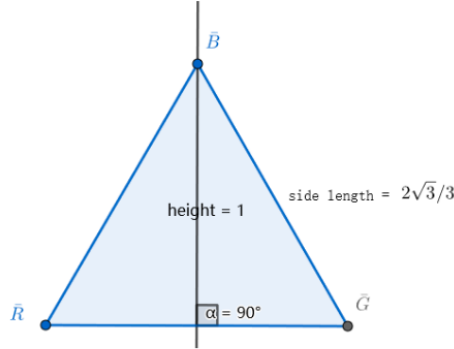# ECE 638 Homework 1 Sample Solution
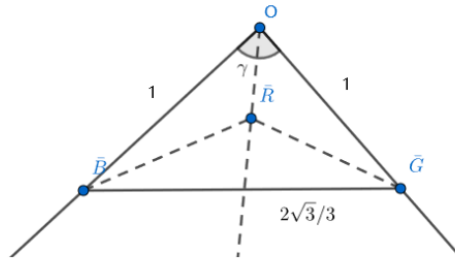
Question 1 (Fanbu)

1   Question 1

Because $r + g + b = 1$, the perpendicular of each side is 1, as shown here.

Since the triangle is an equilateral triangle, the length of each side is: $\bar{B}\tilde{G} = \bar{B}\bar{R} = \bar{R}\tilde{G} = 2\sqrt{3}/3$

Now, let's take a look at the coordinate system. If we put the origin point on top, then the tetrahedron looks like the figure below.

The angle between each of the three axes is the same:

$$\angle\gamma = \angle\bar{B}O\bar{R} = \angle\tilde{G}O\bar{R} = \angle\bar{B}O\tilde{G}$$

<center>1</center>

By the law of cosines, we know that

$$
\begin{aligned}
\cos\gamma &= \frac{(O\bar{B})^2 + (O\tilde{G})^2 - (\bar{B}\tilde{G})^2}{2(O\bar{B})(O\tilde{G})} \\
&= \frac{1 + 1 - \frac{4}{3}}{2} \\
&= \frac{1}{3}
\end{aligned}
$$

Hence the angle between each of the three axes is $\angle\gamma = \arccos\frac{1}{3} \approx 70.53°$

Question 2 (Shenyu)

2   problem 2

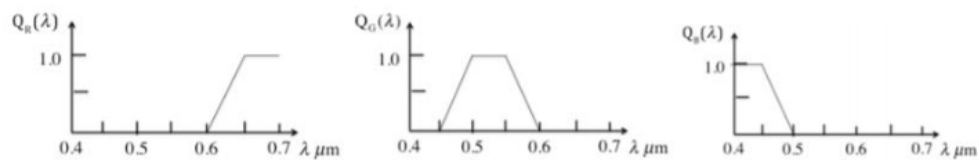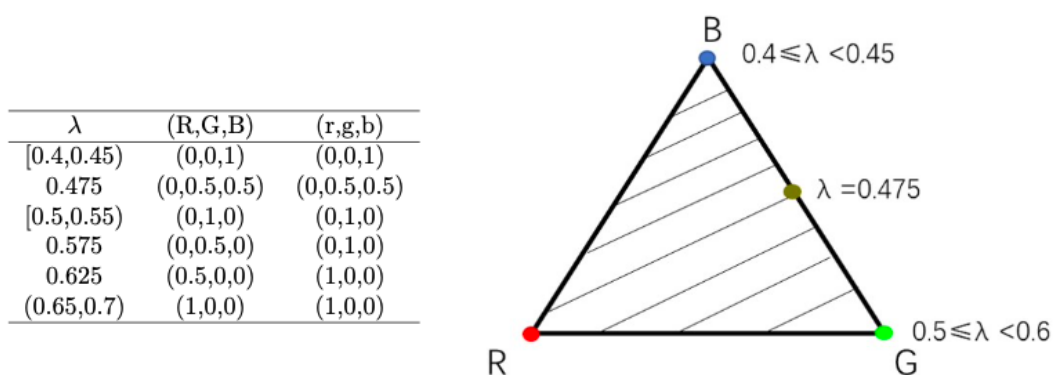Consider a trichromatic sensor with the response functions shown below:

