19 October 2017

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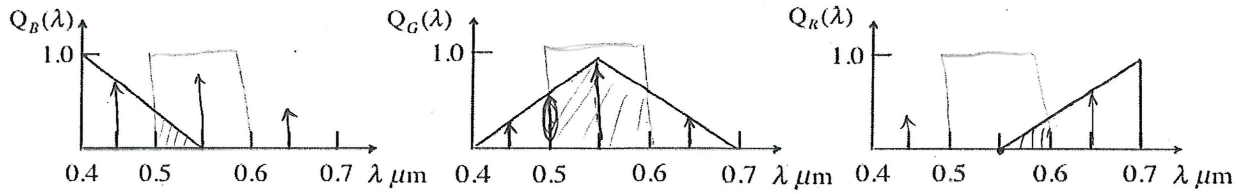
ECE 638 **Exam No. 1** **Fall 2017**

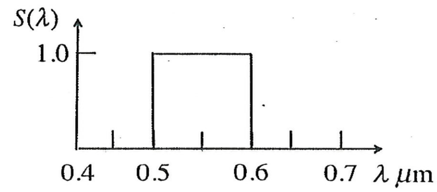
This exam is closed book and closed notes. **No calculators are permitted.** You have 120 minutes to work the following **four** problems that are worth a total of 125 pts. (Please see the last page of the exam for a summary of the point assignment.) To obtain maximum partial credit, be sure to show the complete derivation of your answers. See p. 16 for the formulas to invert a 3×3 matrix, which you will have to do to solve some of these problems.

by Min Zhao

1. (40 pts.) Consider a three-channel sensor with the response functions $[Q_R(\lambda), Q_G(\lambda), Q_B(\lambda)]$ shown below.

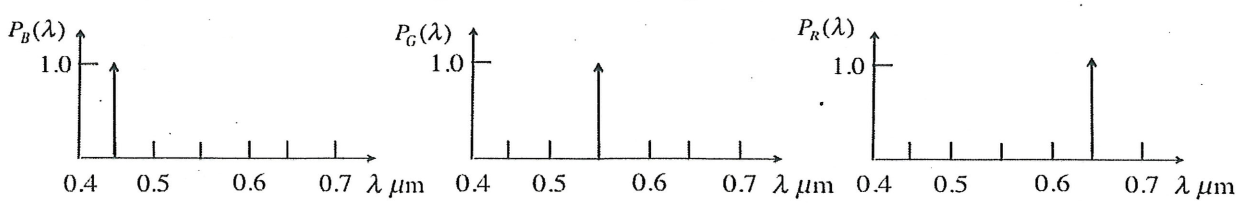


a. (8) Find the response of this sensor to the stimulus $S(\lambda)$ with the spectral power distribution shown below:



b. (8) Carefully sketch the spectral locus in the sensor chromaticity diagram for this sensor, using an equilateral triangle where each chromaticity coordinate is given by the distance from one of the vertices along the direction that is normal to the opposite edge.

Consider the primary set $[P_R(\lambda), P_G(\lambda), P_B(\lambda)]$ with power spectral distribution shown below:



c. (8) Find the amounts of each of the three primaries that will yield a match to the stimulus $S(\lambda)$ from part a), as viewed by the sensor with response functions shown above.

d. (8) Find the color matching functions $[\bar{r}(\lambda), \bar{g}(\lambda), \bar{b}(\lambda)]$ for this primary set.

e. (8) Use the color matching functions as an alternate solution to finding the amounts of the primaries that will match the stimulus $S(\lambda)$ shown above.

1. (continued - 1)

a. $$R_S = \int_{0.5}^{0.6} S(\lambda)\, Q_R(\lambda)\, d\lambda$$

$$= \frac{1}{2} \times 0.05 \times \frac{1}{3} = \frac{1}{2} \times \frac{1}{20} \times \frac{1}{3} = \frac{1}{120}$$

$$G_S = \int_{0.5}^{0.6} S(\lambda)\, Q_G(\lambda)\, d\lambda$$

$$= 2 \times \frac{1}{2} \times \left(\frac{2}{3} + 1\right) \times 0.05$$

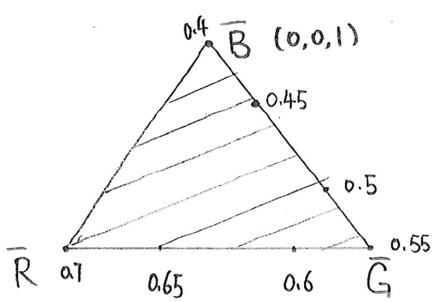
$$= \frac{5}{3} \times \frac{1}{20_4} = \frac{1}{12}$$

$$B_S = \int_{0.5}^{0.6} S(\lambda)\, Q_B(\lambda)\, d\lambda$$

$$= \frac{1}{2} \times 0.05 \times \frac{1}{3} = \frac{1}{2} \times \frac{1}{20} \times \frac{1}{3} = \frac{1}{120}$$

b.

