tr>
<td>Flash (IIC)</td>
<td>0.841</td>
<td>0.110</td>
<td>1.364</td>
<td>100</td>
</tr>
<tr>
<td>Halogen (IIC)</td>
<td>0.843</td>
<td>0.115</td>
<td>1.366</td>
<td>100</td>
</tr>
</tbody>
</table>

– *percentage of random selections with average $\Delta E < 1.5$
• Experiment 2:
  – Measurements for both IDC and IIC were taken under the Halogen light. The spectral reflectance of each sample was reconstructed and evaluated under all the three light sources for IIC.

<table>
<thead>
<tr>
<th>Illuminant</th>
<th>Mean</th>
<th>StdDev</th>
<th>Max</th>
<th>% &lt; 1.5*</th>
</tr>
</thead>
<tbody>
<tr>
<td>D65 (IIC)</td>
<td>1.080</td>
<td>0.118</td>
<td>2.03</td>
<td>98.2</td>
</tr>
<tr>
<td>Flash (IIC)</td>
<td>1.051</td>
<td>0.176</td>
<td>1.966</td>
<td>98.8</td>
</tr>
<tr>
<td>Halogen (IIC)</td>
<td>1.048</td>
<td>0.177</td>
<td>1.972</td>
<td>98.7</td>
</tr>
<tr>
<td>Halogen (IDC)</td>
<td>1.049</td>
<td>0.177</td>
<td>1.980</td>
<td>98.7</td>
</tr>
</tbody>
</table>

  – *percentage of random selections with average $\Delta E < 1.5$
• Spectra for extracted teeth with IIC: The best among 15 reconstructions in a random selection. The $\Delta E$ values were computed for the Halogen light.
Spectra for extracted teeth with IIC: The worst among 15 reconstructions in the same random selection. The $\Delta E$ values were computed for the Halogen light.
Phase III conclusion

• Both IIC and IDC work well with extracted and live teeth.
• The output of IIC can be matched under any illumination condition whereas the output of IDC can only be matched under the illuminant for which the calibration matrix was designed.
• However, a set of training samples which represents the tooth gamut adequately must be carefully picked to ensure good prediction.
Future Work (Phase IV?)

- Further clinical studies
- Highlight removal
- Fluorescence compensation