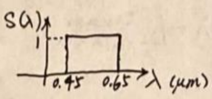
ECE 638 Midterm 1 Sample Solution

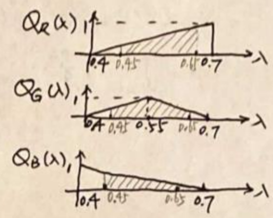
Question 1 (Xiaoyu Ji)

1\. (a) response (R_s, G_s, B_s) can be calculated as below:

$$\begin{cases} R_s = \int S(\lambda) Q_R(\lambda)\, d\lambda \\ G_s = \int S(\lambda) Q_G(\lambda)\, d\lambda \\ B_s = \int S(\lambda) Q_B(\lambda)\, d\lambda \end{cases}$$

where



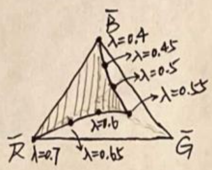


So $R_s = \int S(\lambda) Q_R(\lambda) d\lambda = \int_{0.45}^{0.65} Q_R(\lambda) d\lambda = 0.1$

$G_s = \int S(\lambda) Q_G(\lambda) d\lambda = \int_{0.45}^{0.65} Q_G(\lambda) d\lambda = 0.1333$

$B_s = \int_{0.45}^{0.65} Q_B(\lambda) d\lambda = 0.1$

(b)



λ	(r, g, b)	(R, G, B)
0.4	(0, 0, 1)	(0, 0, 1)
0.45	(0.125, 0.25, 0.625)	($\frac{1}{6}$, $\frac{1}{3}$, $\frac{5}{6}$)
0.5	(0.2, 0.4, 0.4)	($\frac{1}{3}$, $\frac{2}{3}$, $\frac{2}{3}$)
0.55	(0.25, 0.5, 0.25)	(0.5, 1, 0.5)
0.6	(0.4, 0.4, 0.2)	($\frac{2}{3}$, $\frac{2}{3}$, $\frac{1}{3}$)
0.65	(0.625, 0.25, 0.125)	($\frac{5}{6}$, $\frac{1}{3}$, $\frac{1}{6}$)
0.7	(1, 0, 0)	(1, 0, 0)

(c) From the figure of $P_B(\lambda)$, $P_G(\lambda)$, $P_R(\lambda)$, combine with $Q_R(\lambda)$, $Q_G(\lambda)$, $Q_B(\lambda)$

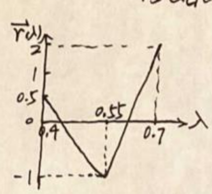
$A = [a_{ij}]$ where $a_{ij} = \int P_j(\lambda) Q_i(\lambda) d\lambda \quad i, j = R, G, B$

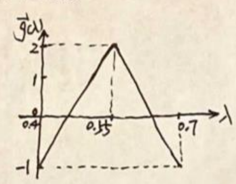
So $A = \begin{bmatrix} \frac{5}{6} & \frac{2}{3} & \frac{1}{6} \\ \frac{1}{3} & \frac{2}{3} & \frac{1}{3} \\ \frac{1}{6} & \frac{1}{3} & \frac{5}{6} \end{bmatrix}$ $\quad A^{-1} = \begin{bmatrix} 2 & -2.25 & 0.5 \\ -1 & 3 & -1 \\ 0 & -0.75 & 1.5 \end{bmatrix}$ (use MATLAB to calculate)

$$P = A^{-1} \vec{C}_T = A^{-1} \begin{bmatrix} R_s \\ G_s \\ B_s \end{bmatrix} = \begin{bmatrix} 2 & -2.25 & 0.5 \\ -1 & 3 & -1 \\ 0 & -0.75 & 1.5 \end{bmatrix} \begin{bmatrix} 0.1 \\ 0.133 \\ 0.1 \end{bmatrix} = \begin{bmatrix} -0.05 \\ 0.2 \\ 0.05 \end{bmatrix}$$