	R	G	B	r	g	b
$\lambda = 0.4$	0	0	1	0	0	1
$\lambda = 0.45$	0	$\frac{1}{3}$	$\frac{2}{3}$	0	$\frac{1}{3}$	$\frac{2}{3}$
$\lambda = 0.5$	0	$\frac{2}{3}$	$\frac{1}{3}$	0	$\frac{2}{3}$	$\frac{1}{3}$
$\lambda = 0.55$	0	1	0	0	1	0
$\lambda = 0.6$	$\frac{1}{3}$	$\frac{2}{3}$	0	$\frac{1}{3}$	$\frac{2}{3}$	0
$\lambda = 0.65$	$\frac{2}{3}$	$\frac{1}{3}$	0	$\frac{3}{2}$	$\frac{1}{3}$	0
$\lambda = 0.7$	1	0	0	1	0	0



1. (continued - 2)

c. $\vec{C_T} = A\vec{P}$

$$\vec{P} = A^{-1}\vec{C_T}$$

$$A = \begin{bmatrix} \frac{2}{3} & 0 & 0 \\ \frac{1}{3} & 1 & \frac{1}{3} \\ 0 & 0 & \frac{2}{3} \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{3}{2} & 0 & 0 \\ -\frac{1}{2} & 1 & -\frac{1}{2} \\ 0 & 0 & \frac{3}{2} \end{bmatrix}$$

$$\vec{C_T} = \begin{bmatrix} R_S \\ G_S \\ B_S \end{bmatrix} = \begin{bmatrix} \frac{1}{120} \\ \frac{1}{12} \\ \frac{1}{120} \end{bmatrix}$$

$$\vec{P} = A^{-1}\vec{C_T} = \begin{bmatrix} \frac{3}{2} & 0 & 0 \\ -\frac{1}{2} & 1 & -\frac{1}{2} \\ 0 & 0 & \frac{3}{2} \end{bmatrix} \begin{bmatrix} \frac{1}{120} \\ \frac{1}{12} \\ \frac{1}{120} \end{bmatrix} = \begin{bmatrix} \frac{1}{80} \\ \frac{3}{40} \\ \frac{1}{80} \end{bmatrix}$$

d. $$\begin{bmatrix} \bar{r}(\lambda) \\ \bar{g}(\lambda) \\ \bar{b}(\lambda) \end{bmatrix} = A^{-1} \begin{bmatrix} Q_R(\lambda) \\ Q_G(\lambda) \\ Q_B(\lambda) \end{bmatrix} = \begin{bmatrix} \frac{3}{2} & 0 & 0 \\ -\frac{1}{2} & 1 & -\frac{1}{2} \\ 0 & 0 & \frac{3}{2} \end{bmatrix} \begin{bmatrix} Q_R(\lambda) \\ Q_G(\lambda) \\ Q_B(\lambda) \end{bmatrix} = \begin{bmatrix} \frac{3}{2}Q_R(\lambda) \\ -\frac{1}{2}Q_R(\lambda) + Q_G(\lambda) - \frac{1}{2}Q_B(\lambda) \\ \frac{3}{2}Q_B(\lambda) \end{bmatrix}$$

e. $$\begin{bmatrix} r' \\ g' \\ b' \end{bmatrix} = \int S(\lambda) \begin{bmatrix} \bar{r}(\lambda) \\ \bar{g}(\lambda) \\ \bar{b}(\lambda) \end{bmatrix} = \int S(\lambda) \begin{bmatrix} \frac{3}{2}Q_R(\lambda) \\ -\frac{1}{2}Q_R(\lambda) + Q_G(\lambda) - \frac{1}{2}Q_B(\lambda) \\ \frac{3}{2}Q_B(\lambda) \end{bmatrix} = \begin{bmatrix} \frac{3}{2} \times \frac{1}{120} \\ -\frac{1}{2} \times \frac{1}{120} + \frac{1}{12} - \frac{1}{2} \times \frac{1}{120} \\ \frac{3}{2} \times \frac{1}{120} \end{bmatrix} = \begin{bmatrix} \frac{1}{80} \\ \frac{3}{40} \\ \frac{1}{80} \end{bmatrix}$$

