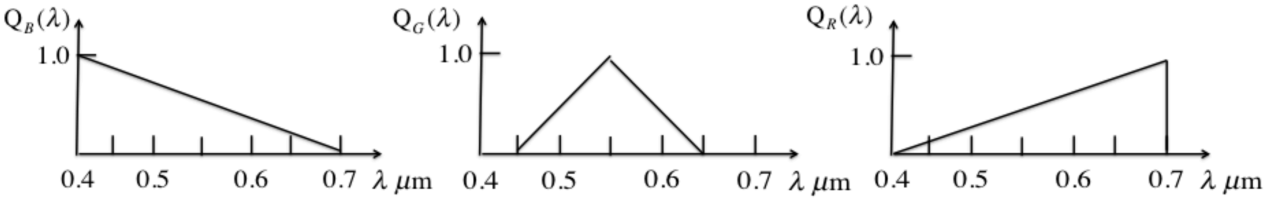
07 November 2019 **Name:** Solution

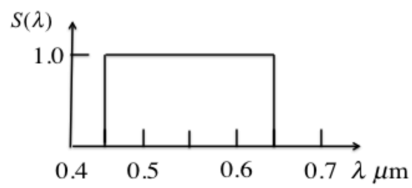
ECE 638 **Exam No. 1** **Fall 2019**

This exam is closed book and closed notes. **No calculators are permitted. Mobile phones must be put away. Smart watches must be put away**. You have 120 minutes to work the following **five** problems that are worth a total of 165 pts. (Please see the last page of the exam for a summary of the point assignment.) To obtain maximum partial credit, be sure to show the complete derivation of your answers. However, you do not need to derive anything that can be found on the provided sheet of formulas. Just cite where you found it. See p. 16 for the formulas to invert a 3×3 matrix, which you will have to do to solve some of these problems.

1. (40 pts.) Consider a three-channel sensor with the response functions $[Q_R(\lambda), Q_G(\lambda), Q_B(\lambda)]$ shown below.

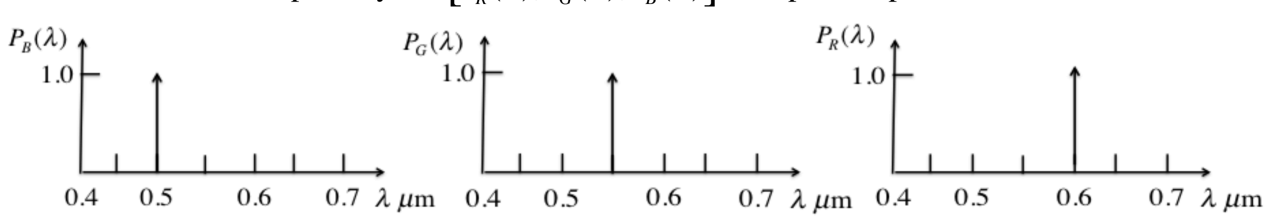


   a. (8) Find the response of this sensor to the stimulus $S(\lambda)$ with the spectral power distribution shown below:



   b. (8) Carefully sketch the spectral locus in the sensor chromaticity diagram for this sensor, using an equilateral triangle where each chromaticity coordinate is given by the distance from one of the vertices along the direction that is normal to the opposite edge.

Consider the primary set $[P_R(\lambda), P_G(\lambda), P_B(\lambda)]$ with power spectral distribution shown below:



   c. (8) Find the amounts of each of the three primaries that will yield a match to the stimulus $S(\lambda)$ from part a), as viewed by the sensor with response functions shown above.

   d. (8) Find the color matching functions $[\bar{r}(\lambda), \bar{g}(\lambda), \bar{b}(\lambda)]$ for this primary set.

   e. (8) Use the color matching functions as an alternate solution to finding the amounts of the primaries that will match the stimulus $S(\lambda)$ shown above.

