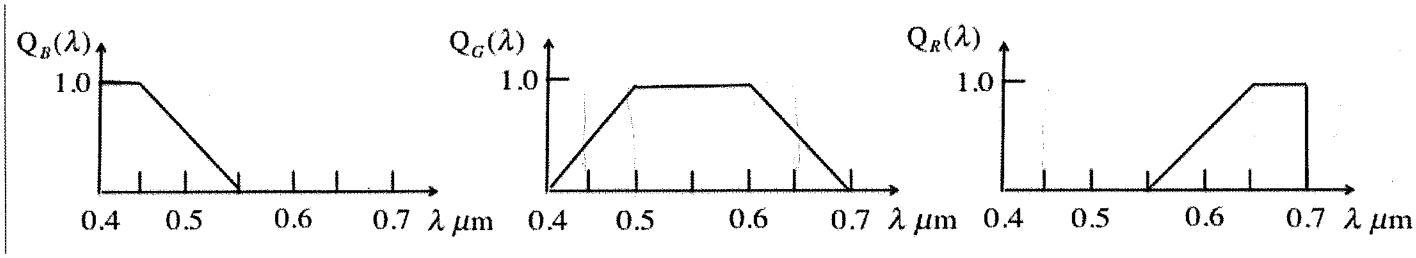
Problems
1 and 2

19 October 2015 **Name:** Chang Liu

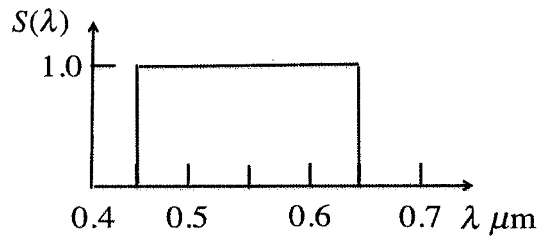
ECE 638 **Exam No. 1** **Fall 2015**

This exam is closed book and closed notes. **No calculators are permitted.** You have 120 minutes to work the following **four** problems that are worth a total of 150 pts. (Please see the last page of the exam for a summary of the point assignment.) To obtain maximum partial credit, be sure to show the complete derivation of your answers.

1. (40 pts.) Consider a three-channel sensor with the response functions $\left[Q_R(\lambda), Q_G(\lambda), Q_B(\lambda)\right]$ shown below.

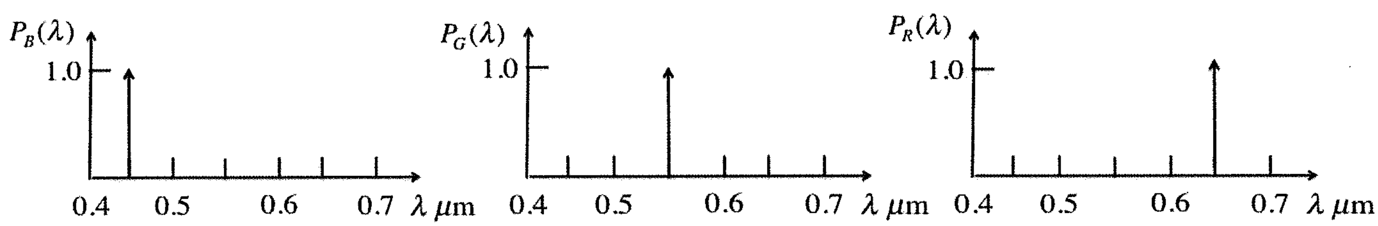


a. (8) Find the response of this sensor to the stimulus $S(\lambda)$ with the spectral power distribution shown below:



b. (8) Carefully sketch the sensor chromaticity diagram for this sensor, using an equilateral triangle where each chromaticity coordinate is given by the distance from one of the vertices along the direction that is normal to the opposite edge.

Consider the primary set $\left[P_R(\lambda), P_G(\lambda), P_B(\lambda)\right]$ with power spectral distribution shown below:



c. (8) Find the amounts of each of the three primaries that will yield a match to the stimulus $S(\lambda)$ from part a), as viewed by the sensor with response functions shown above.

d. (8) Find the color matching functions $\left[\bar{r}(\lambda), \bar{g}(\lambda), \bar{b}(\lambda)\right]$ for this primary set.

e. (8) Use the color matching functions as an alternate solution to finding the amounts of the primaries that will match the stimulus $S(\lambda)$ shown above.