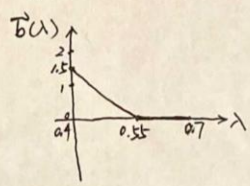
(d) $$\begin{bmatrix} \vec{r}(\lambda) \\ \vec{g}(\lambda) \\ \vec{b}(\lambda) \end{bmatrix} = A^{-1} \begin{bmatrix} Q_R(\lambda) \\ Q_G(\lambda) \\ Q_B(\lambda) \end{bmatrix} = \begin{bmatrix} 2 & -2.25 & 0.5 \\ -1 & 3 & -1 \\ 0 & -0.75 & 1.5 \end{bmatrix} \begin{bmatrix} Q_R(\lambda) \\ Q_G(\lambda) \\ Q_B(\lambda) \end{bmatrix} = \begin{bmatrix} 2Q_R(\lambda) - 2.25Q_G(\lambda) + 0.5Q_B(\lambda) \\ -Q_R(\lambda) + 3Q_G(\lambda) - Q_B(\lambda) \\ -0.75Q_G(\lambda) + 1.5Q_B(\lambda) \end{bmatrix}$$

$$\begin{cases} \vec{r}(\lambda) = 2Q_R(\lambda) - 2.25Q_G(\lambda) + 0.5Q_B(\lambda) \\ \vec{g}(\lambda) = -Q_R(\lambda) + 3Q_G(\lambda) - Q_B(\lambda) \\ \vec{b}(\lambda) = -0.75Q_G(\lambda) + 1.5Q_B(\lambda) \end{cases}$$





$$\Rightarrow \begin{cases} \vec{r}(\lambda) = \begin{cases} 0.5 - 10(x - 0.4) & 0.4 \le x \le 0.55 \\ -4 + 20(x - 0.4) & 0.55 \le x \le 0.7 \end{cases} \\ \vec{g}(\lambda) = \begin{cases} 20(x - 0.4) - 1 & 0.4 \le x \le 0.55 \\ 5 - 20(x - 0.4) & 0.55 \le x \le 0.7 \end{cases} \\ \vec{b}(\lambda) = \begin{cases} 1.5 - 10(x - 0.4) & 0.4 \le x \le 0.55 \\ 0 & 0.55 \le x \le 0.7 \end{cases} \end{cases}$$



(e) $P = A^{-1}\begin{bmatrix} R_s \\ G_s \\ B_s \end{bmatrix} = A^{-1}\int S(\lambda)\begin{bmatrix} Q_R(\lambda) \\ Q_G(\lambda) \\ Q_B(\lambda) \end{bmatrix} d\lambda = \int S(\lambda)\, A^{-1}\begin{bmatrix} Q_R(\lambda) \\ Q_G(\lambda) \\ Q_B(\lambda) \end{bmatrix} d\lambda = \int S(\lambda)\begin{bmatrix} \vec{r}(\lambda) \\ \vec{g}(\lambda) \\ \vec{b}(\lambda) \end{bmatrix} d\lambda$

use $[\vec{r}(\lambda), \vec{g}(\lambda), \vec{b}(\lambda)]$ in (d)

$$P = \int_{0.45}^{0.65} \begin{bmatrix} \vec{r}(\lambda) \\ \vec{g}(\lambda) \\ \vec{b}(\lambda) \end{bmatrix} d\lambda = \begin{bmatrix} -\frac{1}{2}\times 1\times 0.1 + 0 \\ \frac{1}{2}\times 2\times 0.1\times 2 \\ \frac{1}{2}\times 0.1\times \frac{2}{3}\times 1.5 + 0 \end{bmatrix} = \begin{bmatrix} -0.05 \\ 0.2 \\ 0.05 \end{bmatrix}$$