1. (continued - 1)

(a). $R_S(\lambda) = \int S(\lambda)\, Q_R(\lambda)\, d\lambda = \dfrac{(\frac{1}{6}+\frac{5}{6})\times\frac{1}{5}}{2} = \frac{1}{10}$

$G_S(\lambda) = \int S(\lambda)\, Q_G(\lambda)\, d\lambda = \frac{1}{2}\times 1 \times (0.65-0.45) = \frac{1}{10}$

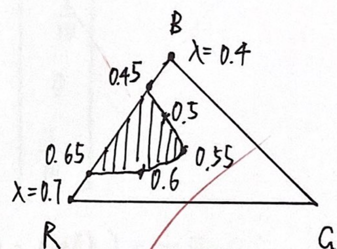
$B_S(\lambda) = \int S(\lambda)\, Q_B(\lambda)\, d\lambda = \dfrac{(\frac{1}{6}+\frac{5}{6})\times\frac{1}{5}}{2} = \frac{1}{10}$

$$C(\lambda) = \begin{bmatrix} \frac{1}{10} \\ \frac{1}{10} \\ \frac{1}{10} \end{bmatrix}$$

~~(b). $A = [a_{ij}]$, where $a_{ij} = \int P_i(\lambda)\, Q_j(\lambda)\, d\lambda$, $i,j = R, G$~~

(b).

λ	R	G	B	r	g	b
0.4	0	0	1	0	0	1
0.45	$\frac{1}{6}$	0	$\frac{5}{6}$	$\frac{1}{6}$	0	$\frac{5}{6}$
0.5	$\frac{2}{6}$	$\frac{1}{2}$	$\frac{4}{6}$	$\frac{2}{9}$	$\frac{1}{3}$	$\frac{4}{9}$
0.55	$\frac{3}{6}$	1	$\frac{3}{6}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$
0.6	$\frac{4}{6}$	$\frac{1}{2}$	$\frac{2}{6}$	$\frac{4}{9}$	$\frac{1}{3}$	$\frac{2}{9}$
0.65	$\frac{5}{6}$	0	$\frac{1}{6}$	$\frac{5}{6}$	0	$\frac{1}{6}$
0.7	1	0	0	1	0	0



1. (continued - 2)

(c). $A = [a_{ij}]$, where $a_{ij} = \int P_i(\lambda) Q_j(\lambda) d\lambda$, $i,j = R,G,B$

$$\cancel{A = \begin{bmatrix} \frac{5}{6} & \frac{1}{2} & \frac{1}{6} \\ & & \\ & & \end{bmatrix}}$$

$$A = \begin{bmatrix} \frac{2}{3} & \frac{1}{2} & \frac{1}{3} \\ \frac{1}{2} & 1 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{2} & \frac{2}{3} \end{bmatrix}$$

~~$P = A^{-1}S$~~

$$A^{-1} = \begin{bmatrix} \frac{5}{2} & -1 & -\frac{1}{2} \\ -1 & 2 & -1 \\ -\frac{1}{2} & -1 & \frac{5}{2} \end{bmatrix}$$

$$P = A^{-1}C^T = \begin{bmatrix} \frac{5}{2} & -1 & -\frac{1}{2} \\ -1 & 2 & -1 \\ -\frac{1}{2} & -1 & \frac{5}{2} \end{bmatrix} \begin{bmatrix} \frac{1}{10} \\ \frac{1}{10} \\ \frac{1}{10} \end{bmatrix} = \begin{bmatrix} \frac{1}{10} \\ 0 \\ \frac{1}{10} \end{bmatrix}$$

(d)
$$\begin{bmatrix} \bar{r}(\lambda) \\ \bar{g}(\lambda) \\ \bar{b}(\lambda) \end{bmatrix} = A^{-1} \begin{bmatrix} Q_R(\lambda) \\ Q_B(\lambda) \\ Q_G(\lambda) \end{bmatrix} = \begin{bmatrix} \frac{5}{2} Q_R(\lambda) - Q_B(\lambda) - \frac{1}{2} Q_G(\lambda) \\ -Q_R(\lambda) + 2Q_B(\lambda) - Q_G(\lambda) \\ -\frac{1}{2} Q_R(\lambda) - Q_B(\lambda) + \frac{5}{2} Q_G(\lambda) \end{bmatrix}$$

(e)
$$\begin{bmatrix} r(\lambda) \\ g(\lambda) \\ b(\lambda) \end{bmatrix} = \int_{0.4}^{0.7} S(\lambda) \begin{bmatrix} \bar{r}(\lambda) \\ \bar{g}(\lambda) \\ \bar{b}(\lambda) \end{bmatrix} = \begin{bmatrix} \frac{1}{10} \\ 0 \\ \frac{1}{10} \end{bmatrix}$$