by Yang Cheng

2. (30 pts.) Consider a finite dimensional model for a linear, bichromatic (2-channel) vision system. Assume that we sample at $N=3$ wavelengths. Suppose that the sensor response matrix is given by

$$\mathbf{S}=\begin{bmatrix} 0.5 & 0 \\ 1 & 1 \\ 0 & 0.5 \end{bmatrix}$$

a. (5) Find the response of this sensor to the stimulus $\vec{n}=\begin{bmatrix} 1 & 0.5 & 1 \end{bmatrix}^T$.

b. (10) Find the projection operator $\mathbf{R}=\mathbf{S}\left(\mathbf{S}^T\mathbf{S}\right)^{-1}\mathbf{S}^T$ for this sensor.

b. (5) Find the fundamental component $\vec{n}^*$ for this stimulus.

c. (5) Find the black-space component $\vec{n}^c$ for the stimulus.

d. (5) Find a metamer $\vec{n}'$ to $\vec{n}$ such that $\vec{n}' \neq \vec{n}$.

a. $q = S^T n = \begin{bmatrix} 0.5 & 1 & 0 \\ 0 & 1 & 0.5 \end{bmatrix}\begin{bmatrix} 1 \\ 0.5 \\ 1 \end{bmatrix}$

$= \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

b. $S = \begin{bmatrix} 0.5 & 0 \\ 1 & 1 \\ 0 & 0.5 \end{bmatrix}$ $\quad S^T = \begin{bmatrix} 0.5 & 1 & 0 \\ 0 & 1 & 0.5 \end{bmatrix}$

$S^T S = \begin{bmatrix} 1.25 & 1 \\ 1 & 1.25 \end{bmatrix} = \begin{bmatrix} \frac{5}{4} & 1 \\ 1 & \frac{5}{4} \end{bmatrix}$

$(S^T S)^{-1} = \frac{1}{\frac{9}{16}} \begin{bmatrix} \frac{5}{4} & -1 \\ -1 & \frac{5}{4} \end{bmatrix}$

$= \frac{16}{9}\begin{bmatrix} \frac{5}{4} & -1 \\ -1 & \frac{5}{4} \end{bmatrix}$

$= \begin{bmatrix} \frac{20}{9} & -\frac{16}{9} \\ -\frac{16}{9} & \frac{20}{9} \end{bmatrix}$

$S(S^T S)^{-1} = \begin{bmatrix} \frac{10}{9} & -\frac{8}{9} \\ \frac{4}{9} & \frac{4}{9} \\ -\frac{8}{9} & \frac{10}{9} \end{bmatrix}$

$R = S(S^T S)^{-1} S^T = \begin{bmatrix} \frac{10}{9} & -\frac{8}{9} \\ \frac{4}{9} & \frac{4}{9} \\ -\frac{8}{9} & \frac{10}{9} \end{bmatrix} \times \begin{bmatrix} 0.5 & 1 & 0 \\ 0 & 1 & 0.5 \end{bmatrix} = \begin{bmatrix} \frac{5}{9} & \frac{2}{9} & -\frac{4}{9} \\ \frac{2}{9} & \frac{8}{9} & \frac{2}{9} \\ -\frac{4}{9} & \frac{2}{9} & \frac{5}{9} \end{bmatrix}$

2. (continued -1)

(c) $\vec{n}^* = R\vec{n}$

$$= \begin{bmatrix} \frac{5}{9} & \frac{2}{9} & -\frac{4}{9} \\ \frac{2}{9} & \frac{8}{9} & \frac{2}{9} \\ -\frac{4}{9} & \frac{2}{9} & \frac{5}{9} \end{bmatrix} \begin{bmatrix} 1 \\ 0.5 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2}{9} \\ \frac{8}{9} \\ \frac{2}{9} \end{bmatrix}$$

(d) $\vec{n}^c = \vec{n} - \vec{n}^* = \begin{bmatrix} 1 \\ 0.5 \\ 1 \end{bmatrix} - \begin{bmatrix} \frac{2}{9} \\ \frac{8}{9} \\ \frac{2}{9} \end{bmatrix} = \begin{bmatrix} \frac{7}{9} \\ -\frac{7}{18} \\ \frac{7}{9} \end{bmatrix}$

