ECE 638 Homework 3 Sample Solution

Question 1 (Ziyue Xiang)

a. The input intensity vs. output luminance for each channel is shown in Figure 1.

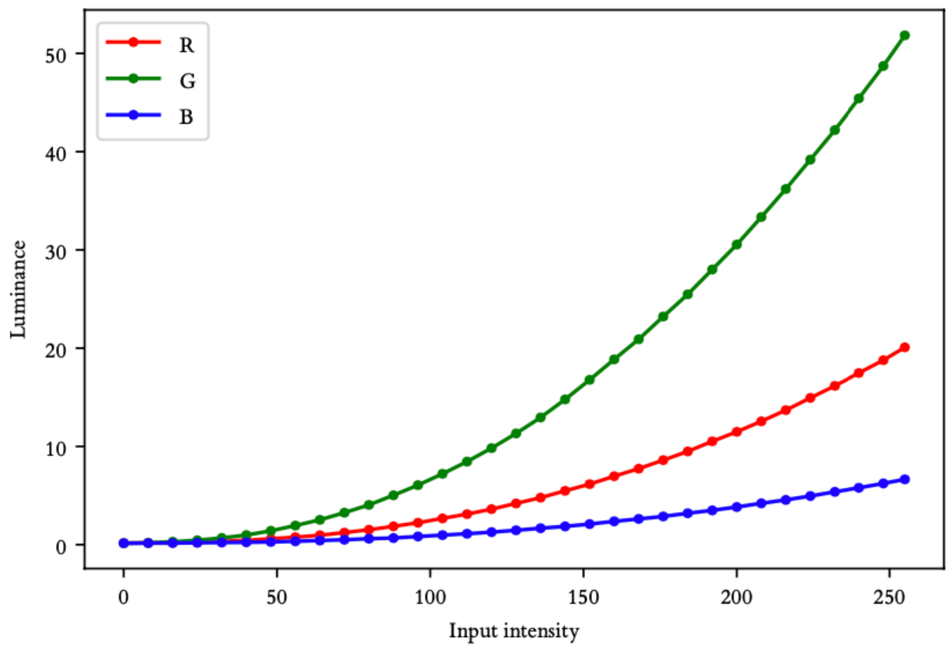


Figure 1: The input intensity vs. output luminance for each channel.

For each channel, we fit a GOG model, which is given by

$$I = \left(a\psi + b\right)^{\gamma}. \quad (1.1)$$

We use the Nelder-Mead optimization algorithm provided by `scipy` package to find the best parameters for each channel. The parameters we found is shown in Table 1. The comparison between the actual curve and the fitted curve is shown in Figure 2.

Channel	a	b	γ
R	3.139	0.209	2.483
G	6.148	−0.019	2.178
B	1.564	0.359	2.904

Table 1: Model parameters found for each channel.