Question 2 (Kennedy Monaco)

$$S = \begin{bmatrix} 0.25 & 1 \\ 1 & 0.5 \\ 0.5 & 0.25 \end{bmatrix}$$

Figure 2.1: *Sensor Response Matrix*

a. Find the response of this sensor to the stimulus $\vec{n} = \begin{bmatrix} 1 & 0.5 & 1 \end{bmatrix}^T$

Solution:
Given the Sensor Response Matrix, the response of a Sensor to a given Stimulus is given by:

$$\vec{q} = S^T \vec{n}$$

$$\vec{q} = S^T \vec{n} = \begin{bmatrix} 0.25 & 1 & 0.5 \\ 1 & 0.5 & 0.25 \end{bmatrix} \begin{bmatrix} 1 \\ 0.5 \\ 1 \end{bmatrix}$$

$$\vec{q} = \begin{bmatrix} 1.25 \\ 1.5 \end{bmatrix}$$

b. Find the projection operator $\mathbf{R} = \mathbf{S}\left(\mathbf{S}^T\mathbf{S}\right)^{-1}\mathbf{S}^T$ for this sensor.

Solution:

The expression is expanded below and was calculated using MATLAB.

$$R = S(S^TS)^{-1}S^T = \begin{bmatrix} 0.25 & 1 \\ 1 & 0.5 \\ 0.5 & 0.25 \end{bmatrix} \left(\begin{bmatrix} 0.25 & 1 & 0.5 \\ 1 & 0.5 & 0.25 \end{bmatrix} \begin{bmatrix} 0.25 & 1 \\ 1 & 0.5 \\ 0.5 & 0.25 \end{bmatrix} \right)^{-1} \begin{bmatrix} 0.25 & 1 & 0.5 \\ 1 & 0.5 & 0.25 \end{bmatrix}$$

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.8 & 0.4 \\ 0 & 0.4 & 0.2 \end{bmatrix}$$

c. Find the fundamental component $\vec{n}^*$ for this stimulus.
Solution:
The fundamental component of a stimulus is given by:

$$\vec{n}^* = R\vec{n}$$

$$\vec{n}^* = R\vec{n} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.8 & 0.4 \\ 0 & 0.4 & 0.2 \end{bmatrix} \begin{bmatrix} 1 \\ 0.5 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0.8 \\ 0.4 \end{bmatrix}$$

$$\vec{n}^* = \begin{bmatrix} 1 \\ 0.8 \\ 0.4 \end{bmatrix}$$

d. Find the black-space component $\vec{n}^c$ for the stimulus.

Solution:

The black-space component of the stimulus is given by:

$$\vec{n}^c = \vec{n} - \vec{n}^*$$

$$\vec{n}^c = \vec{n} - \vec{n}^* = \begin{bmatrix} 1 \\ 0.5 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 0.8 \\ 0.4 \end{bmatrix} = \begin{bmatrix} 0 \\ -0.3 \\ 0.6 \end{bmatrix}$$

$$\vec{n}^c = \begin{bmatrix} 0 \\ -0.3 \\ 0.6 \end{bmatrix}$$

e. Find a metamer $\vec{n}'$ to $\vec{n}$ such that $\vec{n}' \neq \vec{n}$.

Solution:

Any metamer of a stimulus for a sensor can be found as:

$$\vec{n}' = \vec{n}^* + k\vec{n}^c$$

for any constant k.

Let $k = 2$, then a metamer for $\vec{n}$ is:

$$\vec{n}' = \begin{bmatrix} 1 \\ 0.8 \\ 0.4 \end{bmatrix} + 2 \begin{bmatrix} 0 \\ -0.3 \\ 0.6 \end{bmatrix} = \begin{bmatrix} 1 \\ 0.2 \\ 1.6 \end{bmatrix}$$

$$\vec{n}' = \begin{bmatrix} 1 \\ 0.2 \\ 1.6 \end{bmatrix}$$

Question 3 (Seto, Marshia)

a. (5) Using the two images of the blue dishes, as illustrated in Fig. 1 above, calculate and report the average RGB values within each red square.