2. (30 pts.) Consider a finite dimensional model for a linear, bichromatic (2-channel) vision system. Assume that we sample at $N=3$ wavelengths. Suppose that the sensor response matrix is given by

$$\mathbf{S} = \begin{bmatrix} 0.25 & 1 \\ 0.5 & 0.5 \\ 1 & 0.25 \end{bmatrix}$$

a. (5) Find the response of this sensor to the stimulus $\vec{n} = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}^T$.

b. (10) Find the projection operator $\mathbf{R} = \mathbf{S}\left(\mathbf{S}^T\mathbf{S}\right)^{-1}\mathbf{S}^T$ for this sensor.

c. (5) Find the fundamental component $\vec{n}^*$ for this stimulus.

d. (5) Find the black-space component $\vec{n}^c$ for the stimulus.

e. (5) Find a metamer $\vec{n}'$ to $\vec{n}$ such that $\vec{n}' \neq \vec{n}$.

a. $\vec{n} = S^T \vec{n}$

$$= \begin{bmatrix} 0.25 & 0.5 & 1 \\ 1 & 0.5 & 0.25 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.75 \\ 1.5 \end{bmatrix}$$

b. $R = S(S^TS)^{-1}S^T$

$$S^TS = \begin{bmatrix} 0.25 & 0.5 & 1 \\ 1 & 0.5 & 0.25 \end{bmatrix} \begin{bmatrix} 0.25 & 1 \\ 0.5 & 0.5 \\ 1 & 0.25 \end{bmatrix} = \begin{bmatrix} \frac{21}{16} & \frac{3}{4} \\ \frac{3}{4} & \frac{21}{16} \end{bmatrix}$$

$$= \frac{1}{16} \cdot \begin{bmatrix} 21 & 12 \\ 12 & 21 \end{bmatrix}$$

$$(S^TS)^{-1} = 16 \cdot \frac{1}{297} \begin{bmatrix} 21 & -12 \\ -12 & 21 \end{bmatrix}$$

$$S(S^TS)^{-1} = \frac{16}{297} \begin{bmatrix} -\frac{27}{4} & \frac{72}{4} \\ \frac{9}{2} & \frac{9}{2} \\ \frac{72}{4} & -\frac{27}{4} \end{bmatrix} = \frac{4}{33} \begin{bmatrix} -3 & 8 \\ 2 & 2 \\ 8 & -3 \end{bmatrix}$$

2. (continued -1)

$$S(S^TS)^{-1}S^T = \frac{4}{33}\begin{bmatrix} -3 & 8 \\ 2 & 2 \\ 8 & -3 \end{bmatrix}\begin{bmatrix} \frac{1}{4} & \frac{1}{2} & 1 \\ 1 & \frac{1}{2} & \frac{1}{4} \end{bmatrix} = \frac{4}{33}\begin{bmatrix} \frac{29}{4} & \frac{5}{2} & -1 \\ \frac{5}{2} & 2 & \frac{5}{2} \\ -1 & \frac{5}{2} & \frac{29}{4} \end{bmatrix}$$

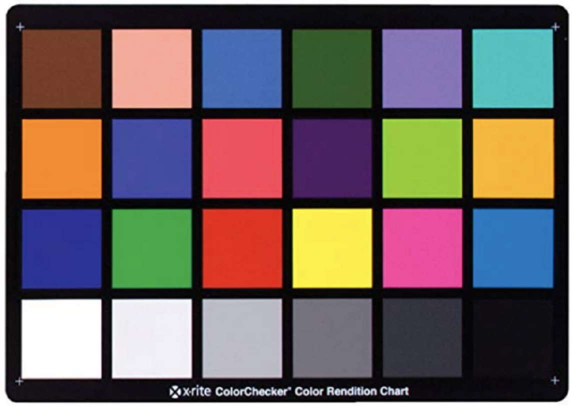
c. $\vec{n}^* = R\vec{n} = \begin{bmatrix} \frac{13}{11} \\ \cancel{13}\ \frac{6}{11} \\ \frac{2}{11} \end{bmatrix}$

