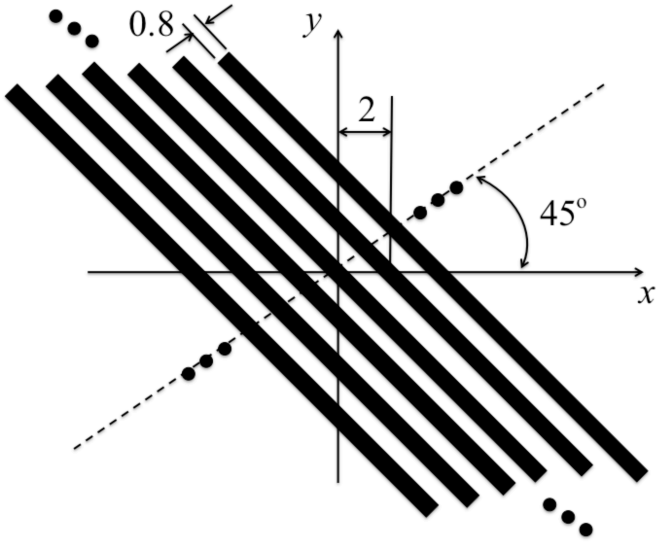
Due Monday, 10/25/21 at 11:59 PM EDT

ECE 638 **Homework No. 4** **Fall 2021**

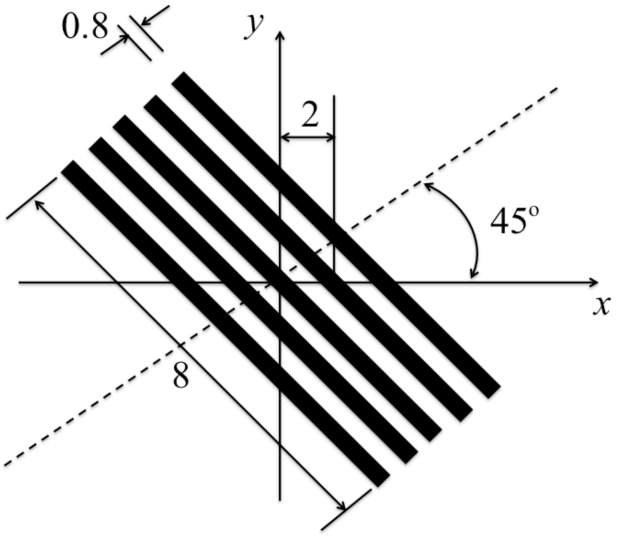
1. a. Consider the signal $f(x,y)$ shown below



which has value 1 in the shaded areas, and value 0 , elsewhere. The bars are infinitely long; and there are infinitely many of them.

Find a simple expression for the CSFT $F(u,v)$ of this signal, and sketch it.

b. Consider the signal $g(x,y)$ shown below



which has value 1 in the shaded areas, and value 0 , elsewhere. Here the bars have length 8; and there are 5 of them.

Find a simple expression for the CSFT $G(u,v)$ of this signal, and sketch it.

Hint: For both parts a. and b., use the rotational property of the 2-D CSFT.

2. Consider the 2-D signal

$$f(x,y) = \begin{cases} (1-|x|)(1-|y|), & 0 \le |x| \text{ and } |y| \le 1 \\ 0, & \text{else} \end{cases}$$

a. Carefully sketch $f(x,y)$.

b. Find a closed-form expression for the 2-D Continuous-Space Fourier Transform (CSFT) $F(u,v)$ of $f(x,y)$ that does not contain any operators.

c. Sketch a fully dimensioned plot for $F(u,v)$. (Here, "fully dimensioned" means that it will be clear where $F(u,v)$ has value 0. At other values for (u,v), your drawing can just be a rough sketch.)

Define a new signal

$$g(x,y) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} f(x-m, y-n)$$

d. Sketch $g(x,y)$ well enough to show that you know what it looks like.

e. Find a closed-form expression for the 2-D Continuous-Space Fourier Transform (CSFT) $G(u,v)$ of $g(x,y)$ that does not contain any operators.

f. Carefully sketch a fully dimensioned plot for $G(u,v)$.

Hint: For this problem, use separability.

3. An imaging system has magnification 2 and point spread function

$$h(x,y) = \text{rect}(x, y/2)$$

a. Find an expression for the image $g(x,y)$ of the object $f(x,y) = \text{rect}(x/5, y/5)$. **Note:** This problem is a lot easier if you take advantage of separability.

b. Sketch both $f(x,y)$ and $g(x,y)$.

4. You have a photograph with an image given by

$$f(x,y) = 1 + \cos\left[2\pi(580x + 20y)\right].$$

a. Sketch what $f(x,y)$ looks like.

You scan this photograph with your new 600 dots/inch flat bed scanner, and display the image on a high resolution 600 dots/inch monitor.

b. Assuming that the scanner acts like an ideal sampler, and the monitor acts like an ideal reconstruction filter with cutoff frequency at 300 dots/inch, find the displayed image $g(x,y)$.

c. Sketch what $g(x,y)$ looks like.

5. Consider a display system

$$f_{\text{display}}(x,y) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} f_{\text{original}}(mX,nX)\, p(x-mX, y-nX)$$

with two possible spot profiles:

$$p_{\text{zoh}}(x,y) = \text{rect}(x/X, y/X)$$

and

$$p_{\text{rcosine}}(x,y) = \begin{cases} \frac{1}{4}\left(1+\cos\left(\pi x/X\right)\right)\left(1+\cos\left(\pi y/X\right)\right), & 0 \le |x| \text{ and } |y| \le X \\ 0, & \text{else} \end{cases}$$

a. Sketch the two possible spot profiles

b. Determine closed-form expressions for the CSFT of these two spot profiles.

Hint: Use separability.

c. Plot these CSFTs along the u-axis only ($v = 0$) on the same graph.

d. Determine a closed-form expression for the CSFT $F_{\text{display}}(u,v)$ in terms of $F_{\text{original}}(u,v)$. Your answer should not contain any operators other than summation signs.

e. In general terms, compare the characteristics of the displayed image for these two different spot profiles, based on the shape of their CSFTs.