1. (continued - 1)

a) $R_S = \int_{0.4}^{0.7} S(\lambda) Q_R(\lambda)\, d\lambda$

$= \frac{1}{2}(0.65-0.55)\cdot 1 = 0.05 = \frac{1}{20}$

$G_S = \int_{0.4}^{0.7} S(\lambda) Q_G(\lambda)\, d\lambda$

$= \frac{1}{2}(\frac{1}{2}+1)(0.5-0.45)\cdot 2 + (0.6-0.5)\cdot 1$

$= \frac{3}{2}\cdot\frac{1}{20} + \frac{1}{10}$

$= \frac{7}{40}$

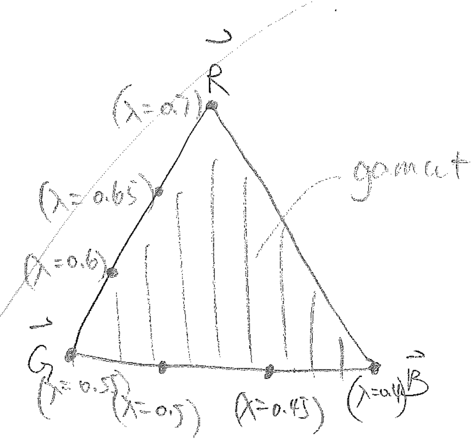
$B_S = \int_{0.4}^{0.7} S(\lambda) Q_B(\lambda)\, d\lambda$

$= \frac{1}{2}(0.55-0.45)\cdot 1 = \frac{1}{20}$

$$C^T = \begin{bmatrix} R_S \\ G_S \\ B_S \end{bmatrix} = \begin{bmatrix} 1/20 \\ 7/40 \\ 1/20 \end{bmatrix}$$

(b)

λ	R	G	B	r	g	b
0.4	0	0	1	0	0	1
0.45	0	1/2	1	0	1/3	2/3
0.5	0	1	1/2	0	2/3	1/3
0.55	0	1	0	0	1	0
0.6	1/2	1	0	1/3	2/3	0
0.65	1	1/2	0	2/3	1/3	0
0.7	1	0	0	1	0	0



1. (continued - 2)

c) $P_R(\lambda) = \delta(\lambda - 0.65)$

$P_G(\lambda) = \delta(\lambda - 0.55)$

$P_B(\lambda) = \delta(\lambda - 0.45)$

$A = [a_{ij}]$ where $a_{ij} = \int P_j(\lambda) Q_i(\lambda) d\lambda$ where $i,j = R, G, B$

$$A = \begin{bmatrix} Q_R(0.65) & Q_R(0.55) & Q_R(0.45) \\ Q_G(0.65) & Q_G(0.55) & Q_G(0.45) \\ Q_B(0.65) & Q_B(0.55) & Q_B(0.45) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 1/2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P = A^{-1} C^T$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ -1/2 & 1 & -1/2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/20 \\ 7/40 \\ 1/20 \end{bmatrix}$$

$$= \begin{bmatrix} 1/20 \\ 5/40 \\ 1/20 \end{bmatrix} = \frac{1}{8}$$

(d) $$\begin{bmatrix} \bar{r}(\lambda) \\ \bar{g}(\lambda) \\ \bar{b}(\lambda) \end{bmatrix} = A^{-1} \cdot Q(\lambda) = \begin{bmatrix} Q_R(\lambda) \\ -\frac{1}{2} Q_R(\lambda) + Q_G(\lambda) - \frac{1}{2} Q_B(\lambda) \\ Q_B(\lambda) \end{bmatrix}$$

e) $$\begin{pmatrix} r \\ g \\ b \end{pmatrix} = \int_{0.4}^{0.7} S(\lambda) \begin{bmatrix} \bar{r}(\lambda) \\ \bar{g}(\lambda) \\ \bar{b}(\lambda) \end{bmatrix} d\lambda = \begin{bmatrix} 1/20 \\ 5/40 \\ 1/20 \end{bmatrix} = \frac{1}{8}$$