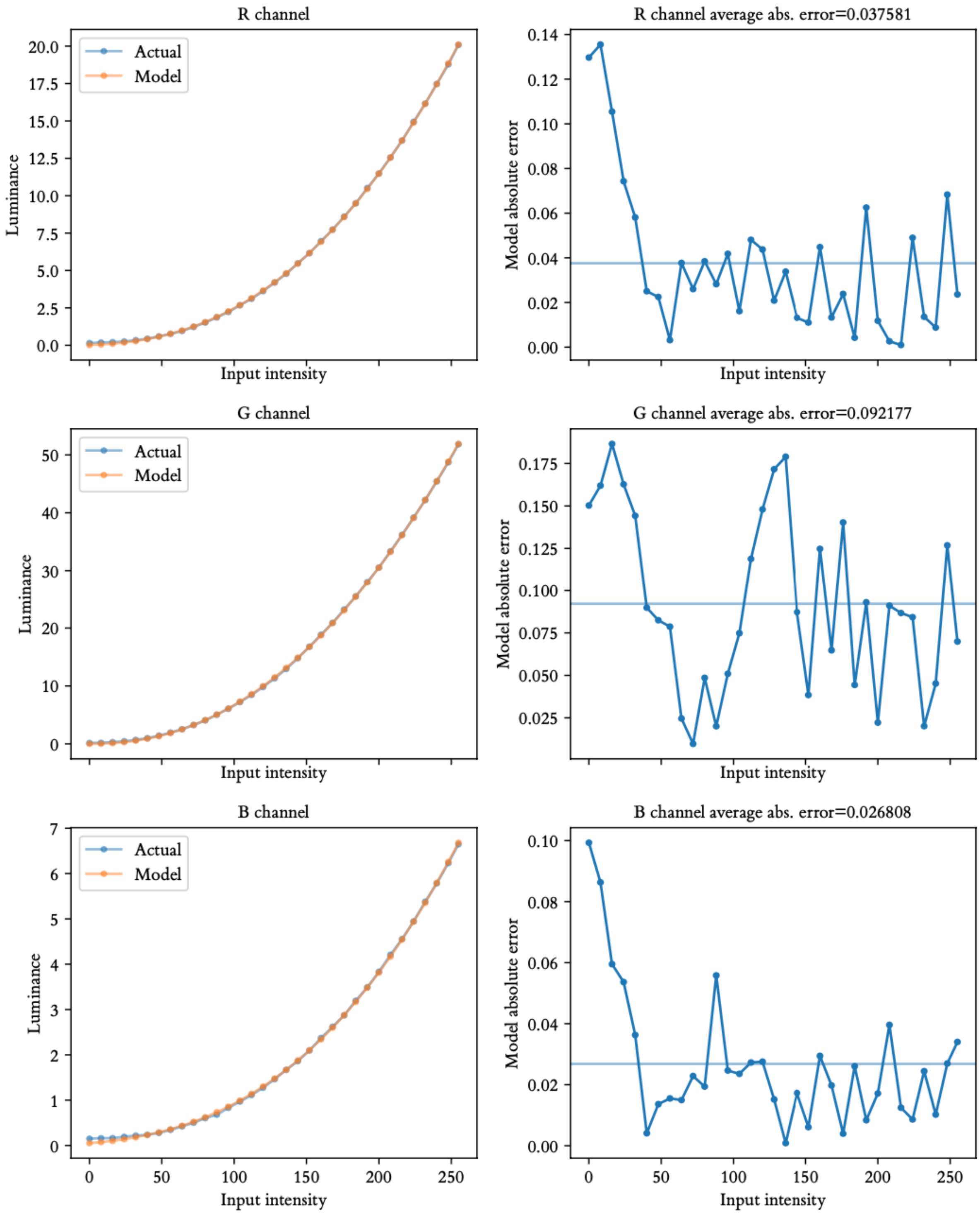


Figure 2: Comparison between actual luminance curve and fitted luminance curve.

b. Using the model found above, we can linearize the input RGB values. We use linear regression to find $\mathbf{T}'$ such that

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \mathbf{T}' \begin{bmatrix} R^l \\ G^l \\ B^l \end{bmatrix} \tag{1.2}$$

The linear regression gives

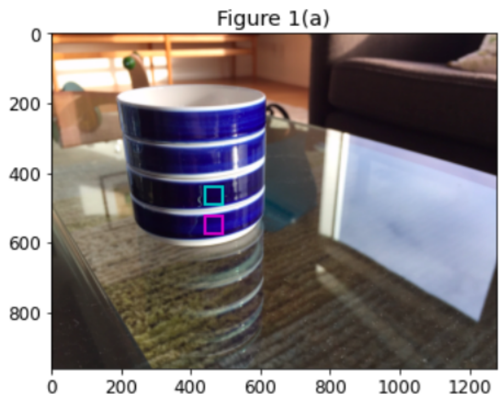
$$\mathbf{T}' = \begin{bmatrix} 0.14429044 & 0.09706078 & 0.05704238 \\ 0.07802181 & 0.20193985 & 0.02717 \\ 0.00744076 & 0.03424976 & 0.29510676 \end{bmatrix}. \tag{1.3}$$