For Fig. 1a:

$$R_{average} = 254.723$$
$$G_{average} = 253.874$$
$$B_{average} = 251.221$$

For Fig. 1b:

$$R_{average} = 203.595$$
$$G_{average} = 190.914$$
$$B_{average} = 184.424$$

b. (5) Using the transformation that you developed in your solution to Problem 2 from HW No. 3, or the posted solution to that problem (be sure to document which you are using), compute the CIE XYZ values for these two patches.

(Using transformation from sRGB.pdf)

$$\begin{bmatrix} X_{D50} \\ Y_{D50} \\ Z_{D50} \end{bmatrix} = \begin{bmatrix} 0.436030342570117 & 0.385101860087134 & 0.143067806654203 \\ 0.222438466210245 & 0.716942745571917 & 0.060618777416563 \\ 0.013897440074263 & 0.097076381494207 & 0.713926257896652 \end{bmatrix} \begin{bmatrix} R_L \\ G_L \\ B_L \end{bmatrix}$$

First, we linearize and de-gamma the sRGB figures (using $\gamma = 2.2$). Then, we use the transformation matrix given in HW3.pdf in Problem 3 Part a (sRGB.pdf) as pasted above to compute the CIE XYZ values.

For Fig. 1a:

$$X_{average} = 0.955$$
$$Y_{average} = 0.991$$
$$Z_{average} = 0.801$$

For Fig. 1b:

$$X_{average} = 0.540$$
$$Y_{average} = 0.545$$
$$Z_{average} = 0.410$$

c. (5) Compute the CIE XYZ xy chromaticity coordinates for these two patches.

Using $X_{average}$, $Y_{average}$, $Z_{average}$ of Fig. 1a and 1b we got above, calculate the xyY values as follows:

$$x = \frac{X}{X + Y + Z}$$
$$y = \frac{Y}{X + Y + Z}$$

$$Y = Y$$

Then using the x and y value we got the chromaticity coordinates as follows:

For Fig. 1a:

$$xy\ chromaticity\ coordinates\ =\ (0.3477, 0.3607)$$

For Fig. 1b:

$$xy\ chromaticity\ coordinates\ =\ (0.3611, 0.3645)$$

d. (5) On Fig. 1(3.11) from Problem 3 of HW No. 2, plot the xy chromaticity points for these two patches.