Figure 3: Sensor Response

i. Plot the spectral locus and chromaticity gamut in the rgb sensor chromaticity plane.
Solve

λ	(R,G,B)	(r,g,b)
[0.4,0.45)	(0,0,1)	(0,0,1)
0.475	(0,0.5,0.5)	(0,0.5,0.5)
[0.5,0.55)	(0,1,0)	(0,1,0)
0.575	(0,0.5,0)	(0,1,0)
0.625	(0.5,0,0)	(1,0,0)
(0.65,0.7)	(1,0,0)	(1,0,0)

spectral locus and chromaticity gamut

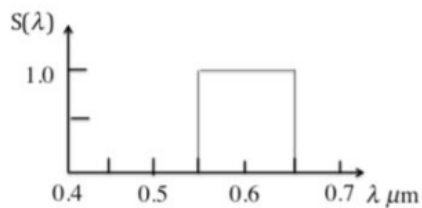ii.Find the response of the sensor to the stimulus.

Figure 4: Stimulus

Solve

$$R_s(\lambda) = \int S(\lambda)Q_R(\lambda)\,\mathrm{d}\lambda = 0.05 * 1/2 = 0.025$$

$$G_s(\lambda) = \int S(\lambda)Q_G(\lambda)\,\mathrm{d}\lambda = 0.05 * 1/2 = 0.025$$

$$B_s(\lambda) = \int S(\lambda)Q_B(\lambda)\,\mathrm{d}\lambda = 0$$

iii. Determine the amounts of each of three monochromatic primaries $P_B(\lambda) = \delta(\lambda - 0.45), P_G(\lambda) = \delta(\lambda - 0.55), P_R(\lambda) = \delta(\lambda - 0.65)$ required to match this stimulus. Is a direct match possible? Why or why not?

Solve

The primary response matrix A is

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Sensor response to stimulus

$$\vec{C_t} = \begin{bmatrix} 0.025 \\ 0.025 \\ 0 \end{bmatrix}$$

Match amount of primaries

$$\vec{p} = A^{-1} * \vec{C_t} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.025 \\ 0.025 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.025 \\ 0.025 \\ 0 \end{bmatrix}$$

So it is possible to perform a direct match.

$$T(\lambda) = 0.025\delta(\lambda - 0.65) + 0.025\delta(\lambda - 0.55)$$

Question 3 (Weichen)

3.

a.