2. (28 pts.) Consider a finite dimensional model for a linear, bichromatic (2-channel) vision system. Assume that we sample at $N = 3$ wavelengths. Suppose that the sensor response matrix is given by

$$\mathbf{S} = \begin{bmatrix} 1 & 0 \\ 0.5 & 0.5 \\ 0 & 1 \end{bmatrix}$$

a. (5) Find the response of this sensor to the stimulus $\vec{n} = \begin{bmatrix} 1 & 2 & 1 \end{bmatrix}^T$.

b. (5) Find the fundamental component $\vec{n}^*$ for this stimulus.

c. (5) Find the black-space component $\vec{n}^c$ for the stimulus.

d. (5) Find a metamer $\vec{n}'$ to $\vec{n}$ such that $\vec{n}' \neq \vec{n}$.

e. (8) Find the projection operator $\mathbf{R}$ for this sensor.

a) $p = S^T \vec{n}$

$$= \begin{bmatrix} 1 & 0.5 & 0 \\ 0 & 0.5 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

+5

b) $\vec{n}^* = S(S^TS)^{-1}S^T\vec{n}$

$$S^TS = \begin{bmatrix} 1 & 0.5 & 0 \\ 0 & 0.5 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0.5 & 0.5 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1.25 & 0.25 \\ 0.25 & 1.25 \end{bmatrix} = \begin{bmatrix} 5/4 & 1/4 \\ 1/4 & 5/4 \end{bmatrix}$$

$$(S^TS)^{-1} = \frac{2}{3} \begin{bmatrix} 5/4 & -1/4 \\ -1/4 & 5/4 \end{bmatrix}$$

$$S(S^TS^{-1}) = \frac{2}{3} \begin{bmatrix} 1 & 0 \\ 0.5 & 0.5 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 5/4 & -1/4 \\ -1/4 & 5/4 \end{bmatrix} = \frac{2}{3} \begin{bmatrix} 5/4 & -1/4 \\ 1/2 & 1/2 \\ -1/4 & 5/4 \end{bmatrix}$$

$$S(S^TS^{-1})S^T = \frac{2}{3} \begin{bmatrix} 5/4 & -1/4 \\ 1/2 & 1/2 \\ -1/4 & 5/4 \end{bmatrix} \begin{bmatrix} 1 & 0.5 & 0 \\ 0 & 0.5 & 1 \end{bmatrix}$$

$$= \frac{2}{3} \begin{bmatrix} 5/4 & 1/2 & -1/4 \\ 1/2 & 1/2 & 1/2 \\ -1/4 & 1/2 & 5/4 \end{bmatrix}$$

2. (continued -1)

$$\vec{n}^* = S(S^TS^{-1})S^T\vec{n}$$

$$= \frac{2}{3}\begin{bmatrix} 5/4 & 1/2 & -1/4 \\ 1/2 & 1/2 & 1/2 \\ -1/4 & 1/2 & 5/4 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$$= \frac{2}{3}\begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 4/3 \\ 4/3 \\ 4/3 \end{bmatrix}$$

+5

(c) $\vec{n}^c = \vec{n} - \vec{n}^* = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} - \begin{bmatrix} 4/3 \\ 4/3 \\ 4/3 \end{bmatrix} = \begin{bmatrix} -1/3 \\ 2/3 \\ -1/3 \end{bmatrix}$ +5

(d) $\vec{n}' = \vec{n}^* + \alpha\vec{n}^c$

$$= \begin{bmatrix} 4/3 \\ 4/3 \\ 4/3 \end{bmatrix} + \alpha\begin{bmatrix} -1/3 \\ 2/3 \\ -1/3 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 4-\alpha \\ 4+2\alpha \\ 4-\alpha \end{bmatrix}$$

where $\alpha \in \mathbb{R}$, $\alpha \neq 1$

+5

(e) $R = S(S^TS)^{-1}S^T = \frac{2}{3}\begin{bmatrix} 5/4 & 1/2 & -1/4 \\ 1/2 & 1/2 & 1/2 \\ -1/4 & 1/2 & 5/4 \end{bmatrix}$ +8

26-27 October 2015 Name: Xianglun Mao

Problems 3 and 4

ECE 638 **Makeup Exam No. 1** **Fall 2015**

This exam is closed book and closed notes. **No calculators are permitted.** You have 30 minutes each (total of 60 minutes if you are redoing both problems) to work the following **two** problems. (Please see the last page of the exam for a summary of the point assignment.) To obtain maximum partial credit, be sure to show the complete derivation of your answers.