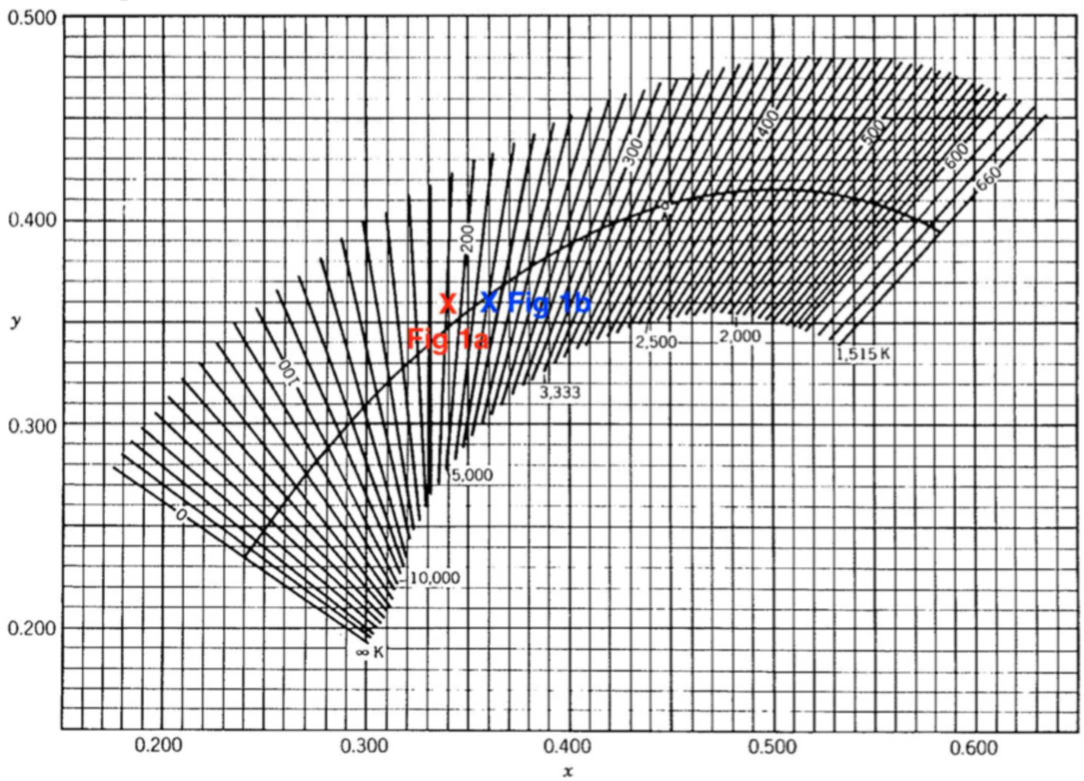


Fig. 1(3.11). **CIE 1931 (x, y)-chromaticity diagram showing isotemperature lines as computed by Kelly (1963).**

Legend:

(0.34, 0.36) Fig. 1a

(0.36, 0.36) Fig. 1b

e. (5) From Fig. 1(3.11), estimate the correlated color temperature of each patch.

For Fig. 1a:

$$color\ temperature\ = \sim 5{,}260\text{K}$$

For Fig. 1b:

$$color\ temperature\ = \sim 4{,}545\text{K}$$

Question 4 (Shenyu Lu)

4(a). The RGB components of the Color Checker shot by iPhone are saved in the file named 'Iphone.excel'. In addition to the RGB values, there is also the ROI information of each patch. I attached a screenshot of the file.

		R	G	B	ROI_position			
1	A1	2.14E+02	205.2275	199.4875	85.74432	53.99811	18.97096	18.97096
2	A2	19.73183	18.57143	17.05263	84.98548	97.25189	19.7298	17.45328
3	A3	101.7193	98.30326	91.95489	83.4678	143.541	19.7298	18.21212
4	A4	215.4085	207.8571	202.4411	81.95013	184.5183	19.7298	18.97096
5	A5	16.54293	16.54293	14.54293	81.19129	230.0486	20.48864	16.69444
6	A6	99.98965	96.88199	92.1118	78.15593	274.0612	22.00631	19.7298

Figure 1 Sampling point of the Color Checker

4(b).

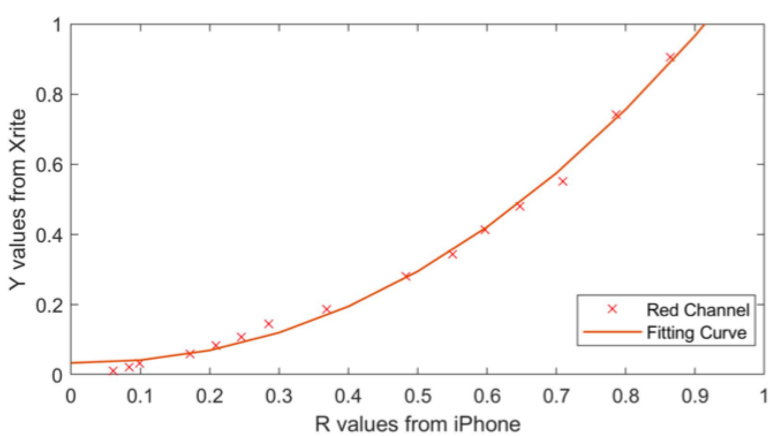


Figure 2 The normalized data from Red channel and the corresponding fitting Curve.