d. $\vec{n}^c = \vec{n} - \vec{n}^* = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} \frac{13}{11} \\ \frac{6}{11} \\ \frac{2}{11} \end{bmatrix} = \begin{bmatrix} -\frac{2}{11} \\ \frac{5}{11} \\ -\frac{2}{11} \end{bmatrix}$

e. $\vec{n}' = n^* + k\vec{n}^c$, we take $k = 2$

$$= \begin{bmatrix} -\frac{2}{11} \\ \frac{5}{11} \\ -\frac{2}{11} \end{bmatrix} \times 2 + \begin{bmatrix} \frac{13}{11} \\ \frac{6}{11} \\ \frac{2}{11} \end{bmatrix} = \begin{bmatrix} \frac{9}{11} \\ \frac{16}{11} \\ -\frac{2}{11} \end{bmatrix}$$

3. (25) This problem deals with the concept of camera calibration. Our overall goal is to determine a mapping from camera RGB to CIE XYZ. Assume that you have a target, such as is shown below, containing patches of constant color for which you have measured the CIE XYZ value of each patch using a spectrophotometer. You have used your camera to acquire an image of this target. You have segmented this image into individual patches, and computed the average camera RGB values for each patch.



a. (8) The first step is to convert the gamma-corrected RGB values to linear RGB. Describe in detail what is the assumed form of the mapping from camera RGB to linear RGB. How would you determine this mapping from the data that you obtained, as described above. Which patches would you use for this stage of the calibration process?

b. (9) The second step is to determine a mapping from linear RGB to CIE XYZ. Describe in detail what is the assumed form for this mapping, and how you would determine the parameters of this mapping from the data that you obtained, as described above. Which patches would you use for this stage of the calibration process?

c. (8) Finally, describe specifically how you would determine the accuracy of the overall calibration mapping that you obtained in parts a) and b) above?

1(a) To get the parameter γ for gamma correction, we use the gray scale patches, which is the last row of the target, to get the gray balance curve,

The value from linear RGB to sRGB is that

$f_{out} = 255 \times \left(\frac{f_{In}}{255}\right)^{1/\gamma}$, and we use the spectrophotometer measured Y as lightness, along with the given gray scale value from the target to get the ~~grave~~ gray balancing curves and using GOG (or other algorithm) to estimate the value of γ

3. (continued - 1)

(b) There is a linear transformation ~~to~~ T from ~~CIE XYZ~~ linear RGB to CIE XYZ ~~to CIE XYZ~~, to get the parameter T, we use the first three rows ~~and~~ in the target and the parameter ~~γ~~ γ we estimated from last step.

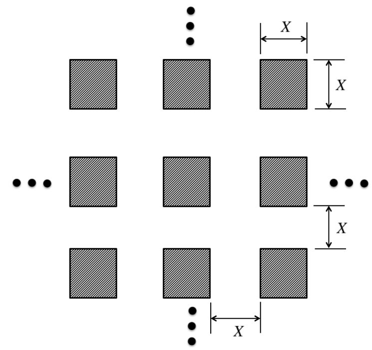
~~For each color we form two matrix~~ First we transform each patches from gamma-corrected RGB to linear RGB using the parameter γ and $R_L = 255\left(\frac{R_V}{255}\right)^{\gamma}$ (similar for G, B)

Then we form two matrice using linear RGB of each patch ~~and~~ ~~each~~ $A = \begin{bmatrix} R_{L1} & G_{L1} & B_{L1} \\ R_{L2} & G_{L2} & B_{L2} \\ & \vdots & \\ R_{LN} & G_{LN} & B_{LN} \end{bmatrix}$ and CIE XYZ of each patch $B = \begin{bmatrix} X_1 & Y_1 & Z_1 \\ & \vdots & \\ X_N & Y_N & Z_N \end{bmatrix}$

And use regression, ~~to~~ we estimate the transform matrix T as $T = (A^T A)^{-1} A^T B$

(c) We use the measured XYZ, ~~a~~ and ~~the~~ from sRGB we can use our T and γ to computed a estimated XYZ, then we transform both into $L^*a^*b^*$ space, and computed the delta E as error estimation

4. (40) Consider the doubly periodic binary pattern $g(x,y)$ shown below:



Here, $g(x,y)=1$ within the shaded squares, and $g(x,y)=0$ elsewhere.

a. (5) Using standard operators and standard functions, find a simple expression for $g(x,y)$.
b. (5) Using standard transform relations, find an expression for the Continuous-Space Fourier Transform (CSFT) $G(u,v)$ of $g(x,y)$. Your answer should not contain any operators other than summation signs.
c. (5) Sketch $G(u,v)$ with sufficient detail to indicate that you know what it looks like. Be sure to dimension all quantities.