(e) $\vec{n}' = \vec{n}^* + \alpha \vec{n}^c$

$$= \begin{bmatrix} \frac{2}{9} \\ \frac{8}{9} \\ \frac{2}{9} \end{bmatrix} + \alpha \begin{bmatrix} \frac{7}{9} \\ -\frac{7}{18} \\ \frac{7}{9} \end{bmatrix}$$

e.g. let $\alpha = 2$

$$\vec{n}' = \begin{bmatrix} \frac{16}{9} \\ \frac{1}{9} \\ \frac{16}{9} \end{bmatrix}$$

by Xiaoyu Xiang

3. (25) This problem deals with the concept of correlated color temperature.
   a. (5) Explain in detail what the plot below represents.
   b. (5) What is the meaning of the solid line that goes from coordinates (0.24, 0.235) to (0.585, 0.385)?
   c. (5) How is it determined?
   d. (10) For the given spectral power distribution $I(\lambda)$ of an illuminant, provide a detailed procedure and equations for calculating the correlated color temperature for $I(\lambda)$.

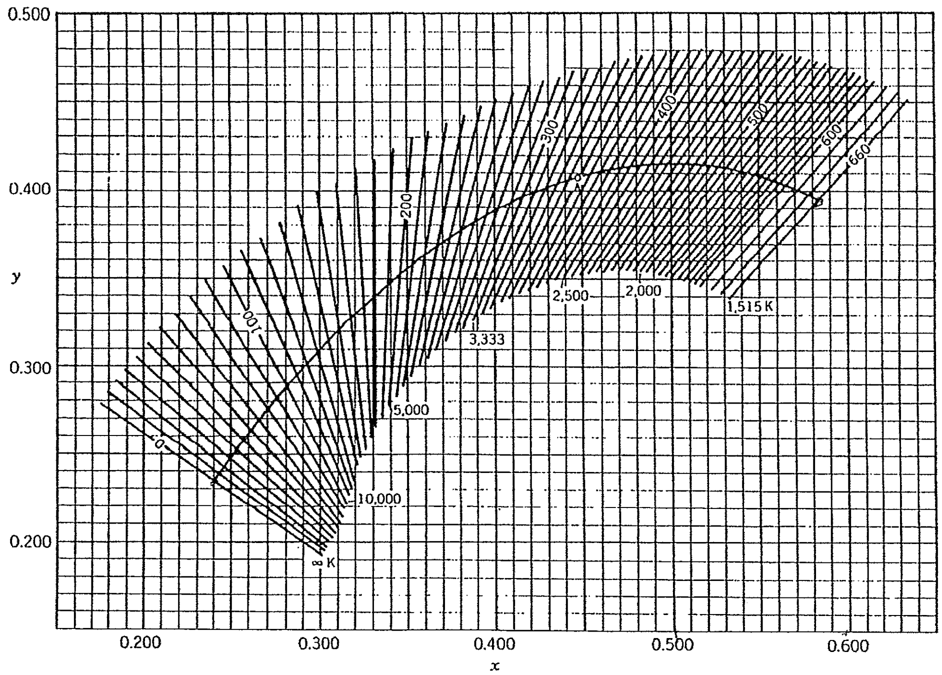


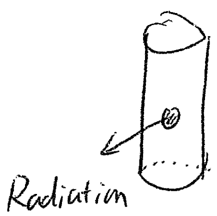
Fig. 1(3.11). CIE 1931 (x, y)-chromaticity diagram showing isotemperature lines as computed by Kelly (1963).

3. a. The plot is showing the black-body radiation's (x,y)-chromaticity diagram plot. For the radiation emitted by a black-body at a given temperature (K), there is a corresponding (x,y) of it, which is the x and y axis of this plot.

b. The solid line is plotted with the black-body radiation of temperature ranging from 1515 K ~ ∞ K. For each temperature, there is a corresponding (x,y) of the light radiated by the black-body, and we can plot ~~plotted~~ it on the x-y chromaticity diagram. The solid line is acquired by connecting all the points. The line is the curve of black-body radiation