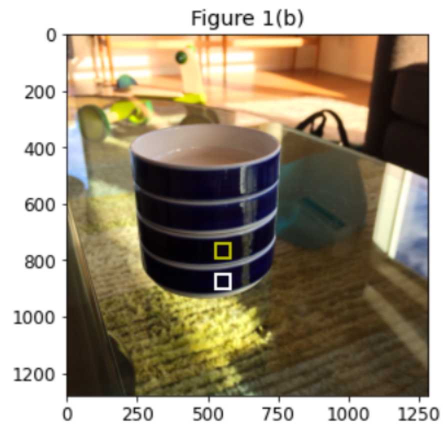
The matrix that transforms from XYZ to RGB is given by

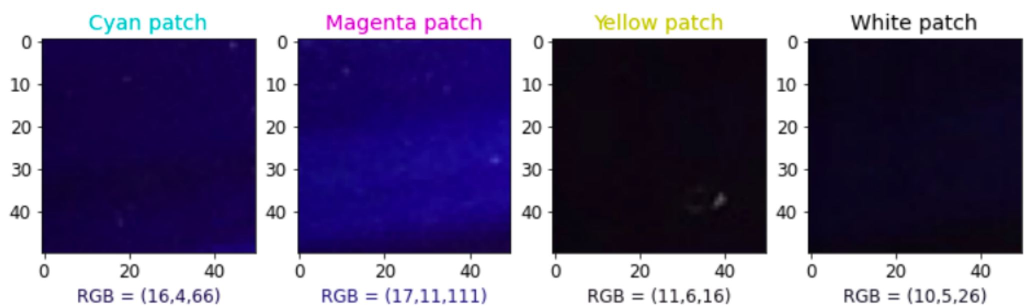
$$\mathbf{T} = \mathbf{T}'^{-1} = \begin{bmatrix} 9.28791115 & -4.22565516 & -1.4062488 \\ -3.61340806 & 6.6744885 & 0.08394104 \\ 0.18518469 & -0.66808881 & 3.41431892 \end{bmatrix}. \tag{1.4}$$

The mean $\Delta E_{a^*b^*} = 2.205100130180881$; the maximum $\Delta E_{a^*b^*} = 9.89850868036168$.

Question 2 (Tianqi Guo)







Cyan patch

R_s	G_s	B_s	R_l	G_l	B_l
16.43	4.20	66.64	0.63	0.04	13.56
X	Y	Z	L*	a*	b*
2.23	0.99	9.69	8.94	35.02	-54.96

Magenta patch

R_s	G_s	B_s	R_l	G_l	B_l
17.42	11.07	111.01	0.74	0.34	42.45
X	Y	Z	L*	a*	b*
6.53	2.98	30.35	19.97	48.74	-81.29

Yellow patch

R_s	G_s	B_s	R_l	G_l	B_l
11.81	6.35	16.06	0.33	0.11	0.62
X	Y	Z	L*	a*	b*
0.27	0.19	0.45	-1.72	9.11	-10.69

White patch

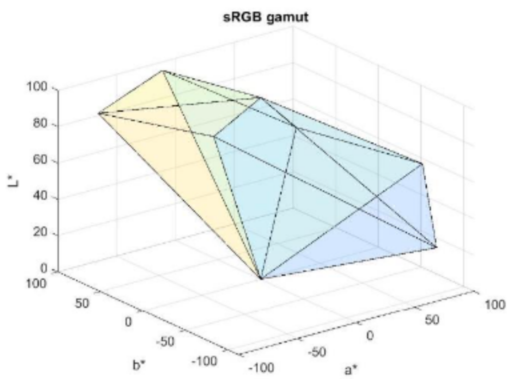
R_s	G_s	B_s	R_l	G_l	B_l
10.30	5.99	26.46	0.22	0.07	1.81
X	Y	Z	L*	a*	b*
0.38	0.21	1.30	-1.15	15.24	-24.60