Now consider a new pattern $h(x,y)$ defined as follows:

$$h(x,y)=\begin{cases} g(x,y), & |x|<3X \text{ and } |y|<3X \\ 0, & \text{else} \end{cases}$$

d. (5) Using standard transform relations, find an expression for the CSFT $H(u,v)$ of $h(x,y)$. Your answer should not contain any operators other than summation signs.
e. (5) Sketch $H(u,v)$ with sufficient detail to indicate that you know what it looks like. Be sure to dimension all quantities.

Finally, consider a continuous-tone image $f(x,y)$. Let $f_{\text{screen}}(x,y)$ denote a new image defined as follows:

$$f_{\text{screen}}(x,y)=\begin{cases} f(x,y), & |x-m2X|<X/2 \text{ and } |y-n2X|<X/2 \text{ for all integer pairs } (m,n) \\ 0, & \text{else} \end{cases}$$

f. (5) Using standard transform relations, find an expression for the CSFT $F_{\text{screen}}(u,v)$ of $f_{\text{screen}}(x,y)$ in terms of $F(u,v)$ the CSFT of $f(x,y)$. Your answer should not contain any operators other than summation signs.
g. (5) Assume simple illustrative examples for $f(x,y)$ and $F(u,v)$. Sketch $f_{\text{screen}}(x,y)$ and $F_{\text{screen}}(u,v)$ with sufficient detail to indicate that you know what they look like. Be sure to dimension all quantities.
h. (5) Determine conditions on $F(u,v)$ under which it is possible to reconstruct $f(x,y)$ exactly from $f_{\text{screen}}(x,y)$. Determine the process that needs to be applied to $f_{\text{screen}}(x,y)$ to reconstruct $f(x,y)$.

4. (continued - 1)

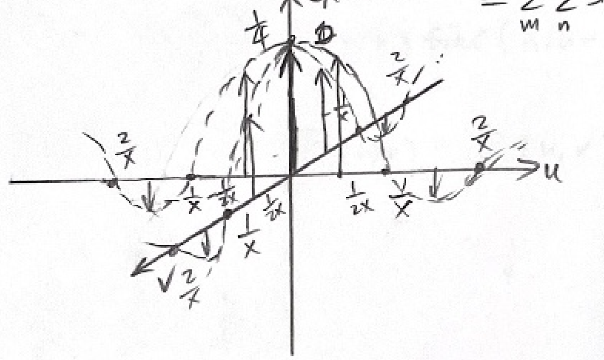
a. $g(x,y) = rep_{2X,2X}\left(rect\left(\frac{x}{X}, \frac{y}{X}\right)\right)$

b. $G(u,v) = CTFT[g(x,y)] = \frac{1}{4X^2} comb_{\frac{1}{2X},\frac{1}{2X}}\left(X^2 sinc(Xu, Xv)\right)$

$= \frac{1}{4} comb_{\frac{1}{2X},\frac{1}{2X}}(sinc(Xu, Xv))$

$= \sum_m \sum_n sinc(u,v)\, \delta\left(u - \frac{m}{2X}, v - \frac{n}{2X}\right)$

c.

