

Be sure to turn in all your Matlab, C, or Python code for the problems below.

1. Spectral Neugebauer models applied to digital halftoning

Consider a two-colorant system with colorants C and M . Suppose that these colorants have ideal block spectral reflectances given by

$$P_C(\lambda) = \begin{cases} 1, & 400 < \lambda \leq 600, \\ 0, & \text{else.} \end{cases} \quad P_M(\lambda) = \begin{cases} 1, & 400 < \lambda \leq 500, 600 < \lambda \leq 700 \\ 0, & \text{else.} \end{cases}$$

These colorants are printed on ideal white media with spectral reflectance given by

$$P_W(\lambda) = \begin{cases} 1, & 400 < \lambda \leq 700, \\ 0, & \text{else.} \end{cases}$$

Throughout this problem, the units of λ are nanometers. Suppose that the fractional area coverage of each of the two colorants C and M within a patch of size $X \times X$ is a_C and a_M , respectively.

Suppose we have an instrument that measures the spatially averaged spectral reflectance within this $X \times X$ patch. So the instrument returns the function

$$S(\lambda) = \frac{1}{X^2} \int_{-X/2}^{X/2} \int_{-X/2}^{X/2} R(x, y, \lambda) dx dy$$

where $R(x, y, \lambda)$ denotes the spectral reflectance of the printed patch as a function of the spatial coordinates (x, y) , and thus is dependent on the halftoning algorithm that is used to print the patch.

We wish to consider two distinct special cases: (1) full *dot-off-dot* printing where there is no overlap of the two colorants anywhere within the $X \times X$ square, and (2) full *dot-on-dot* printing, where the two colorants overlap to the maximum extent possible, i.e. if $a_C < a_M$, then C is always printed with M , and never occurs alone; and if $a_M < a_C$, then M is always printed with C , and never occurs alone. For a given mixture (a_C, a_M) , let $S_{\text{dot-off-dot}}(\lambda)$ and $S_{\text{dot-on-dot}}(\lambda)$ denote the spatially averaged spectral reflectances returned by our instrument for these two cases.

- a. Sketch typical dot-off-dot and dot-on-dot halftone patterns for the case where $a_C = a_M = 0.5$.
- b. Determine $S_{\text{dot-off-dot}}(\lambda)$ and $S_{\text{dot-on-dot}}(\lambda)$ for the case where $a_C = a_M = 0.5$.
- c. Assuming that the viewer cannot resolve the structure of the printed halftone patterns, i.e. the patches appear to have a single uniform color, how would you describe the visual appearance of the printed patch for the two cases described above where we have full dot-off-dot and full dot-on-dot halftoning with $a_C = a_M = 0.5$?
- d. Sketch typical dot-off-dot and dot-on-dot halftone patterns for the case where $a_C = 0.5, a_M = 0.25$.

- e. Determine $S_{\text{dot-off-dot}}(\lambda)$ and $S_{\text{dot-on-dot}}(\lambda)$ for the case where $a_c = 0.5, a_M = 0.25$.
2. A first-order stationary random signal $X(t)$ obeys the following first-order probability density function:

$$f_X(x) = \begin{cases} |x|, & |x| < 1 \\ 0, & \text{else} \end{cases}$$

- a. Sketch $f_X(x)$
- b. Design a two-output-level, minimum-mean-squared error, non-uniform quantizer $Q(X)$ for $X(t)$. **Hint:** Be sure to exploit symmetry considerations as much as possible.
- c. Compute the ensemble-averaged, mean-squared error $\phi = E\{[Q(X) - X]^2\}$ that will result when this quantizer is used to quantize $X(t)$.
3. In class, we discussed quantization, which is important in many applications of imaging systems. Earlier, we also discussed sampling theory. The purpose of this problem is to use sampling theory to analyze the effect of quantization. For this problem, we will consider a uniform quantizer with step-size Δ . Let X denote the input to the quantizer; and let Y denote the output. The quantization operation can be expressed as $Y = Q(X) = k\Delta$, where $(k - \frac{1}{2})\Delta < X \leq (k + \frac{1}{2})\Delta$, and k is integer-valued.