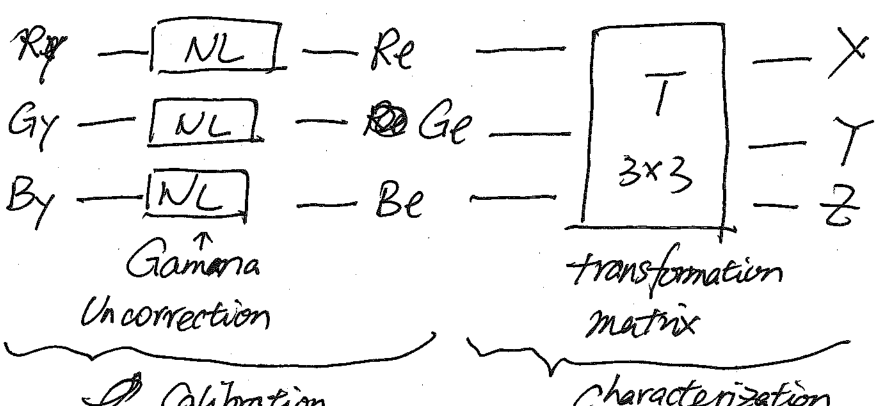
3. (28) You have just purchased the Epson Perfection V730 scanner shown below.

You would like to characterize and calibrate it so that you can determine the CIE *XYZ* coordinates of the content that you scan.

a. (8) Describe in detail how you would model the relationship between the *RGB* output of the scanner and the CIE *XYZ* values of each scanned pixel. Include a block diagram of your model.
b. (5) Describe in detail all items of equipment that you will need to complete this task.
c. (6) Describe in detail each step in your characterization procedure that you will need to carry out in order to determine the parameters of the model described in part (a) of this question.
d. (5) Discuss the factors that may limit the accuracy that you are ultimately able to achieve in determining the CIE *XYZ* coordinates of your scanned content.
e. (4) Explain why you wanted to do this in the first place. Who cares about CIE *XYZ*? Why is it important?

Solution:

a) Here, we suppose to use the regression based model, two steps are needed, calibration and characterization.



JPA comment: I would describe **characterization** as the process of determining both NL_i, i = R, G, B and T for the scanner; and calibration as the process of implementing the model shown here to obtain CIE XYZ for a specific scanner RGB

3. (continued - 1)

Here, R_Y, G_Y, B_Y refer to the device dependent, gamma corrected output value we obtained from the scanner.

R_e, G_e, B_e refer to the linearlized value. 8

X, Y, Z values are the CIE XYZ values.

NL refers to the non-linear relationship between R_Y, G_Y, B_Y and R_e, G_e, B_e.

See comment at bottom of preceding page.

Calibration: Linearlization or Gray balancing. device independent

Characterization: Transformed the linear RGB values to the CIE XYZ color space.

b). Scanner is used to obtain the R_Y, G_Y, B_Y output value.

X-Rite Spectrophotometer will be used to determine the ~~illuminan~~ transformation matrix T. and the gray balancing curves. 5

Target ~~will be use~~ with neutral gray patches with increasing illuminant and different color patches will be used in calibration/Characterization steps (eg. Kodak Q60). using the X-Rite spectrophotometer.

c) In the calibration step, we firstly obtain the Y (illuminant) from the target (Kodak Q60 target), and then try to fit it with R_Y, G_Y, B_Y using the low-power curve-fitting model. $R_e = a \times (\frac{R_Y}{255})^b + c$, $R_e = Y$.

Here, $b \approx 2.2$. 6

After this step, we will obtain the linearlized R_e, G_e, B_e value based on R_Y, G_Y, B_Y.