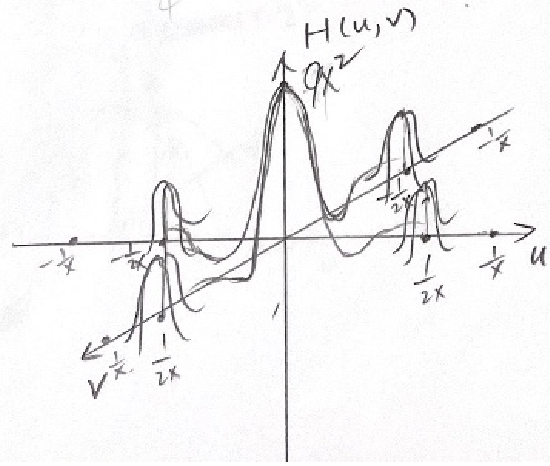


d. Given $h(x,y) = g(x,y) \cdot rect\left(\frac{x}{6X}, \frac{y}{6X}\right)$

$\Rightarrow H(u,v) = G(u,v) ** 36X^2 sinc(6Xu, 6Xv)$

$= 9X^2 \sum_m \sum_n sinc(u,v)\, sinc\left(6X\left(u - \frac{m}{2X}\right), 6X\left(v - \frac{n}{2X}\right)\right)$

e.



4. (continued - 2)

f. Given $f_{screen}(x,y) = rep_{2X,2X}\left(rect\left(\frac{x}{X},\frac{y}{X}\right)\cdot f(x,y)\right)$

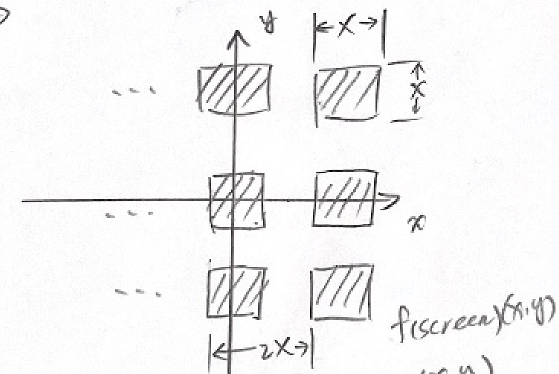
$\Rightarrow F_{screen}(u,v) = \frac{1}{4X^2} comb_{\frac{1}{2X},\frac{1}{2X}}\left(X^2 sinc(Xu, Xv) ** F(u,v)\right)$

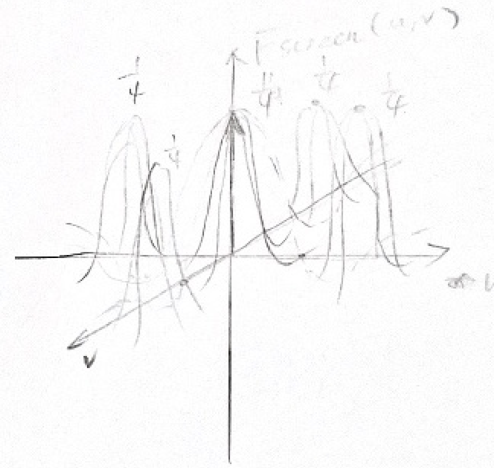
$= \frac{1}{4} comb_{\frac{1}{2X},\frac{1}{2X}}\left(sinc(Xu,Xv) ** F(u,v)\right)$

$= \frac{1}{4}\sum_m \sum_n F(u,v)\, sinc\left(X\left(u-\frac{m}{2X}\right), X\left(v-\frac{n}{2X}\right)\right)$

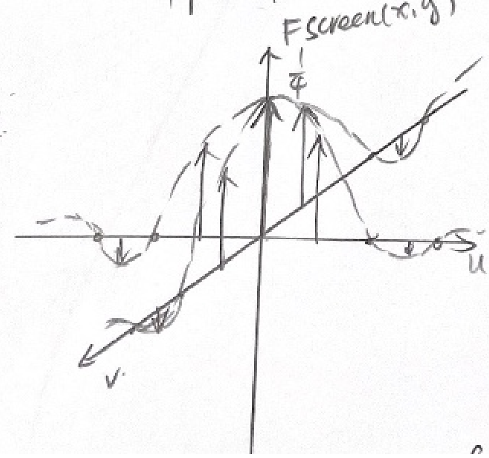
g. let $f(x,y) = 1 \Rightarrow F(u,v) = \delta(u,v)$

$\Rightarrow$





h.