In class, we discussed the fact that the quantizer error $D = Y - X$ can be approximated as a random variable that is uniformly distributed on the interval $[-\frac{1}{2}\Delta, \frac{1}{2}\Delta]$. Assume that X is a random variable with probability density function $f_X(x)$. The purpose of the first part of this problem is to find an exact expression for the probability density $f_Y(y)$ of the quantizer output in terms of $f_X(x)$. From the expression given in the preceding paragraph, it follows that

$$f_Y(y) = \sum_{k=-\infty}^{\infty} p_k \delta(y - k\Delta),$$

where

$$\begin{aligned} p_k &= \Pr\{Y = k\Delta\} \\ &= \int_{(k-\frac{1}{2})\Delta}^{(k+\frac{1}{2})\Delta} f_X(x) dx \end{aligned}$$

Now, let us see how we can use sampling theory to obtain an alternate expression for $f_Y(y)$ in terms of $f_X(x)$. Recall that we define the comb operator as

$$\text{comb}_T[f(x)] = \sum_{k=-\infty}^{\infty} f(kT)\delta(x - kT).$$

- a. Show that the density function for the quantizer output Y may be expressed as $f_Y(y) = \text{comb}_\Delta[w(x) * f_X(x)]$,

where * denotes convolution and $w(x)$ is an appropriately chosen function.

- b. Determine what is the function $w(x)$.

Let

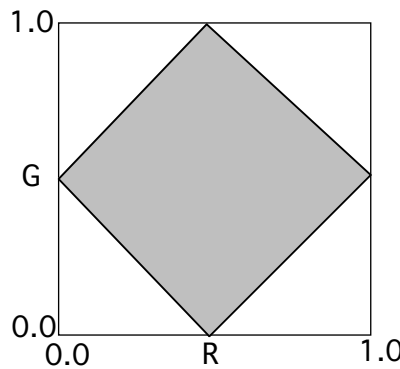
$$\varphi_X(u) = E\{e^{-j2\pi uX}\}$$

denote the characteristic function for the random variable X , where $E\{\cdot\}$ denotes statistical expectation and u is the frequency variable.

- c. Show that the characteristic function for the output Y of the quantizer may be expressed in terms of the characteristic function of the input X , as

$$\varphi_Y(u) = \frac{1}{\Delta} \sum_{\ell=-\infty}^{\infty} W(u - \frac{\ell}{\Delta}) \varphi_X(u - \frac{\ell}{\Delta})$$

4. Consider a bichromatic (duotone) image with histogram shown below



Assume that the histogram has constant value within the shaded area.

- a. Find the first four splits and determine the output colors for the following two methods of color image quantization:
 - i. median cut
 - ii. binary splitting
- b. Consider the application of SSQ to the above figure.
 - i. Suppose that we split the R axis first, and quantize it into 4 levels. Determine the optimal cell boundaries. You may use a built-in function for Lloyd-Max quantization to solve this part of the problem. But be sure to indicate what function you are using.
 - ii. Now suppose that we further split the leftmost column into 3 levels along the G axis. Determine the optimal cell boundaries and the output colors for this column. You may again use the built-in function for Lloyd-Max quantization to solve this part of the problem.

5. Uniform scalar quantization vs. vector quantization

- a. Write a program in either C, Python, or Matlab to perform uniform scalar quantization of color images. Turn in print-outs of your code, as well as printed images for $N = 256, 64,$ and 16 colors for the image `woman_color.tff`, which may be downloaded from the “Data Files” directory of the course website. Also, compute the mean-squared error for each of these cases
- b. Write a program in either C, Python, or Matlab to perform LBG quantization of color images. Don't worry about making the code efficient. Turn in print-outs of your code, as well as printed images for $N = 256, 64,$ and 16 colors for the image `woman_color.tff`. Also, compute the mean-squared error for each of these cases.
- c. On the same axes, plot the rate-distortion curves for your uniform scalar and LBG quantizers. Discuss the relative performance of the two quantizers.