3. (continued - 2)

with the scanner

In the characterization step, we firstly obtain N (e.g. 240) different color patches from the target, ~~and~~ Then we use $\boxed{\text{X-Rite Spectrophotometer}}$ to determine the CIE XYZ values of those color patches.

Note that this device provides its own illuminant.

We obtain

$$A = \begin{bmatrix} Re_1 & Ge_1 & Be_1 \\ \vdots & \vdots & \vdots \\ Re_N & Ge_N & Be_N \end{bmatrix}, \quad B = \begin{bmatrix} X_1 & Y_1 & Z_1 \\ \vdots & \vdots & \vdots \\ X_N & Y_N & Z_N \end{bmatrix}$$

testing data

Then $T = (A^TA)^{-1}A^TB$

Thus, we can obtain the target XYZ value based on any given Re, Ge, Be.

d). The accuracy of this model directly related to the parameters that we used in this model, for example the estimated transformation T. So that the quality of the target (space variation between color patches) and ~~the~~ different illuminant conditions we used in the scanner and the X-Rite spectrophotometer will all affect the accuracy of this model. 5

e). CIE XYZ ~~values are~~ is the device independent color space, while the RGB values we obtained from any other devices (e.g. scanners) are in the device dependent color space. We can only compare the colors in the device independent color space because ~~the~~ they will not be affected by any other factors. And thus we can explicitly display the colors in the display device. 4

3. (continued - 3)

4. (54 pts.) Consider the 2-D signal

$$f(x,y)=\begin{cases} \cos(\pi x)\cos(\pi y), & 0\leq|x|\leq 1/2 \text{ and } 0\leq|y|\leq 1/2 \\ 0, & \text{else} \end{cases}$$

a. (6) Carefully sketch $f(x,y)$.

b. (6) Find a closed-form expression for the 2-D Continuous-Space Fourier Transform (CSFT) $F(u,v)$ of $f(x,y)$ that does not contain any operators.

c. (6) Carefully sketch a fully dimensioned plot for $F(u,v)$.

Define a new signal

$$g(x,y)=\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} f(x-m,y-n)$$

d. (6) Sketch $g(x,y)$ well enough to show that you know what it looks like.

e. (6) Find a closed-form expression for the 2-D Continuous-Space Fourier Transform (CSFT) $G(u,v)$ of $g(x,y)$ that does not contain any operators.

f. (6) Carefully sketch a fully dimensioned plot for $G(u,v)$.

Define a new signal

$$h(x,y)=\begin{cases} g(x,y), & 0\leq|x|\leq 5 \text{ and } 0\leq|y|\leq 5 \\ 0, & \text{else} \end{cases}$$

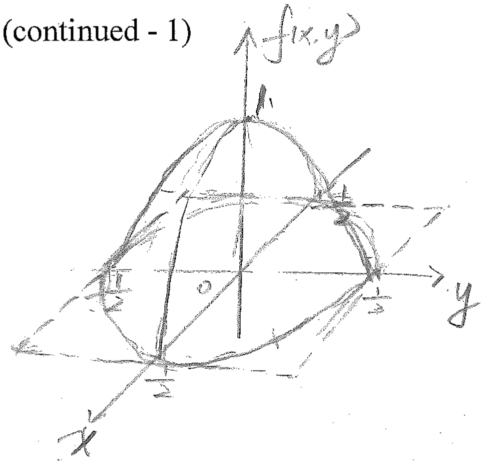
g. (6) Sketch $h(x,y)$ well enough to show that you know what it looks like.

h. (6) Find a closed-form expression for the 2-D Continuous-Space Fourier Transform (CSFT) $H(u,v)$ of $h(x,y)$ that does not contain any operators.

i. (6) Carefully sketch a fully dimensioned plot for $H(u,v)$.