4. (continued - 2)

f) $f_{screen}(x,y) = f(x,y)\, rep_{2X,2X}\left[rect\left(\frac{x}{X}, \frac{y}{X}\right)\right]$

Thus

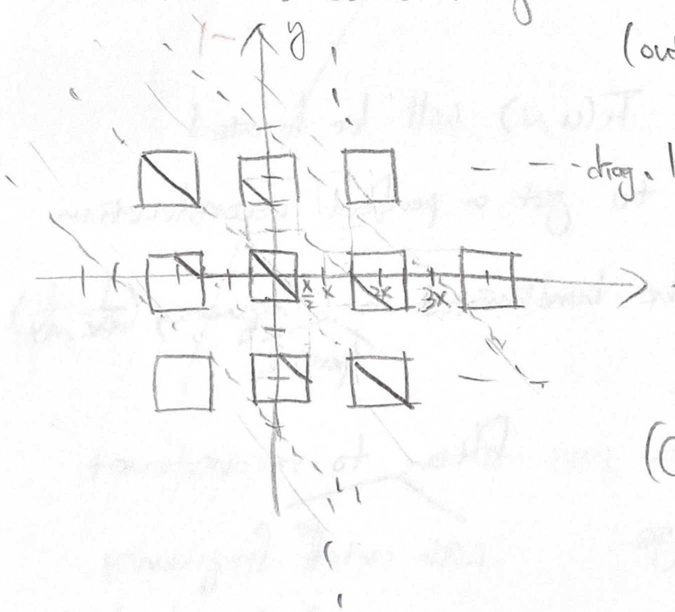
$$F_{screen}(u,v) = F(u,v) ** \left[\frac{1}{4}\sum_{k,l} sinc(Xu, Xv)\, \delta\left(u-\frac{k}{2X}, v-\frac{l}{2X}\right)\right]$$

$$= \frac{1}{4}\sum_{k,l} sinc\left(\frac{k}{2}, \frac{l}{2}\right) F\left(u-\frac{k}{2X}, v-\frac{l}{2X}\right)$$

g) Let us use $f(x,y) = \cos\left(2\pi\left(\frac{x}{3X} + \frac{y}{3X}\right)\right)$

Then the screened image will look like below:

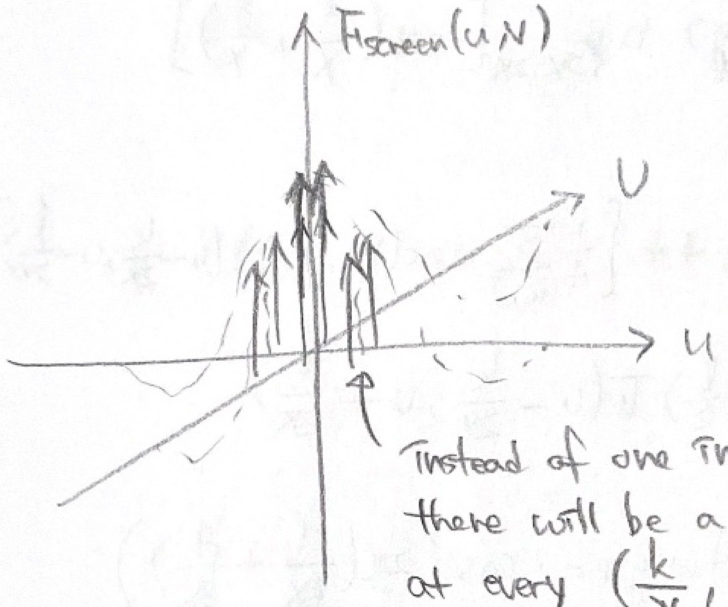
(outside the squares, $f_{screen} = 0$. --- diag. line indicates $f_{screen} = \pm 1$)



(Continued)

g) (Continued)

And $F_{screen}(u,v)$ will look like below:



Instead of one impulse as in e), there will be a pair of impulse at every $\left(\frac{k}{2X}, \frac{l}{2X}\right)$

i) Since each copy of $F_I(u,v)$ will be located every $\left(\frac{k}{2X}, \frac{l}{2X}\right)$, to get a perfect reconstruction $F_I(u,v)$ needs to be bandlimited for spatial frequency $\left(\frac{1}{4X}, \frac{1}{4X}\right)$

We can apply a 2-D low pass filter with cutoff frequency $\left(\frac{1}{4X}, \frac{1}{4X}\right)$ to reconstruct the original image.