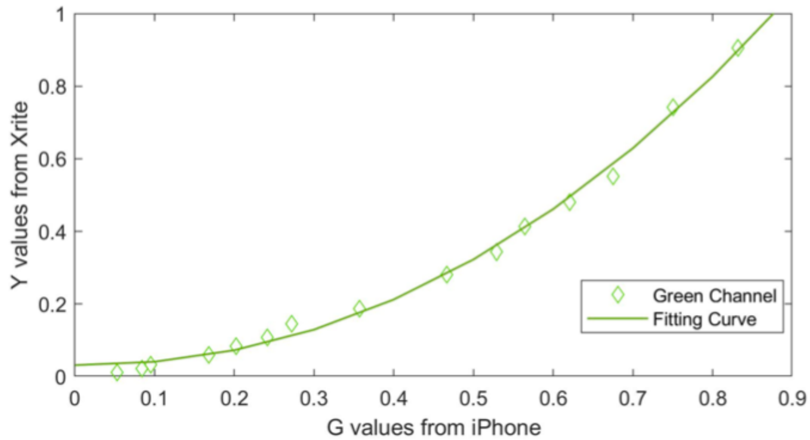


Figure 3 The normalized data from Green channel and the corresponding fitting Curve.

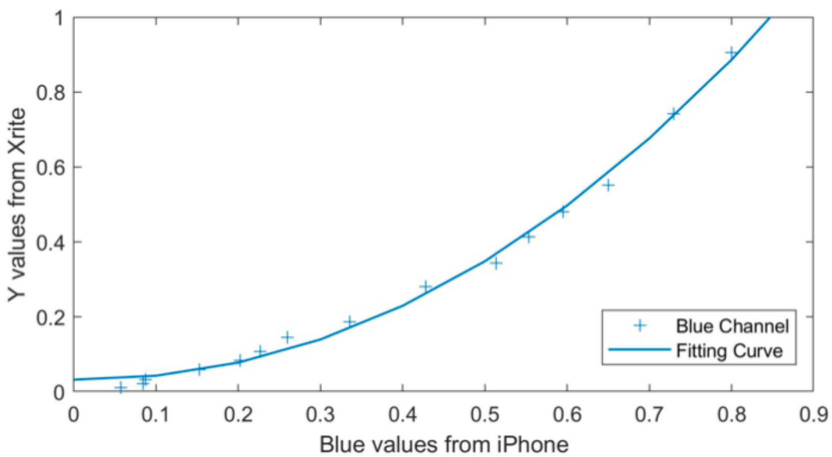


Figure 4 The normalized data from Blue channel and the corresponding fitting Curve.

The model is given by:

$$Y = a * (\frac{R}{255})^{\gamma} + c$$

$$Y = a * (\frac{G}{255})^{\gamma} + c$$

$$Y = a * (\frac{B}{255})^{\gamma} + c$$

The parameters for each curve are following

	a	γ	c
R	1.16	2.167	0.03262
G	1.281	2.137	0.03152
B	1.369	2.115	0.03262

4(c) Performing linearization on RGB channels and normalized those values. The matrix of linear RGB is called $\mathbf{L_{RGB}}$

The matrix of CIE XYZ corresponding the same patch of the linear RGB is called $\mathbf{M_{xyz}}$

What we want to find is

$$\mathbf{M}_{\mathrm{xyz}} = \boldsymbol{T}\mathbf{L}_{RGB}$$

Where $\mathbf{M}_{\mathrm{xyz}} = \begin{bmatrix} \mathrm{X_{patch1}} & \cdots & \mathrm{X_{patch140}} \\ \mathrm{Y_{patch1}} & \cdots & \mathrm{Y_{patch140}} \\ \mathrm{Z_{patch1}} & \cdots & \mathrm{Z_{patch140}} \end{bmatrix}$, $\mathbf{L}_{RGB} = \begin{bmatrix} \mathrm{R_{Linear\_patch1}} & \cdots & \mathrm{R_{Linear\_patch140}} \\ \mathrm{G_{Linear\_patch1}} & \cdots & \mathrm{G_{Linear\_patch140}} \\ \mathrm{B_{Linear\_patch1}} & \cdots & \mathrm{B_{Linear\_patch140}} \end{bmatrix}$