4. (continued - 1)

Solution: a)



b) $F(u,v) = CSFT\{f(x,y)\}$

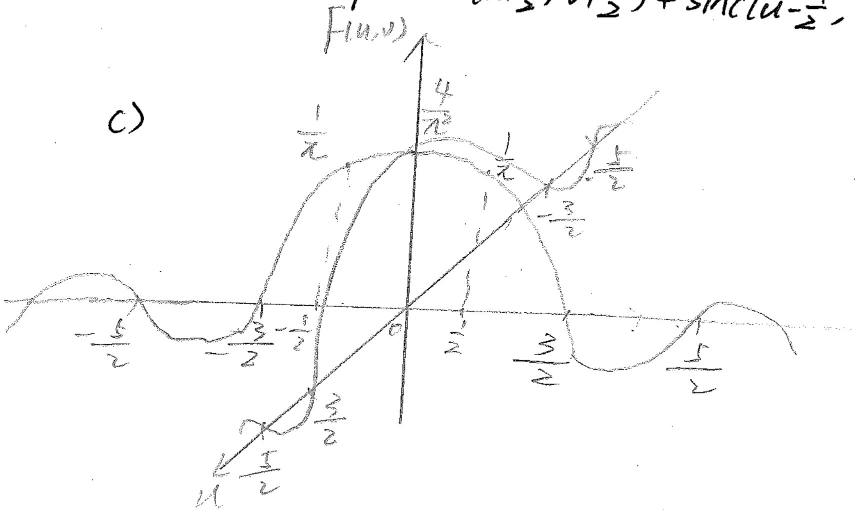
$= CSFT\{\cos(\pi x)\cdot\cos(\pi y)\} * CSFT\{rect(x,y)\}$

$= \{\frac{1}{2}[\delta(u+\frac{1}{2})+\delta(u-\frac{1}{2})]\cdot\frac{1}{2}[\delta(v+\frac{1}{2})+\delta(v-\frac{1}{2})]\} * sinc(u,v)$

$= \frac{1}{4}[\delta(u+\frac{1}{2},v+\frac{1}{2})+\delta(u-\frac{1}{2},v+\frac{1}{2})+\delta(u+\frac{1}{2},v-\frac{1}{2})+\delta(u-\frac{1}{2},v-\frac{1}{2})] * sinc(u,v)$

$= \frac{1}{4}[sinc(u+\frac{1}{2},v+\frac{1}{2})+sinc(u-\frac{1}{2},v+\frac{1}{2})+sinc(u+\frac{1}{2},v-\frac{1}{2})+sinc(u-\frac{1}{2},v-\frac{1}{2})]$

c)

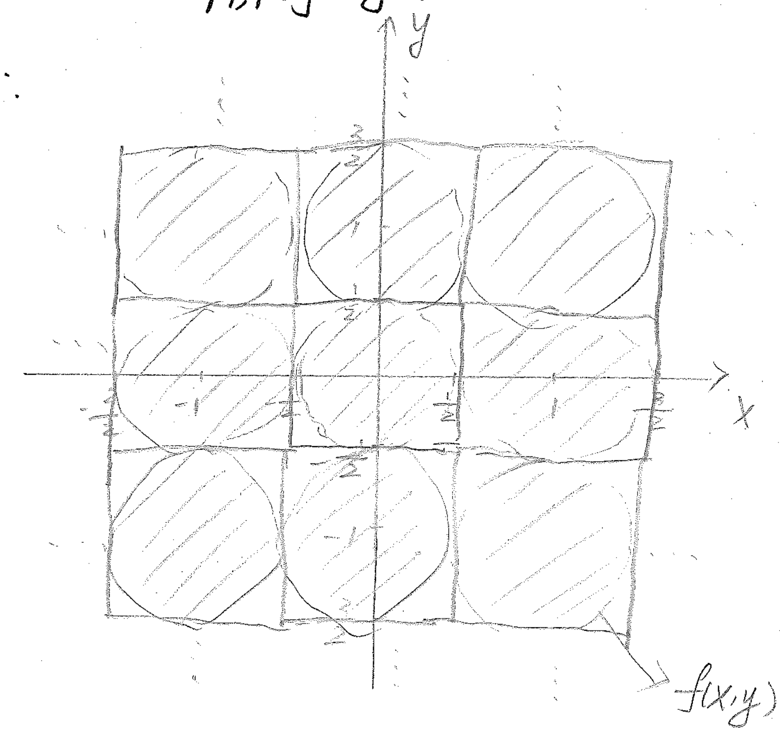


4. (continued - 2)