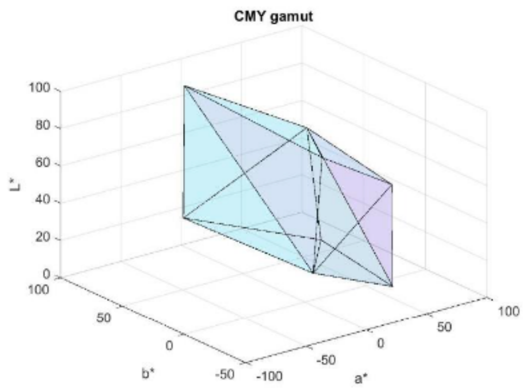
```
E_Lab_1 = ((L_a1-L_a2)**2 + (a_a1-a_a2)**2 + (b_a1-b_a2)**2)**0.5
E_Lab_2 = ((L_b1-L_b2)**2 + (a_b1-a_b2)**2 + (b_b1-b_b2)**2)**0.5
print('ΔLab of Figure 1a = ', E_Lab_1)
print('ΔLab of Figure 1b = ', E_Lab_2)
```

```
ΔLab of Figure 1a =  31.666703696368824
ΔLab of Figure 1b =  15.20774045921641
```

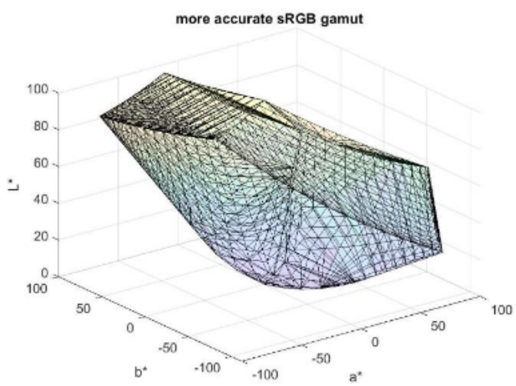
Question 3 (Kennedy F. Monaco)

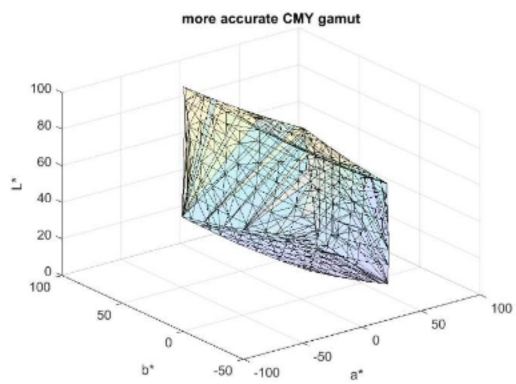
a. **Plot sRGB and CMY Indigo gamuts in L*a*b* space:**