The least squares method is applied to approximate the transformation matrix **T**:

$$\mathbf{T} = \mathbf{M}_{\mathrm{xyz}}\mathbf{L}_{RGB}^{\mathrm{T}}(\mathbf{L}_{RGB}\mathbf{L}_{RGB}^{\mathrm{T}})^{-1}$$

Which is equal to

$$\mathbf{T} = \begin{bmatrix} 0.6798 & 0.4650 & 0.1998 \\ 0.3261 & 0.9745 & 0.1340 \\ 0.0435 & 0.0197 & 1.1654 \end{bmatrix}$$

4(d) The formula that used to transform CIE XYZ to CIE L*a*b* under D50 illuminant is

$$L^* = 116(\frac{Y}{1})^{\frac{1}{3}} - 16$$

$$a^* = 500[(\frac{X}{0.942})^{\frac{1}{3}} - (\frac{Y}{1})^{\frac{1}{3}}]$$

$$b^* = 200[(\frac{Y}{1})^{\frac{1}{3}} - (\frac{Z}{0.8249})^{\frac{1}{3}}]$$

For $\frac{Y}{1}, \frac{X}{0.942}, \frac{Z}{0.8249} > \left(\frac{6}{29}\right)^3$

Otherwise:

$$L^* = 116f(\frac{Y}{1}) - 16$$

$$a^* = 500\left[f\left(\frac{X}{0.942}\right) - f\left(\frac{Y}{1}\right)\right]$$

$$b^* = 200\left[f\left(\frac{Y}{1}\right) - f\left(\frac{Z}{0.8249}\right)\right]$$

Where $f(x) = \frac{x}{3*\left(\frac{6}{29}\right)^2} + \frac{4}{29}$

4(e) The average error and the standard deviation of the error in units of DELTA E_76.

$$\Delta E^*_{ab} = \sqrt{(L_{real} - L_{transform})^2 + (a_{real} - a_{transform})^2 + (b_{real} - b_{transform})^2}$$

The subscript *real* is obtained by X-Rite instrument, the subscript *transform* is obtained by Transformation matrix.

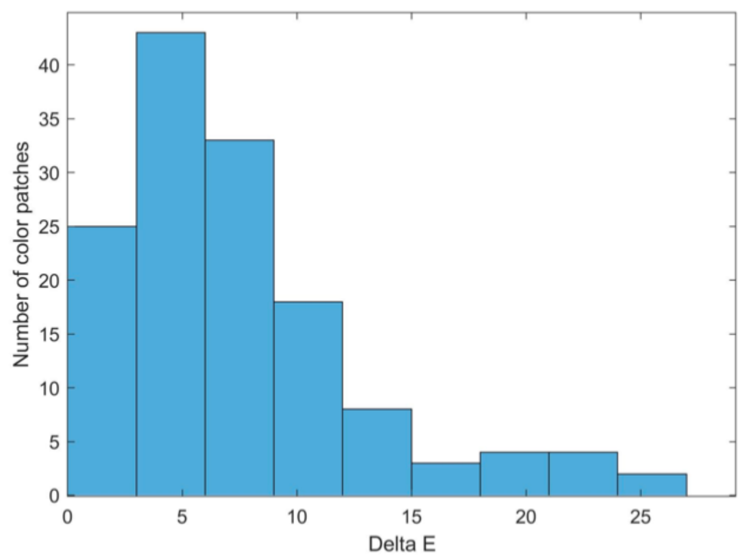


Figure 5 Histogram of ΔE^*_{ab}

Statistic	
Average ΔE^*_{ab}	7.5460
Standard deviation	5.3861