3. (continued - 1)

c. In the temperature range (like 1,515k ~> 10,000k). we sample some points at a certain temperature. By heating the black-body (shown in the figure below) to the temperature, there would be light emitted from the hole of black-body. We measure the (x,y) of this light, and plot it in the x-y chromaticity diagram. By connecting all the sample points together, we get the solid line, which is the black-body radiation curve.



d. For a given $I(\lambda)$: we can acquire the CIE X. Y. Z coordinates:

$$X = \int \bar{x}(\lambda) I(\lambda) d\lambda$$

$$Y = \int \bar{y}(\lambda) I(\lambda) d\lambda$$

$$Z = \int \bar{z}(\lambda) I(\lambda) d\lambda.$$

color matching functions

$\bar{x}(\lambda)$. $\bar{y}(\lambda)$. $\bar{z}(\lambda)$ is 3 batches of X. Y. Z.

And we calculate the (x,y) with the following equations:

$$x = \frac{X}{X+Y+Z}$$

$$y = \frac{Y}{X+Y+Z}.$$

According to the (x,y) we get, we plot it on the Fig (3.11) in the previous page, and read out the nearest temperature, which is the correlated color temperature for $I(\lambda)$.

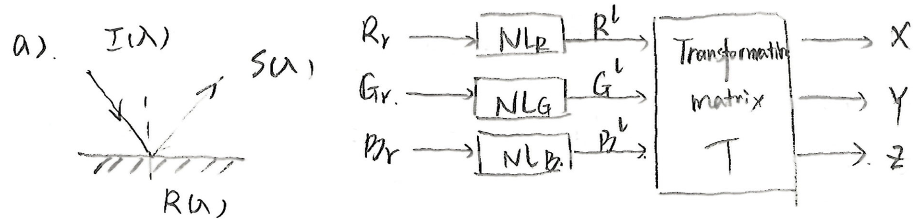
Version 1 by Ruiting Shao

4. (30) You have a page of paper that has been painted with a uniform color. You use a spectroradiometer to determine the spectral reflectance $R(\lambda)$ of this page. You would like to display a color patch on your monitor that indicates what this patch would look like under an illuminant with spectral power distribution $I(\lambda)$. The inputs to your monitor are a 3-tuple of RGB values for each pixel.

   a. (15) Describe in detail with complete equations (parameter values need not be specified) the mapping from $R(\lambda)$ to the RGB 3-tuple value that you will input to the monitor, such that it will display a color that looks the same to the human viewer as the painted sheet of paper.

   b. (15) Describe in detail how you would determine the values of the unknown parameters in your answer to part (a) above.

a).



where $S(\lambda) = I(\lambda) \cdot R(\lambda)$. thus we got the stimulus.

Here we use a regression model to mimic the entire work.

And for the entire system it contains 2 steps. calibration and characterization.

For calibration, we we first to select grayscale patches (using Y) from the color patches (Kodak Q60) to do the gray curve fitting

$$R_L = a\left(\frac{R_r}{255}\right)^b + c \qquad \text{where we can set } R_L = Y.$$

get from Kodak Q60 ← then use the different color patches put into the monitor to get all $[R_r, G_r, B_r]$ values, and apply the above equation, we can get all $[R^l, G^l, B^l]$ values.

thus we can use the least square fit method to solve the following equation:

$$\text{let } A = [R^l, G^l, B^l]_{N\times 3},$$

$$B = [X \;\; Y \;\; Z]_{N\times 3}.$$

$$B = A \cdot T.$$

$$\text{thus } T = (A^T A)^{-1} A^T B.$$

get the XYZ values just based on xyz color matching function and S(lambda) directly

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \int S(\lambda) \begin{bmatrix} \bar{x}(\lambda) \\ \bar{y}(\lambda) \\ \bar{z}(\lambda) \end{bmatrix} d\lambda$$

⇐ Once we get the $S(\lambda)$, we use CIE XYZ color matching function to get the correspong $[X\ Y\ Z]$ value. then apply the inverse model to get corresponding (R, G, B) value that is need to monitor. i.e.

$$\begin{bmatrix} R^l \\ G^l \\ B^l \end{bmatrix} = T^{-1} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

then apply $R_r = \left(\frac{(R_l - c)}{a}\right)^{1/b} \times 255$

4. (continued - 1)

$$G_r = \left(\frac{G^l - c}{a}\right)^{\frac{1}{b}} \times 255$$

$$B_r = \left(\frac{B^l - c}{a}\right)^{1/b} \times 255$$

thus we can get the output of the monitor is $\begin{bmatrix} R_r \\ G_r \\ B_r \end{bmatrix}$