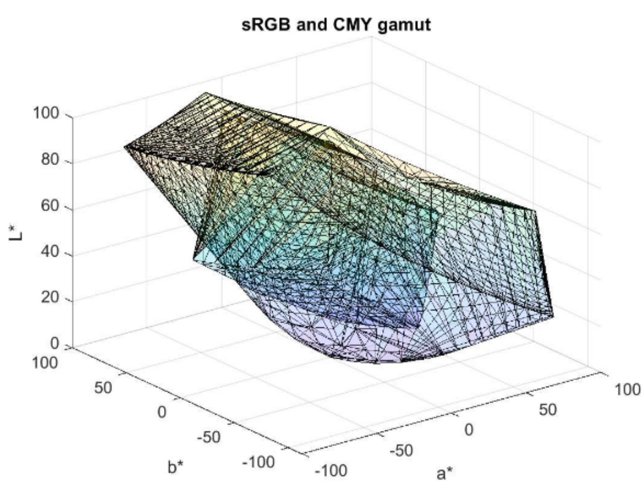


b. **Plot more accurate sRGB and CMY Indigo gamuts in L*a*b* space:**

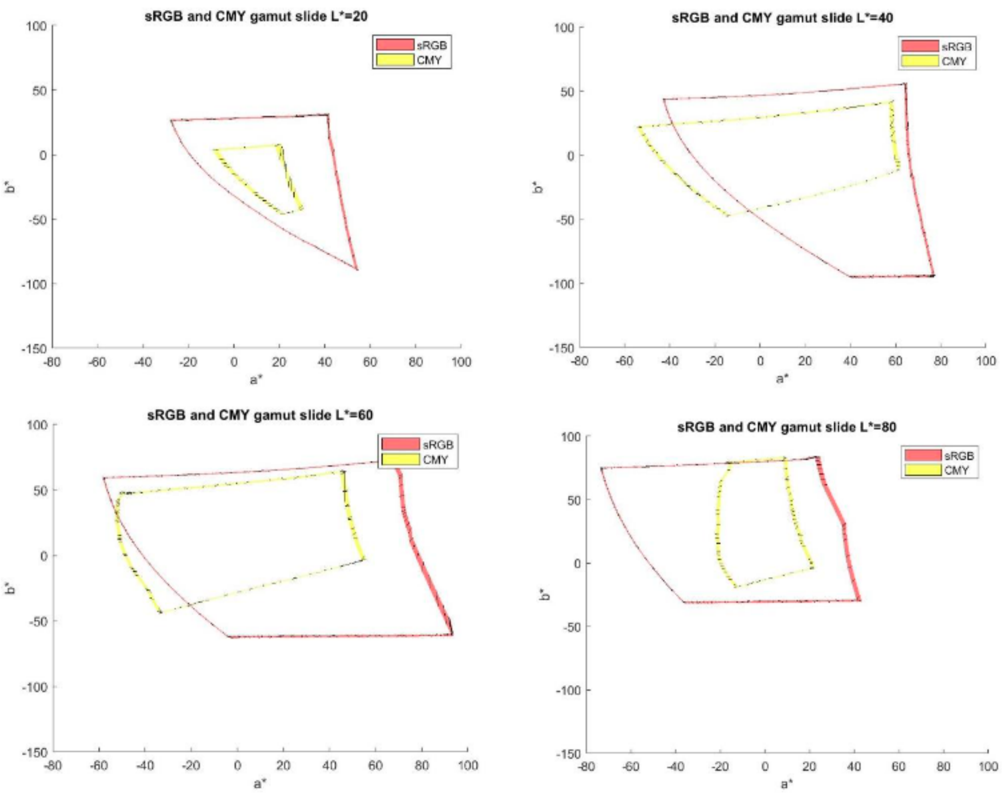




Show both gamuts on the same axes:



c. Plot slices of two gamuts for constant L*=[20,40,60,80]:



d. From the figure two gamuts put in the same axes, we can see that sRGB gamut is larger than CMY gamut in L*a*b* space. So sRGB gamut covers more color than CMY indigo gamut.

When the largest chroma value achieved, in sRGB gamut, the hue angle is around -45 or 135 degree. In CMY gamut, the hue angle is around 45 or 135 degree. As the luminance change the hue angle of largest chroma value might shift. A major difference can be found in slice L*=80, hue angle in sRGB gamut is around 135 degree, but in slice L*=40, the angle is -45 degree. Generally speaking these two gamuts shows different sensitivity for colors.

As L* increase from low value to high value, the largest chroma value shift from -45 hue angle to 135 hue angle, that shows these two gamuts include more yellow and green color when the luminance is high, and include more red color when luminance is low.

Please include the legend in the figure, such as this sample solution.