$$\begin{bmatrix} r(\lambda) \\ g(\lambda) \\ b(\lambda) \end{bmatrix} = A^{-1} \begin{bmatrix} Q_R(\lambda) \\ Q_G(\lambda) \\ Q_B(\lambda) \end{bmatrix}$$
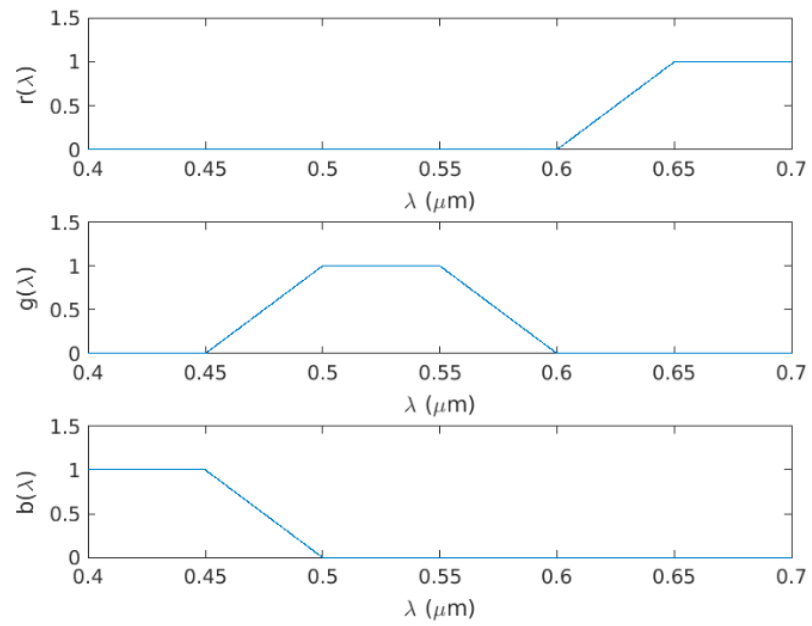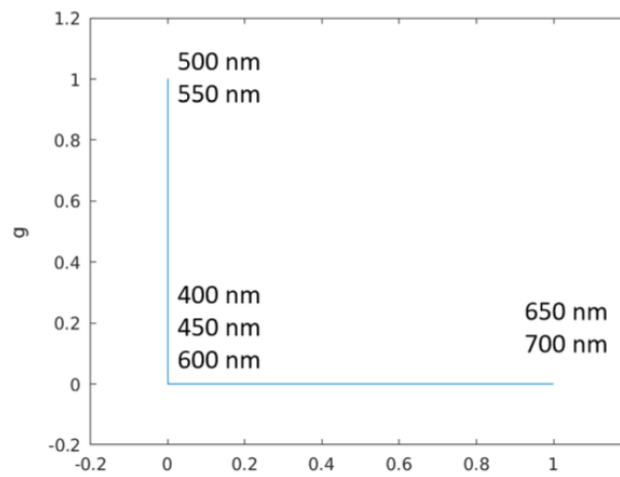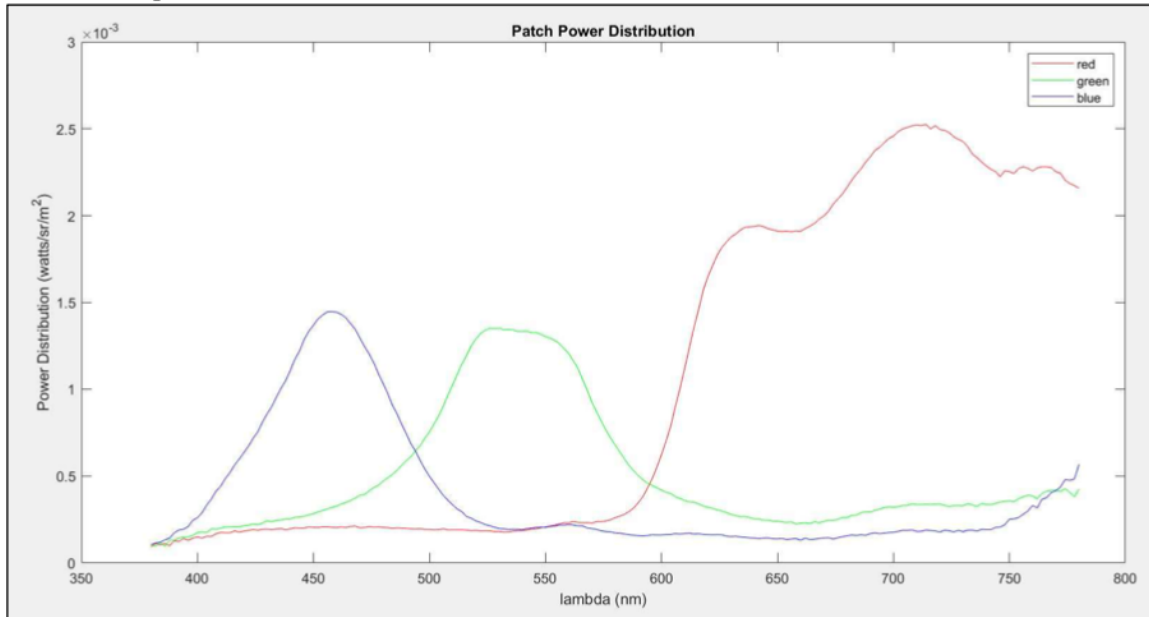
Fig.2: Color matching functions

b.