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5. (30) You have a 4x4 pixel continuous-tone image with constant value 0.25 in units of absorptance (0 = white, and 1 = black). You wish to halftone it using error diffusion with the weights given below. Assuming that we process the pixels in the continuous-tone image by scanning row-to-row, from left-to-right within each row, we diffuse 1/2 of the error to the next pixel to be processed on the same line, and 1/2 of the error to the pixel immediately below the pixel that is currently being processed, as indicated in the figure below, where the "X" indicates the pixel that is currently being processed.

	X	0.5
	0.5	

a. (25) Following the scheme described above, generate the 4x4 pixel halftone image corresponding to the 4x4 pixel constant-value continuous-tone image described above. Be sure to show in detail the sequence of steps that you use to obtain your result, since the manner in which boundary conditions are handled will lead to different answers that may be equally valid.

b. (5) Calculate the average absorptance of your halftone image, and compare it to the absorptance of the continuous-tone original image.

a. let the threshold be 0.5

①

0	3/8	1/4	1/4
3/8	1/4	1/4	1/4
1/4	1/4	1/4	1/4
1/4	1/4	1/4	1/4

②

0	0	7/16	1/4
3/8	7/16	1/4	1/4
1/4	1/4	1/4	1/4
1/4	1/4	1/4	1/4

③

0	0	0	15/32
3/8	7/16	15/32	1/4
1/4	1/4	1/4	1/4
1/4	1/4	1/4	1/4

④

0	0	0	0
3/8	7/16	15/32	31/64
1/4	1/4	1/4	1/4
1/4	1/4	1/4	1/4

⑤

0	0	0	0
0	10/16	15/32	31/64
7/16	1/4	1/4	1/4
1/4	1/4	1/4	1/4

⑥

0	0	0	0
0	1	9/32	31/64
7/16	1/16	1/4	1/4
1/4	1/4	1/4	1/4

⑦

0	0	0	0
0	1	0	10/16
7/16	1/16	25/64	1/4
1/4	1/4	1/4	1/4

⑧

0	0	0	0
0	1	0	1
7/16	1/16	25/64	1/16
1/4	1/4	1/4	1/4

⑨

0	0	0	0
0	1	0	1
0	9/32	25/64	1/16
15/32	1/4	1/4	1/4

⑩

0	0	0	0
0	1	0	1
0	0	34/64	1/16
15/32	25/64	1/4	1/4

⑪

0	0	0	0
0	1	0	1
0	0	1	-11/64
15/32	25/64	1/64	1/4

⑫

0	0	0	0
0	1	0	1
0	0	1	0
15/32	25/64	1/64	21/128

⑬

0	0	0	0
0	1	0	1
0	0	1	0
0	10/16	1/64	21/128

⑭

0	0	0	0
0	1	0	1
0	0	1	0
0	1	-11/64	21/128

⑮

0	0	0	0
0	1	0	1
0	0	1	0
0	1	0	-1/128

⑯

0	0	0	0
0	1	0	1
0	0	1	0
0	1	0	0

$-\frac{22}{128} + \frac{21}{128}$

0.25

$\frac{1}{4} + \frac{1}{8}$

$\frac{3}{8}$

$\frac{3}{16} + \frac{4}{16}$

$\frac{7}{32} + \frac{8}{32}$

$\frac{15}{64} + \frac{16}{64}$

$\frac{7}{16} + \frac{3}{16}$

$1 - \frac{10}{16} = \frac{6}{16}$

$\frac{6}{32}$

$\frac{8}{32} - \frac{6}{32}$

$\frac{9}{64} + \frac{31}{64}$

$\frac{10}{16}$

$\frac{16}{64} + \frac{9}{64}$

$\frac{6}{16}$

$\frac{30}{64}$

$\frac{7}{32} + \frac{8}{32}$

$\frac{15}{32}$

$\frac{9}{64} + \frac{25}{64}$

$+ \frac{16}{64}$

$\frac{32}{64}$

21

5. (continued - 1)

b. the average absortance of ~~be~~ the halftone : $\frac{4}{16} = 0.25$

which is equal to the average absortance of the continuous-tone image ~~a~~ : $\frac{0.25 \times 16}{16} = 0.25$