(d) $g(x,y) = \sum_m \sum_n f(x-m, y-n)$

$= rep_{1,1}[f(x,y)]$

$\therefore$



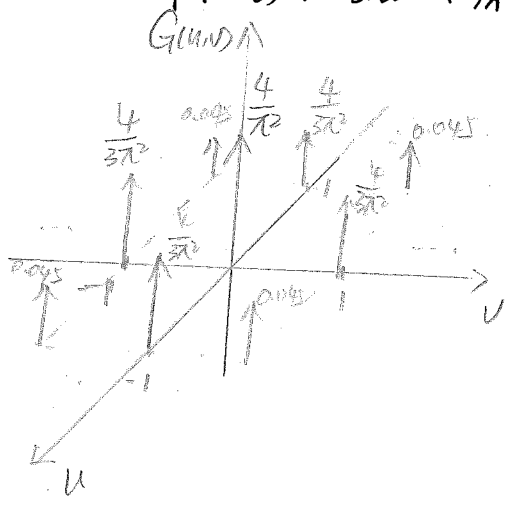
(e) $G(u,v) = CSFT\{g(x,y)\}$

$= comb_{1,1}[F(u,v)]$

$= \sum_k \sum_l F(k,l)\,\delta(u-k, v-l)$

where $F(k,l)$ is shown in (b).

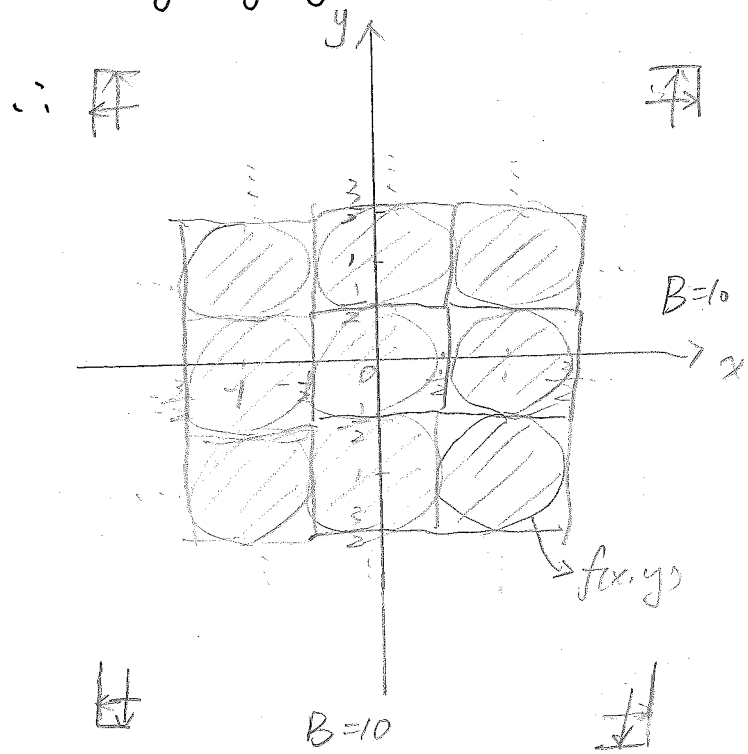
(f)



4. (continued - 3)

(g) $h(x,y) = g(x,y)\, \mathrm{rect}\left(\frac{x}{10}, \frac{y}{10}\right)$

$\therefore$



limited by $0 \le |x| \le 5$, $0 \le |y| \le 5$

(h) $H(u,v) = \mathrm{CSFT}\{h(x,y)\}$

$$= \mathrm{CSFT}\{G(u,v)\} * \mathrm{CSFT}\{\mathrm{rect}(\tfrac{x}{10}, \tfrac{y}{10})\}$$

$$= \left\{\sum_k \sum_l F(k,l)\, \delta(u-k, v-l)\right\} * 100\, \mathrm{sinc}(10u, 10v)$$

$$= 100 \sum_k \sum_l F(k,l) \cdot \mathrm{sinc}(10(u-k), 10(v-l)).$$

where $F(k,l)$ shows in (b).

(i).