Since we know when we try to display on the monitor, it well always do gamma uncorrection γ to the input source. So to make the output of monitor to seen as HVS. the input source should do gamma correction, i.e.

$$R_r = 255 \left(\frac{R}{255}\right)^{1/\gamma}$$

thus. $R = \left(\frac{R_r}{255}\right)^{\gamma} \cdot 255$. where $\gamma = 2.2$

do the same thing for G. B component.

b) First we will use Kodak Q60 patches: different grayscale patches to determine its Y value through X-Rite spectrophotometer

Use these value and monitor display these. patches to get R.G.B. fitting the curve

$$R_l = a\left(\frac{R_r}{255}\right)^b + c \qquad \text{where use } R_L = Y.$$

thus we get the nonliearzationship between (R_r, G_r, B_r) and (R_L, G_L, B_L).

Then we use Kodak Q60 color patches: 240 different color patches using X-Rite Spectrophotometer to determine their $[X\ Y\ Z]_{240\times3}$ values and. Use the monitor to display these patches to get $[R_r, G_r, B_r]_{240\times3}$

Then we can use the up equation to get their linearized value $[R_L, G_L, B_L]_{240\times3}$, Then fit into the equation:

$$\underbrace{[R_L, G_L, B_L]_{240\times3}}_{A}\, T_{3\times3} = \underbrace{[X\ Y\ Z]_{240\times3}}_{B}$$

using least-square method to solve $T_{3\times3}$.

$$T = (A^T A)^{-1} A^T B$$

Version 2 by Yin Wang

4. (30) You have a page of paper that has been painted with a uniform color. You use a spectroradiometer to determine the spectral reflectance $R(\lambda)$ of this page. You would like to display a color patch on your monitor that indicates what this patch would look like under an illuminant with spectral power distribution $I(\lambda)$. The inputs to your monitor are a 3-tuple of RGB values for each pixel.

   a. (15) Describe in detail with complete equations (parameter values need not be specified) the mapping from $R(\lambda)$ to the RGB 3-tuple value that you will input to the monitor, such that it will display a color that looks the same to the human viewer as the painted sheet of paper.

   b. (15) Describe in detail how you would determine the values of the unknown parameters in your answer to part (a) above.

Soln:

a) The problem is based on the theorem that if we want display a color from a uniform color space to a monitor, we need to gamma-correct it.

$$R(\lambda) \equiv \frac{I(\lambda)}{S(\lambda)}.$$

Since we have a paper which is painted in uniform color space.

we can use the equation $X = \int I(\lambda)x(\lambda)\,d\lambda$ to find $(X\,Y\,Z)$ values for spectroradiometer

$$Y = \int I(\lambda)y(\lambda)\,d\lambda$$

$$Z = \int I(\lambda)z(\lambda)\,d\lambda.$$

and convert it back to linear RGB color space. by multiply transform matrix $T_{3\times3}$

Then use gamma correction $R = 255\left(\frac{R_L}{255}\right)^{1/\gamma}$, $B = 255\left(\frac{B_L}{255}\right)^{1/\gamma}$, $G = 255\left(\frac{G_L}{255}\right)^{1/\gamma}$ to get the (R, G, B) values so that the monitor will display the color same to the human viewer.

Note: All the values we need to do the calibration could be obtained from spectroradiometer.

b) For the transform matrix T and gamma(γ). they can be determined by using Kodak Q60 target with increasing illuminant bar at the bottom. Set Y (Illuminant) as output. we can determine the gray balance curve for R.G.B channel respectively. From the gray balance curve, we would know the gamma value. Also. From Kodac Q60 colored patch. we could know X.Y Z. values and RGB values for each colored patch. After transform RGB to linear RGB. we would have two matrices

$$A = \begin{bmatrix} R_{L1} & G_{L1} & B_{L1} \\ \vdots & \vdots & \vdots \\ R_{LN} & G_{LN} & B_{LN} \end{bmatrix} \quad B = \begin{bmatrix} X_1 & Y_1 & Z_1 \\ \vdots & \vdots & \vdots \\ X_N & Y_N & Z_N \end{bmatrix} \quad \underline{T = (A^TA)^{+}A^TB}$$

And $I(\lambda)$ could be obtained from spectroradiometer.
XYZ could be measured. from spectroradiometer as well