Fig.3: Spectral locus in the primary chromaticity (r,g)

Question 4 (Kennedy)

Problem 4

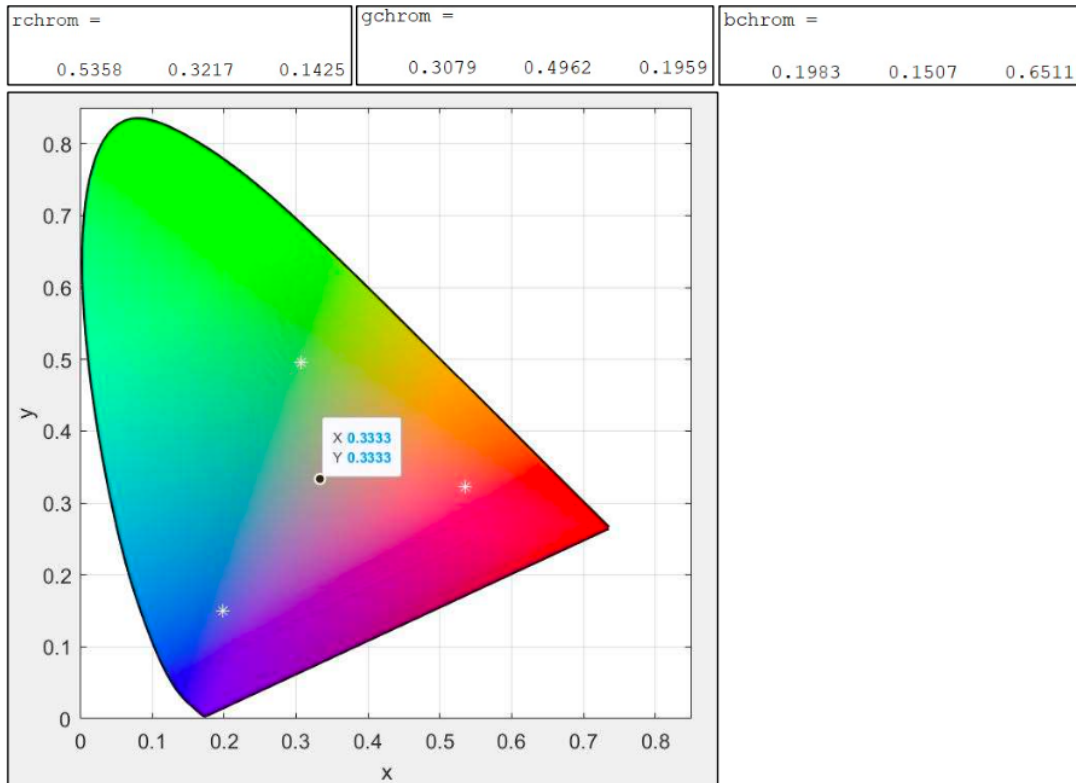## Part A: Plot Spectral Power Distributions

According to the plot of the Spectral Power Distributions of the red, green, and blue patches, the red patch has the highest power distribution over the largest range of wavelengths (~620 to 775). The green and blue patches have similar peak power, but the green patch covers a slightly larger range of wavelengths (~500 to 575) than the blue patch (~440 to 470).

Part B: CIE X, Y, Z Coordinates

rCIE =		
0.0791	0.0475	0.0210

gCIE =		
0.0588	0.0947	0.0374

bCIE =		
0.0352	0.0267	0.1155

Part C: CIE Chromaticity

rchrom =			gchrom =			bchrom =		
0.5358	0.3217	0.1425	0.3079	0.4962	0.1959	0.1983	0.1507	0.6511

The chromaticity coordinate of the Red Patch lies about midway between Equal Energy (1/3, 1/3) and the pure red end of the spectral locus, which indicates a reddish hue with medium saturation. The chromaticity coordinate of the Green Patch lies slightly closer to Equal Energy than the pure green curve of the spectral locus, which indicates a greenish hue with medium saturation. Finally, the chromaticity coordinate of the Blue Patch lies slightly closer to the pure blue end of the spectral locus than Equal Energy, which indicates a blue hue with medium saturation.

Part D: Chromaticity Value Comparison

Color Patch	Coordinate	Computed	PR-705	Percent Error
Red	x	0.5358	0.5359	0.018%
	y	0.3217	0.321	0.218%
Green	x	0.3079	0.3082	0.097%
	y	0.4962	0.496	0.040%
Blue	x	0.1983	0.1985	0.101%
	y	0.1507	0.1507	0.000%